High-Resolution Modelling With Bi-Dimensional Shallow Water Equations Based Codes – High-Resolution Topographic Data Use For Flood Hazard Assessment Over Urban And Industrial Environments –

Abily Morgan\textsuperscript{a,*}, Delestre Olivier\textsuperscript{a,b}, Bertrand Nathalie\textsuperscript{c}, Duluc Claire-Marie\textsuperscript{c} & Gourbesville Philippe\textsuperscript{a}

\textsuperscript{a}URE 005 I-CiTy, University of Nice Sophia Antipolis, 930 Route des Colles, 06903 Sophia Antipolis Cedex, France
\textsuperscript{b}Laboratory J.A. Dieudonné UMR 7351, University of Nice Sophia Antipolis
\textsuperscript{c}Institute for Radioprotection and Nuclear Safety (IRSN), PRP-DGE, SCAN, BEHRIG, BP17, 92262 Fontenay-aux-Roses Cedex, France.

Abstract

The availability of new generation of High-Resolution (HR) topographic datasets combined with high performance computing resources opens the door to HR hydraulic simulations for risk assessment. LiDAR and photo-interpreted datasets are promising for HR Digital Elevation Model (DEM) generation, allowing inclusion of fine (infra-metric) aboveground structures influencing overland flow hydrodynamic in urban environment. Nonetheless, if topographic data is one key input for free surface hydraulic modelling using standard 2D Shallow Water Equations (SWEs) based codes, several categories of technical and numerical challenges arise to use HR dataset within numerical modelling. This proceeding explores the new possibilities, advantages and limits of HR topographic data use with 2D SWE based numerical modelling tools for flood hazard assessment and proposes an original method for uncertainty assessment. The concepts of HR topographic data and 2D SWE based numerical modelling are reviewed. Using LiDAR and photo-interpreted datasets, different 2D SWEs based codes (Mike 21, Mike 21 FM, TELEMAC-2D, FullSWOF_2D) and strategies are tested to encompass HR DEM in intense rainfall and river flood events simulations ranging from industrial site scale to a megacity district scale (Nice, France). Tools and methods for assessing uncertainties related to HR DEM use with 2D SWE based codes are developed to perform a spatial global sensitivity analysis related to HR topographic data use. Computed sensitivity indices maps quantify the importance and spatial variability of uncertainties introduced by modeller choices regarding ways HR topographic information are integrated in models, compared to measurement errors. Impact of thin aboveground features inclusion, even at a decreased resolution, appears as a crucial asset in flood risk assessment on urban area, but requires providing caution to decision makers along with models’ results.

Keywords: Urban flood; Saint-Venant; LiDAR; Photo-interpreted dataset; uncertainties.
1. Introduction

High-resolution (infra-metric) topographic datasets, including LiDAR and photo-interpreted classified datasets, are becoming available at large range of spatial extent, such as municipality or industrial site scale [1, 2]. This category of dataset is promising for High-Resolution (HR) Digital Elevation Model (DEM) generation, allowing inclusion of fine above-ground structures which might influence overland flow hydrodynamic in urban environment (Fig. 1). DEMs are one key input data in Hydroinformatics, for practitioner willing to perform free surface hydraulic modelling using standard 2D SWEs based numerical codes (e.g. modeller wishing to assess flood hazard). Models approximating 2D Shallow Water Equations (SWEs) solution using HR description of the urban environment are therefore getting increasingly used in practical engineering applications to understand or to predict surface flow properties during an extreme flood event [3-5]. Indeed, an improvement of the flood phenomena description is expected by better describing the physical properties of the urban environment in the HR DEM encompassing the information of detailed above ground features that influence overland flow paths and hydrodynamic. Nonetheless, several categories of conceptual, technical and numerical challenges arise from this type of data use with standard 2D SWEs numerical codes. Moreover, sources of uncertainties others than those related to the quality of the topography description exists in the modelling approach. Consequently, limits regarding these points worth being recalled and balanced with the rendering aspects of the HR models results.

Using HR topographic datasets provided by the municipality of Nice (France), research teams of IRSN and I-City have carried out for several years different types of HR flood modelling over industrial or urban sites [1, 2, 6]. The aim was to test, from a practical perspective, the feasibility, the added value and the limits of 2D SWE based HR urban flood modelling approaches. Two types of phenomena generating flooding issues were tested for HR modelling: (i) intense runoff and (ii) river flood event [6]. Tests were purposely critics in terms of over-passing the SWEs original framework and the study cases were voluntary highly challenging for standards codes as they introduced: huge number of computational points, rainfall runoff over steep slope, wet/dry transition and flow regime changes occurrences. Three scales of spatial extent are tested, from a small industrial site scale to a city district scale (Nice low Var valley, France). Several numerical modelling tools based on 2D SWEs were used, from commercial (Mike 21, Mike 21 FM from DHI) to open source (TELEMAC 2D, FullSWOF_2D) codes. The aim was not to benchmark the codes, but to extensively compare possibilities and limitations of their use for HR urban flood modelling.

The objective of this paper is to evaluate for practitioners the possibilities and limits of High-Resolution (HR) topographic data use within standard categories of 2D hydraulic numerical modelling tools for flood hazard assessment purpose. Moreover, an illustration of modern practices to assess uncertainty applied to the specific problematic of HR topographic data inclusion is presented and discussed. Section 2 introduces the background of
the theoretical framework and the numerical challenges of SWEs solving, in order to raise questions up regarding validity of the approach in the context of HR modelling over complex environments. Section 3 presents a Global Sensitivity Approach (GSA) applied to specifically study uncertainties related to HR topographic data use and inclusion in 2D SWE based codes. Conclusions are introduced in section 3, limits and caution on uncertainties aspects with 2D SWEs based HR models are presented.

2. Concept of 2DSWE based modelling vs issues of HR topographic data use in urban flood modelling

2.1. Reminder on background of 2D SWE based modelling approach

The SWE system is a set of non-linear, time dependent partial differential equations of hyperbolic type aiming at describing the flow free surface properties. From a conceptual point of view, basics behind the simplified idealistic situation that conducted de Saint-Venant [7, 8] is to switch from local detailed scale to a more macroscopic one (several hundred meters). Then, at such a scale the only forces which are considered are gravity, inertial and resistance forces. Therefore, simplification introduced by de Saint-Venant are that (i) the water surface is the same over one cross section (1D), (ii) it can be considered that flow has one privileged direction and that the flow velocity is the same over one vertical, (iii) hydrostatic pressure hypothesis and (iv) energy losses can be represented using empirical formula (Chézy like formulas). Originally, validity of this simplified framework is for a flow along an inclined channel of constant slope and cross sections. Moreover, what has been conceptualized in the SWEs system is energy losses related to resistance (friction) against channel boundary. It has to be emphasized here that this empirical formulation of energy losses introducing one parameter in the SWEs system has been found to be empirically valid for steady-state flow over experimental channel [7].

From a mathematical point of view the solution of the SWEs can be approximated over a calculation domain of finite length only if the problem is well-posed. Well-posed problem requires that the solution exists, is unique and that the initial condition that is a “function of the solution” over the domain at time \( t=0 \) is known. Moreover one boundary condition has to be specified for each characteristic that enters the domain at the boundaries during the time of calculation [7, 9]. The number of characteristics entering the domain is function of the sign of the eigenvalues of the system, which depends on the flow regime. If the flow is supercritical (upstream control), both eigenvalues are imposed upstream. If the flow is subcritical one is imposed upstream and the other one downstream. Beside for simple cases (e.g. canal or backwater curve influence), in real practical cases with the objective to assess flood event extent in 2D, these conditions are seldom fully achieved, due to incomplete knowledge of these initial and boundary conditions. Transcritical flow occurrences lead to a division of the solution domain in two subdomains separated by a stationary discontinuity. Indeed, transcritical condition leads to sign change in the slowest eigenvalue, leading to a so called shock speed. Hyperbolic properties of SWEs allow discontinuous solutions such as hydraulic jump [10] also called Riemann problem [9].

As a key reminder regarding above mentioned aspects, it is impossible to exactly solve the SWEs, but only in the best case to approximate solution of the system, if the system is well posed, to guaranty from a mathematical point of view condition of existence of the solution. In fact, in practical cases the boundary and initial conditions are not well known. Moreover a HR description of an urban environment will make sharp topography gradient arise in the computation grid (mesh) where overland flow occurs with flow regime changes and frequent wet/dry transitions. This goes beyond the framework for which SWEs hypotheses were conceptualized. This section introduced important basic aspects of 2D SWEs. Moreover, as presented next, from a numerical point of view, not all the numerical methods are equally able to properly handle aspects.

2.2. Numerical challenges and HR flood modelling with standard 2D SWE codes

Objective of numerical approaches used in the SWEs codes is to approximate the solution (when existing) of equations as faithfully as possible by a method where the unknowns are the values of hydraulic variables (water depth and velocities or discharges) at a finite number of points (nodes) of the studied domain, and in a finite number of instances during the considered period of time (spatial and temporal discretization). The feasibility, the performances and the relevance of HR flood modelling have been tested with a selection of different codes approximating the 2D SWEs, based on various spatial discretization strategies (structured and non-structured) and
having different numerical approaches. Table 1 summarizes the main properties of the tested codes. The feasibility of these tools use for the specific purpose of HR flood modelling has been confirmed for three study cases and various 2D SWE based codes [1, 2, 6]. In standard applications, the codes encounter different level of numerical issues regarding treatment of (i) steep slope or high gradient occurrence, (ii) treatment of flow regime changes, (iii) wetting/drying treatment and (iv) permanent regime occurrences. HR modelling enhances the effects of these issues as detailed below.

Table 1. Overview of several standard codes used for HR urban flood modelling where numerical issues are handled differently.

<table>
<thead>
<tr>
<th>Code</th>
<th>Numerical method</th>
<th>Spatial discretization</th>
<th>Computation time (relative)</th>
<th>Flow regime changes</th>
<th>Inclusion of hydraulic structures (e.g. weirs)</th>
<th>Overland flow connection</th>
<th>Wetting / Drying with sewer system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mike 21</td>
<td>Finite differences (ADI)</td>
<td>Structured (straightforward to use); non optimized of computational points</td>
<td>Good</td>
<td>stable; not accurate</td>
<td>Through topography or empirical formulas</td>
<td>Threshold (possible mass creation)</td>
<td>Possible</td>
</tr>
<tr>
<td>Mike 21 FM</td>
<td>Finite volumes (Roe)</td>
<td>Non-structured (not easy to build if complicated topography); optimized number of computational points</td>
<td>reasonable</td>
<td>Stable; not accurate</td>
<td>Through topography/ empirical formulas</td>
<td>Threshold (possible mass creation)</td>
<td>Possible</td>
</tr>
<tr>
<td>TELEMAC 2D</td>
<td>Finite elements (SUPG)</td>
<td>Non-structured (not easy to build if complicated topography); optimized number of computational points</td>
<td>reasonable</td>
<td>Stable</td>
<td>Through topography/ through empirical formulas (not through GUI here)</td>
<td>Threshold (possible mass creation)</td>
<td>Not available</td>
</tr>
<tr>
<td>FullSWOF</td>
<td>Finite volumes (well balanced with hydrostatic reconstruction)</td>
<td>Non-structured (not easy to build if complicated topography); optimized number of computational points</td>
<td>Important</td>
<td>Stable</td>
<td>Through topography</td>
<td>Not ok</td>
<td>Not available</td>
</tr>
</tbody>
</table>

The HR description of an industrial or urban environment will make sharp topography gradient arise in the computation grid where overland flow occurs and has to be computed. This makes the validity of the resolution of the momentum equation questionable in these specific cases. Reduction of the spatial discretization might reduce these effects [11]. However, due to fine features inclusions, these steep gradients occur anyway.

Transcritical flow, wet/dry transitions and steady states are difficult to handle for numerical approaches. Explanations and methods adaptations to these numerical challenges are explained here. The hyperbolic property of the SWEs allowing mathematical existence of discontinuous solution (e.g. hydraulic jump) is not handled equally by the numerical schemes. Thus, flow regime changes treatments which are likely to occur in HR flood modelling
application might not be treated properly. As a reminder, both eigenvalues propagate downstream in case of a supercritical flow, whereas one eigenvalue propagates downstream and the other one upstream, in case of subcritical flow regime. Therefore, for a numerical flux, the information has to be considered depending on where it is coming from (upwinding). For the boundary conditions of the system, scheme imposes one of the conservative variables following the inflow characteristic (generally a discharge upstream and a water level downstream) and the other variable is calculated thanks to the other characteristic coming from the inside of the domain. Many Riemann solvers or numerical fluxes exist like the Godunov solver, which requires heavy workload for its implementation or approximating Riemann solvers like Rusanov, Roe, HLL, etc. These approximating solvers upwind the fluxes depending on where the information comes from. Moreover, most of these solvers check the Rankine-Hugoniot relation and are therefore able to treat discontinuities in the solution. For transcritical flows, when Fr=1, (when a subcritical flow becomes supercritical through a critical point) one of the eigenvalue is null and a stationary wave occurs. Some of the Riemann solvers can provide a solution (non-entropic solution) with a non-physical discontinuity. Roe solvers have this default; methods exist to correct this default.

Wet/dry transitions lead to the situation where at one side of the interface the water depth is positive and on the other water depth is null. It is well known that with a centered finite differences scheme the positivity of the solution cannot be guaranteed. This might occur as well with the finite volume numerical fluxes. [12] proved that HLL and Rusanov solvers are positivity preserving. Another commonly used treatment is to fix a low threshold value to fill up the dry cell to allow the computation to ensure the positivity at wet/dry transition (e.g. this solution is used with Mike 21, Mike 21 FM codes). Drawback of such treatment is the possible mass creation in the system.

Stationary/permanent regime states lead to numerical difficulties for the numerical fluxes computation. For instance if a hydrostatic equilibrium is reached, there is an equilibrium between the flux of the pressure term and the sources term that include the topography ($z$). This represents an issue for preserving steady states at rest that can create spurious oscillation [13], due to the upwind treatment of the hydrostatic fluxes term that is not applied to the topography fluxes term that is still centered. Solution of a so called well-balanced method [14] is to upwind the computation of the topography fluxes the same way as hydrostatic pressure fluxes are upwinded. Nevertheless, this will affect the positivity preservation property of the scheme and a technique has to be implemented to ensure positivity preservation of the scheme. Hydrostatic reconstruction can solve this issue [15]. Codes such as TELEMAC-2D or FullSWOF_2D method are based on well-balanced scheme properties including a rewriting of the SWEs using a hydrostatic reconstruction leading to an oscillation free and permanently positivity solution [15, 12].

As a remark, it has to be emphasized that, when properly optimized and parameterized in the sense of treatment of the above mentioned numerical difficulties (which involve different degrees of efforts depending on code properties), most of the codes provide comparable results for flood water level estimations. However, sources of uncertainty, other than the one resulting from the numerical aspects, can significantly impact the results variability (e.g. input parameters such at the topography) and deserve to be studied applying suited methodologies.

3. HR topographic dataset use and associated uncertainties

LiDAR or photogrammetry technologies settled on an aerial vector are well suited to gathered HR topographic datasets. Qualitative differences between LiDAR and Photogrammetric based HR datasets rely in the interpretation/classification possibilities that are more important in photogrammetry. Photo-interpreted dataset offers a broader range of possibilities for HR DEM design, in accordance with descriptions of the above-ground structure that will influence overland flow. Indeed classification of above-ground features being more extensive in photo-interpreted datasets, it will allow hydraulic modeller to design its HR DEM having a control on which elevation information should be included in it. This is especially relevant for complex environment such as urban and industrial sites, where an important diversity of above-ground elements exists. Optimal use of HR DEM in standard 2D numerical modelling tools appeared challenging in terms of feasibility of data integration within modelling tools [1, 6]. With HR topographic datasets, spatial discretization, often leads to operational choices from the modeller to reach an optimal balance between dataset ease of use, accuracy and time consumption aspects. Impact of errors in HR topographic dataset and modeller choices in HR topographic data integration effects on flood modelling results are used as a framework to analyze uncertainty through the application of a Global Sensitivity Analysis [16].

Global Sensitivity Analysis (GSA) approaches are based on variance decomposition procedure [17]. GSA has been used in various fields such as for 1D free surface hydraulic modelling [18, 19]. These approaches allow to
explore the space of uncertain input parameters and are suited for models having non linear effects [20]. Moreover, GSA can cope with models having spatially varying inputs [16, 21]. Limitations to GSA application are related to: (i) the definition of the distribution that characterizes the input parameter uncertainties which will impact both the output uncertainty and the sensitivity of the output to the uncertain input parameters [17, 21]; (ii) the fact that this type of approach, is affected by the so-called curse of dimensionality which makes its application highly computational resources demanding.

A GSA has been implemented in [16], following standard steps used for such type of approach [20]. A simplified way to see this method can be presented as follow: defining the problem notably by choosing uncertain parameters and output of interest (step A); assessing probability density function of uncertain parameters (step B); propagating uncertainty, here using a random sampling approach (step C); ranking contribution to the output variance of the main effect of each input parameters (step D). First steps of the approach (A and B) are the most subjective. For our study purpose, steps A and B are treated as follow.

For uncertainty related to topographic data error, a spatially uniform parameter is considered. This parameter (var. E) is an error randomly introduced for every point of the highest resolution DEM (1m) following a normal distribution N (0; 0.2).

For uncertainties related to modeller choices when including HR data in hydraulic code, two variables are considered. First one (Var. S) is a categorical ordinal parameter having values representing the level of detail of flow direction impacting above ground feature includes in DSM. S1 is a DTM (Digital Terrain Model) only, S2 is S1 plus buildings, S3 is S2 plus walls and S4 is S3 plus concrete street structures (sidewalks, roar-curbs, etc.). Last parameter (Var. R) represents choices made by modeller for the resolution is discretized in the model. In FullSWOF_2D, the grid cells are regular. This parameter, Var. R, can have five discrete values from 1 to 5m. At 1m resolution, numbers of computational points of the grid is above 17.5 million and at the 5m resolution grid size is 700,000 computational points.

A total of 2,000 DEMs were generated and used in the implementation of the GSA. A coupling between a parametric environment (Prométhée) and a 2D free surface modelling code (FullSWOF_2D) has been completed over a High Performance-Computing structure. The uncertainty propagation (step C) is carried out using a Monte-Carlo approach for random sampling in the DEM database. Sobol indices (step D) are used for the variance based ranking of input parameters over output variance. Sobol indices are defined as follow:

$$S_i(\Xi) = \frac{\text{Var}[E(Y|\Xi_i)]}{\text{Var}(Y)},$$

where $S_i$ is the Sobol index of parameter $i$, $E$ is the Esperance, $Y$ is the output. First-order Sobol indices indicate the contribution to the output variance of the main effect of each input parameters [20]. 1,500 simulations were computed using a total of 400,000 CPU hours. Results consist of a database of maps of the maximal water depth calculated at each point of the domain (Fig. 2a). Results were checked locally over 20 points of interest. Illustration of local results at one point of interest (point 8) is presented (Fig. 2b). Convergence is checked using ratio of the standard deviation over the mean as criterion. Convergence is observed for all points when size of the random sampling got higher than 900-1,000. This is observable with all the points. Distribution of maximal water depth is analyzed. Point 8 variance is 0.71 which is the maximum variance value for all of the 20 points (average variance of the maximal water depth is 0.51). Depending on points of interest, the output distribution is either normal or can be bi-modal. When the distributions got bi-modal, the analyze has shown that var. S1, which is the use of a DTM (no inclusion of above ground elements), is mostly responsible for this mixed distribution. Convergence of 1st order Sobol indices is illustrated for point 8. Over the 20 points of interest, var. S and var. R (modeller choices for HR data inclusion) are always the parameters contributing the most to the output variance.

The Sobol index maps were calculated at a 5m spatial resolution (Fig. 2c). Results confirm the local sensitivity analysis in the sense that var. R and var. S are parameters having the highest Sobol index value. The main conclusions of the spatial repartition between var. S and var. R are: for var. R is the most important parameter over sloppy areas, when the maximal water depth is relatively low and when the flood has not cross densely urbanized areas; for var. S was predominant in output variance contribution when densely urbanized areas are flooded.

The uncertainty analysis lead to: output variability quantification, nonlinear behavior of the model observation and enhancement of the spatial heterogeneity of the output variance. Result stresses out the point that even though other input hydraulic parameters were supposed to be fully known (set-up as constant) in the simulations, the uncertainty related to HR topographic data use plays a major role in results quality and deserved to be assessed and understood. The spatial distribution of $S_i$ illustrates the major influence of the modeller choices, when using the HR
topographic data in 2D hydraulic models (var. S and var. R) with respect to the influence of HR dataset accuracy (var. E). Spatial variation of the Si ranking was clearly observable. Moreover, it is possible to link the spatial distribution of the Si to the properties of the model, especially with the physical properties of represented urban sector topography.

![Diagram](image)

**Fig. 2.** Local uncertainty analysis and GSA at local and spatial levels over one fourth of the computational domain.

### 4. Conclusive aspects

The background of the theoretical framework of SWEs was summarized in order to raise questions up regarding validity of the approach of HR 2D SWEs based modelling over complex environments. As the framework of this type of application is different from the one for which SWEs have originally been designed for, the expected limits that might be encountered for HR topographic data use in standards codes were enhanced. Quantification of uncertainty through GSA goes in the direction of improvement of state of the art, compared to quantification of uncertainty based on expert opinion only. GSA, by ranking uncertain parameters allows practical approaches to better investigate on the uncertainty to better understand mechanisms leading to models output variability. Indeed, even if GSA results can vary from one approach to another at least, it helps modeller to have a better understanding of its model limitation, and provide effective strategies to improve model. Drawbacks are related to computational cost and remaining subjectivity of the approach. In a global perspective, research is active to reduce the computational burden of GSA approaches. For instance, more parsimonious sampling strategies should be tested. Lastly it has to be enhanced that HR modelling rely on the use of datasets representing a reality (above ground feature elevation information) that highly evolve with time. Therefore, survey and update of dataset to cope with the pretended high resolution of models results is necessary, not forgetting to mention the fact that other sources on uncertainties in the modelling approach are numerous (e.g. conceptual, numerical, input data, etc.) and should be explained to decision makers when HR model results are provided.
Acknowledgment

Photogrammetric and photo-interpreted dataset used for this study have been kindly provided by Nice Côte d’Azur Metropolis for research purpose. DHI kindly provided license for their codes. This work was granted access to the HPC and visualization resources of the “Centre de Calcul Interactif” hosted by University Nice Sophia Antipolis.

References