The generalized second law in irreversible thermodynamics for the interacting dark energy in a non-flat FRW universe enclosed by the apparent horizon

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ABSTRACT

We investigate the validity of the generalized second law in irreversible thermodynamics in a non-flat FRW universe containing the interacting dark energy with cold dark matter. The boundary of the universe is assumed to be enclosed by the dynamical apparent horizon. We show that for the present time, the generalized second law in nonequilibrium thermodynamics is satisfied for the special range of the energy transfer constants.

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1. Introduction

Type Ia supernovae observational data suggest that the universe is dominated by two dark components: dark matter and dark energy [1]. Dark matter (DM), a matter without pressure, is mainly used to explain galactic curves and large-scale structure formation, while dark energy (DE), an exotic energy with negative pressure, is used to explain the present cosmic accelerating expansion. However, the nature of DE is still unknown, and people have proposed some candidates to describe it. The cosmological constant, Λ, is the most obvious theoretical candidate of DE, which has the equation of state $\omega = -1$. Astronomical observations indicate that the cosmological constant is many orders of magnitude smaller than estimated in modern theories of elementary particles [2]. Also the "fine-tuning" and the "cosmic coincidence" problems are the two well-known difficulties of the cosmological constant problems [3].

There are different alternative theories for the dynamical DE scenario which have been proposed by people to interpret the accelerating universe. i) The scalar-field models of DE including quintessence [4], phantom (ghost) field [5], K-essence [6] based on earlier work of K-inflation [7], tachyon field [8], dilatonic ghost condensate [9], quintom [10], and so forth. ii) The DE models including Chaplygin gas [11], braneworld models [12], holographic DE models [13], and agegraphic DE models [14], etc.

Besides, as usually believed, an early inflation era leads to a flat universe. This is not a necessary consequence if the number of e-foldings is not very large [15]. It is still possible that there is

a contribution to the Friedmann equation from the spatial curvature when studying the late universe, though much smaller than other energy components according to observations. Therefore, it is not just of academic interest to study a universe with a spatial curvature marginally allowed by the inflation model as well as observations. Some experimental data have implied that our universe is not a perfectly flat universe and that it possesses a small positive curvature [16].

In the semiclassical quantum description of black hole physics, it was found that black holes emit Hawking radiation with a temperature proportional to their surface gravity at the event horizon and they have an entropy which is one quarter of the area of the event horizon in Planck unit [17]. The temperature, entropy and mass of black holes satisfy the first law of thermodynamics [18]. On the other hand, it was shown that the Einstein equation can be derived from the first law of thermodynamics by assuming the proportionality of entropy and the horizon area [19]. The study on the relation between the Einstein equation and the first law of thermodynamics has been generalized to the cosmological context where it was shown that the first law of thermodynamics on the apparent horizon R_h can be derived from the Friedmann equation and vice versa if we take the Hawking temperature $T_h = 1/2\pi R_h$ and the entropy $S_h = \pi R_h^2$ on the apparent horizon [20]. Furthermore, the equivalence between the first law of thermodynamics and Friedmann equation was also found for gravity with Gauss–Bonnet term and the Lovelock gravity theory [20,21].

Besides the first law of thermodynamics, a lot of attention has been paid to the generalized second law (GSL) of thermodynamics in the accelerating universe driven by dark energy. The generalized second law of thermodynamics is as important as the first law, governing the development of the nature [22–29]. For the different DE models like the generalized Chaplygin gas [27], the holographic

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DE [28,30], the braneworld scenarios [31], and the new agegraphic DE [32], people showed that the GSL for the universe containing the DE and DM enclosed by the dynamical apparent horizon is always satisfied throughout the history of the universe for any spatial curvature.

Note that in [28], the authors investigated the validity of the first and the generalized second law of thermodynamics for both apparent and event horizon for the case of holographic DE with DM in a flat universe. They showed that in contrast to the case of the apparent horizon, both the first and second law of thermodynamics break down if one consider the universe to be enveloped by the event horizon with the usual definitions of entropy and temperature. They argued that the break down of the first law can be attributed to the possibility that the first law may only apply to variations between nearby states of local thermodynamic equilibrium, while the event horizon reflects the global spacetime properties. Author of [33] cleared that even by redefining the event horizon measured on the sphere of the horizon as the system's IR cut-off for a holographic DE model in a non-flat universe, the GSL cannot be satisfied.

One of another interesting issue in the cosmological context, is the study of interaction between DE and DM. The choice of the interaction between both components was to get a scaling solution to the coincidence problem such that the universe approaches a stationary stage in which the ratio of DE and DM becomes a constant [34]. Das et al. [35] computed leading-order corrections to the entropy of any thermodynamic system due to small statistical fluctuations around equilibrium. They obtained a general logarithmic correction to black hole entropy. Wang et al. [36] studied a thermodynamical description of the interaction between holographic DE and DM. Resorting to the logarithmic correction to the equilibrium entropy [35] they arrived to an expression for the interaction term which was consistent with the observational tests. Pavón and Wang [37] considered a system composed of two subsystems (DM and DE) at different temperatures. In virtue of the extensive property, the entropy of the whole system is the sum of the entropies of the individual subsystems which (being equilibrium entropies) are just functions of the energies of DE and DM even during the energy transfer process. Zhou et al. [38] have further employed the second law of thermodynamics to study the coupling between the DE and DM in the universe by resorting to the nonequilibrium entropy of extended irreversible thermodynamics.

Here our aim is to extend the work of Zhou et al. [38] and investigate the validity of the generalized second law in irreversible thermodynamics for the interacting DE with DM in a non-flat FRW universe enclosed by the dynamical apparent horizon. The apparent horizon is more appropriate than the event horizon on defining the thermodynamics in the universe [27]. This Letter is organized as follows. In Section 2, we study the DE model in a non-flat universe which is in interaction with the cold DM. In Section 3, we investigate the validity of the generalized second law in irreversible thermodynamics for the universe enclosed by the apparent horizon. Section 4 is devoted to conclusions.

2. Interacting DE and DM

We consider the Friedmann–Robertson–Walker (FRW) metric for the non-flat universe as

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right), \tag{1}$$

where $k = 0, 1, -1$ represent a flat, closed and open FRW universe, respectively. Observational evidences support the existence of a closed universe with a small positive curvature ($\Omega_k \sim 0.02$)

[16]. The first Friedmann equation for the non-flat FRW universe takes the form

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3}(\rho_x + \rho_m), \tag{2}$$

where we take $G = 1$. Also ρ_x and ρ_m are the energy density of DE and DM, respectively. Let us define, as usual, the fractional energy densities as

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{8\pi\rho_m}{3H^2}, \qquad \Omega_x = \frac{\rho_x}{\rho_{cr}} = \frac{8\pi\rho_x}{3H^2},$$

$$\Omega_k = \frac{k}{a^2 H^2}, \tag{3}$$

then, the first Friedmann equation can be written as

$$\Omega_m + \Omega_x = 1 + \Omega_k. \tag{4}$$

We consider a universe containing an interacting DE density ρ_x and the cold DM (CDM), with $\omega_m = 0$. The energy equations for DE and CDM are

$$\dot{\rho}_x + 3H(1 + \omega_x)\rho_x = -\Gamma, \tag{5}$$

$$\dot{\rho}_m + 3H\rho_m = \Gamma, \tag{6}$$

where following [38], we choose $\Gamma = 3H\lambda\rho_x$ as an interaction term with λ a small, dimensionless, positive quantity. For $\Gamma > 0$ the energy proceeds from DE to DM. Although this expression for the interaction term may look purely phenomenological but different Lagrangians have been proposed in support of it [39]. Note that choosing the H in the Γ-term is motivated purely by mathematical simplicity. Because from the continuity equations, the interaction term should be proportional to a quantity with units of inverse of time. For the latter the obvious choice is the Hubble factor H. The dynamics of interacting DE models with different Γ-classes have been studied in ample detail by [40]. It should be emphasized that this phenomenological description has proven viable when contrasted with observations, i.e., SNIa, CMB, large-scale structure, $H(z)$, and age constraints [41], and recently in galaxy clusters [42].

The deceleration parameter is given by

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right). \tag{7}$$

Taking the time derivative in both sides of Eq. (2), and using Eqs. (3), (4), (5) and (6), we get

$$q = \frac{1}{2}(1 + \Omega_k + 3\Omega_x\omega_x). \tag{8}$$

3. Generalized second law in irreversible thermodynamics

Here, we study the validity of the generalized second law (GSL) in irreversible (nonequilibrium) thermodynamics. According to the GSL in equilibrium thermodynamics, entropy of matter and fluids inside the horizon plus the entropy of the horizon do not decrease with time [28]. In equilibrium thermodynamics, irreversible fluxes such as energy transfers play no part and they do not enter the entropy function which is defined for equilibrium states only. However, in nonequilibrium extended thermodynamics such fluxes enter the entropy function [38].

The entropy of the DE and CDM which are in interaction with each other are given by Gibb's equation [25,26,28]

$$T_x dS_x = dQ_x = dE_x + P_x dV, \tag{9}$$

$$T_m dS_m = dQ_m = dE_m, \tag{10}$$

where like [28], $V = 4\pi R_h^3/3$ is the volume containing the DE and CDM with the radius of the horizon R_h. And

$$E_x = \rho_x V, \qquad E_m = \rho_m V, \tag{11}$$

$$P_x = \omega_x \rho_x = \frac{3H^2}{8\pi} \omega_x \Omega_x. \tag{12}$$

Also T_x and T_m are the temperatures of DE and CDM, respectively, and following [38] are given by

$$T_x = T_{eq} e^{-3(\omega_x + \lambda)(x - x_{eq})}, \tag{13}$$

$$T_m = T_{eq} \frac{r}{r_{eq}} e^{-[2+3(\omega_x + \lambda)](x - x_{eq})}, \tag{14}$$

where $r = \Omega_m / \Omega_x$ and $x = \ln a$. The subscript 'eq' indicates the value taken by the corresponding quantity when DE and DM are in thermal equilibrium. Note that in Eqs. (13) and (14), λ and ω_x are constants. We limit ourselves to the assumption that the thermal system including the DE and CDM bounded by the horizon in the absence of interaction remain in equilibrium so that the temperature of the system must be uniform and the same as the temperature of its boundary. This requires that the temperature T of the both DE and CDM inside the horizon in the absence of interaction should be in equilibrium with the Hawking temperature T_h associated with the horizon, so we have $T_{eq} = T_h = 1/(2\pi R_h)$. In the presence of interaction, when the temperature of the system differs from that of the horizon, there will be spontaneous heat flow between the horizon and the fluid and the thermal equilibrium will no longer hold [25,26].

Taking the derivative in both sides of (9) and (10) with respect to cosmic time t, and using Eqs. (2), (3), (4), (5), (6), (11) and (12), we obtain

$$\dot{Q}_x = 4\pi R_h^2 (\dot{R}_h - HR_h)(1 + \omega_x)\rho_x - 4\pi R_h^3 H\lambda\rho_x, \tag{15}$$

$$\dot{Q}_m = 4\pi R_h^2 (\dot{R}_h - HR_h)\rho_m + 4\pi R_h^3 H\lambda\rho_x. \tag{16}$$

Also in addition to the entropies of DE and CDM in the universe, there is a geometric entropy on the horizon $S_h = \pi R_h^2$ [28]. The evolution of this horizon entropy is obtained as

$$\dot{S}_h = 2\pi R_h \dot{R}_h. \tag{17}$$

In the presence of interaction between DE and CDM inside the universe enveloped by the horizon, the GSL in irreversible thermodynamics can be obtained by extending Eq. (13) in [38] as

$$\dot{S}^* = \frac{\dot{Q}_m}{T_m} + \frac{\dot{Q}_x}{T_x} - A_x \dot{Q}_x \ddot{Q}_x - A_h \dot{Q}_h \ddot{Q}_h + \dot{S}_h, \tag{18}$$

where A_x and A_h are the energy transfer constants between DE and DM inside the universe and between the universe and the horizon, respectively. Since the overall system containing the universe and the horizon is isolated, one has $\dot{Q}_h = -(\dot{Q}_m + \dot{Q}_x)$. For an isolated universe only, i.e. $\dot{Q}_h = 0$ and $\dot{S}_h = 0$, then Eq. (18) reduces to Eq. (13) in [38].

Here we assume the boundary of the universe to be enveloped by the dynamical apparent horizon. Hence, define $\tilde{r} = ar$, the metric (1) can be rewritten as $ds^2 = h_{ab} dx^a dx^b + \tilde{r}^2 d\Omega^2$, where $x^a = (t, r)$, $h_{ab} = \text{diag}(-1, a^2/(1 - kr^2))$. By definition, $h^{ab}\partial_a\tilde{r}\partial_b\tilde{r} = 0$, the location of the apparent horizon in the FRW universe is obtained as $\tilde{r} = R_h = (H^2 + k/a^2)^{-1/2}$ [43]. For $k = 0$, the apparent horizon is same as the Hubble horizon.

Cai and Kim [20] proofed that the Friedmann equations in Einstein gravity are derived by applying the first law of thermodynamics to the dynamical apparent horizon, R_h, of a FRW universe with any spatial curvature in arbitrary dimensions and assuming that the apparent horizon has an associated entropy S_h and Hawking temperature T_h as $S_h = \pi R_h^2$, $T_h = 1/2\pi R_h$. In the braneworld scenarios, the Friedmann equations also can be written directly in the form of the first law of thermodynamics, at the apparent

horizon with the Hawking temperature on the brane, regardless of whether there is the intrinsic curvature term on the brane or a Gauss–Bonnet term in the bulk [31]. Recently the Hawking radiation with temperature $T_h = 1/2\pi R_h$ on the apparent horizon of a FRW universe with any spatial curvature has been observed in [43]. The Hawking temperature is measured by an observer with the Kodoma vector inside the apparent horizon [43].

For the dynamical apparent horizon

$$R_h = H^{-1}(1 + \Omega_k)^{-1/2}, \tag{19}$$

if we take the derivative in both sides of (19) with respect to cosmic time t, then we obtain

$$\dot{R}_h = \frac{3(1 + \Omega_k + \Omega_x\omega_x)}{2(1 + \Omega_k)^{3/2}}. \tag{20}$$

Using Eqs. (19) and (20) one can get

$$\dot{R}_h - HR_h = \frac{(1 + \Omega_k + 3\Omega_x\omega_x)}{2(1 + \Omega_k)^{3/2}}. \tag{21}$$

Substituting Eqs. (19), (20) and (21) in Eqs. (15), (16) and (17) reduce to

$$\dot{Q}_x = \frac{3}{2(1 + \Omega_k)^{5/2}}\big(q(1 + \omega_x) - \lambda(1 + \Omega_k)\big)\Omega_x, \tag{22}$$

$$\dot{Q}_m = \frac{3}{2(1 + \Omega_k)^{5/2}}\big(q\Omega_m + \lambda\Omega_x(1 + \Omega_k)\big), \tag{23}$$

$$\dot{S}_h = \frac{3\pi}{H(1 + \Omega_k)^2}(1 + \Omega_k + \Omega_x\omega_x). \tag{24}$$

Taking the time derivative of Eqs. (22), (23), and using

$$\dot{\Omega}_k = 2Hq\Omega_k, \tag{25}$$

$$\dot{\Omega}_x = H\big[2(q + 1) - 3(1 + \omega_x + \lambda)\big]\Omega_x, \tag{26}$$

we get

$$\ddot{Q}_x = \frac{3H}{2(1 + \Omega_k)^{5/2}}\Bigg\{\Bigg[-2(q + 1) + \frac{3q\Omega_k}{(1 + \Omega_k)} + 3(1 + \omega_x + \lambda)\Bigg]\lambda(1 + \Omega_k) + \Bigg[2q(q + 1) - \frac{5q^2\Omega_k}{(1 + \Omega_k)} + \frac{\dot{q}}{H} - 3q(1 + \omega_x + \lambda)\Bigg](1 + \omega_x)\Bigg\}\Omega_x, \tag{27}$$

$$\ddot{Q}_m = \frac{3H}{2(1 + \Omega_k)^{5/2}}\Bigg\{\Bigg[2(q + 1) + \frac{3q(1 - \Omega_k)}{(1 + \Omega_k)} - 3(1 + \omega_x + \lambda)\Bigg]\lambda(1 + \Omega_k)\Omega_x + \Bigg[2q(q + 1) - \frac{5q^2\Omega_k}{(1 + \Omega_k)} + \frac{\dot{q}}{H} - 3q\Bigg]\Omega_m\Bigg\}, \tag{28}$$

where

$$\dot{q} = Hq\Omega_k + \frac{3}{2}H\Omega_x\omega_x\big[2(q + 1) - 3(1 + \omega_x + \lambda)\big]. \tag{29}$$

Using Eqs. (13), (14), (22), (23), (27) and (28), we obtain

$$\dot{Q}_x\ddot{Q}_x = \frac{9H\Omega_x^2}{4(1 + \Omega_k)^5}\Bigg\{\Bigg[2(q + 1) - \frac{3q\Omega_k}{(1 + \Omega_k)} - 3(1 + \omega_x + \lambda)\Bigg]\big(q(1 + \omega_x) - \lambda(1 + \Omega_k)\big)^2$$

$$+ \left[\frac{\dot{q}}{H} - \frac{2q^2 \Omega_k}{(1+\Omega_k)} \right] (1+\omega_x)\big(q(1+\omega_x)$$
$$- \lambda(1+\Omega_k) \big) \bigg\}, \tag{30}$$

$$\dot{Q}_h \ddot{Q}_h = \frac{9Hq}{4(1+\Omega_k)^5} \left\{ \left[\frac{(2-3\Omega_k)}{(1+\Omega_k)} q^2 + \frac{\dot{q}}{H} - q \right] \right.$$
$$\times (1+\Omega_k+\Omega_x\omega_x)^2$$
$$\left. - 3q\Omega_x\omega_x(1+\omega_x+\lambda)(1+\Omega_k+\Omega_x\omega_x) \right\}, \tag{31}$$

$$\frac{\dot{Q}_m}{T_m} + \frac{\dot{Q}_x}{T_x} = \frac{3}{2T_x(1+\Omega_k)^{5/2}} \left\{ \frac{r_{eq}}{r} e^{2(x-x_{eq})} \big[\lambda(1+\Omega_k)\Omega_x \right.$$
$$+ q(1+\Omega_k-\Omega_x) \big]$$
$$\left. + \big[q(1+\omega_x) - \lambda(1+\Omega_k) \big]\Omega_x \right\}. \tag{32}$$

Using $x - x_{eq} = \ln(a/a_{eq}) = \ln(\frac{1+z_{eq}}{1+z})$ and Eqs. (13) and (19), one can rewrite Eq. (32) as

$$\frac{\dot{Q}_m}{T_m} + \frac{\dot{Q}_x}{T_x} = \frac{3\pi}{H(1+\Omega_k)^3} \left(\frac{1+z_{eq}}{1+z} \right)^{3(\omega_x+\lambda)} \left\{ \frac{r_{eq}}{r} \left(\frac{1+z_{eq}}{1+z} \right)^2 \right.$$
$$\times \big[\lambda(1+\Omega_k)\Omega_x + q(1+\Omega_k-\Omega_x) \big]$$
$$\left. + \big[q(1+\omega_x) - \lambda(1+\Omega_k) \big]\Omega_x \right\}. \tag{33}$$

Using Eqs. (8), (24), (29), (30), (31), and (33), and taking $\omega_x = -1$, $\lambda = 0.3$, $z_{eq} = 5.56 \times 10^7$, $r_{eq} = 1.09 \times 10^5$ [38], $\Omega_x = 0.72$, and $\Omega_k = 0.02$ for the present time, i.e. $z = 0$, we get

$$q = -0.57,$$

$$\dot{q} = 0.03H,$$

and

$$\frac{\dot{Q}_m}{T_m} + \frac{\dot{Q}_x}{T_x} = 1.91 \times 10^4 H^{-1},$$

$$\dot{Q}_x \ddot{Q}_x = -6.40 \times 10^{-4} H,$$

$$\dot{Q}_h \ddot{Q}_h = 11.76 \times 10^{-4} H,$$

$$\dot{S}_h = 2.72 H^{-1}.$$

Therefore, the GSL in irreversible thermodynamics, i.e. Eq. (18), for the present time yields

$$\dot{S}^* = H^{-1}\big(1.91 \times 10^4 + 6.40 \times 10^{-4} A_x H^2$$
$$- 11.76 \times 10^{-4} A_h H^2 + 2.72 \big),$$
$$= H^{-1}\big(1.91 \times 10^4 + 6.40 \times 10^{-4} \bar{A}_x$$
$$- 11.76 \times 10^{-4} \bar{A}_h + 2.72 \big), \tag{34}$$

where we define $\bar{A} := AH^2$. If we set $\bar{A}_x = \bar{A}_h$, then Eq. (34) shows that when $\bar{A}_x = \bar{A}_h \leqslant 3.56 \times 10^7$, the GSL in nonequilibrium thermodynamics is respected, i.e. $\dot{S}^* \geqslant 0$, for the present time.

4. Conclusions

Here the GSL in nonequilibrium thermodynamics for the interacting DE with CDM in a non-flat FRW universe is investigated. The boundary of the universe is assumed to be enveloped by the dynamical apparent horizon. The dynamical apparent horizon in comparison with the cosmological event horizon, is a good boundary for studying cosmology, since on the apparent horizon there is the well-known correspondence between the first law of thermodynamics and the Einstein equation [27]. In the other words, the Friedmann equations describe local properties of spacetimes and the apparent horizon is determined locally, while the cosmological event horizon is determined by global properties of spacetimes [20]. We assumed that when the DE and DM evolve separately, each of them remain in thermal equilibrium with the Hawking temperature on the dynamical apparent horizon. We found that for the present time, the GSL in irreversible thermodynamics is respected for the special range of the energy transfer constants.

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