Mechanism of unconventional aerodynamic characteristics of an elliptic airfoil

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Abstract The aerodynamic characteristics of elliptic airfoil are quite different from the case of conventional airfoil for Reynolds number varying from about $10^4$ to $10^6$. In order to reveal the fundamental mechanism, the unsteady flow around a stationary two-dimensional elliptic airfoil with 16% relative thickness has been simulated using unsteady Reynolds-averaged Navier–Stokes equations and the $\frac{\gamma}{\gamma_0}$ transition turbulence model at different angles of attack for flow Reynolds number of $5 \times 10^5$. The aerodynamic coefficients and the pressure distribution obtained by computation are in good agreement with experimental data, which indicates that the numerical method works well. Through this study, the mechanism of the unconventional aerodynamic characteristics of airfoil is analyzed and discussed based on the computational predictions coupled with the wind tunnel results. It is considered that the boundary layer transition at the leading edge and the unsteady flow separation vortices at the trailing edge are the causes of the case. Furthermore, a valuable insight into the physics of how the flow behavior affects the elliptic airfoil’s aerodynamics is provided.

1. Introduction

As the elliptic airfoil is applied on canard rotor/wing (CRW) aircraft, more and more attention has been paid to the performance of this kind of airfoil in relatively low-Reynolds-number flows in recent years. In practice, people are especially interested in the elliptic airfoil with relatively large thickness.

Kwon and Park conducted an experimental study of flow over an elliptic airfoil with 16% relative thickness for the Reynolds number of $3 \times 10^5$ and found that its aerodynamic characteristics were very different from the case of conventional airfoil. Furthermore, in order to examine the influence of the Reynolds numbers, Zhan et al. performed a series of experimental studies of flow over the same elliptic airfoil for a range of Reynolds number from $5 \times 10^5$ to $2.5 \times 10^6$ in the low speed wind tunnel in Northwestern Polytechnical University, by varying the wind speed from 10 m/s to 50 m/s. They also found the unconventional aerodynamic characteristics of elliptic airfoil at the Reynolds number of $5 \times 10^5$. Firstly, lift coefficient $C_L$ increased nonlinearly with the angle of attack $\alpha$, while at small angles of attack, the lift increased fast as $\alpha$ increased; secondly, unlike the conventional symmetrical airfoil, the minimum drag coefficient $C_D$ was obtained at $\alpha = 4^\circ$ rather than...
$\alpha = 0^\circ$; thirdly, the variation of the pitching moment about the quarter chord was very irregular and severe and two inflection points were found in the pitching moment coefficient $C_m$ curve. The numerical results obtained from Reynolds-averaged Navier–Stokes (RANS) equations coupled with S–A and $k-\omega$ shear stress transport (SST) fully turbulence models show apparent discrepancies compared with experimental data, which can be seen clearly in Fig. 1. It indicates that these unconventional aerodynamic characteristics are difficult to capture using traditional method.

The flow separation commonly occurs in engineering practices. In aviation, the designers always try to avoid separation or control it on aircraft surface. The flow past an elliptic airfoil has been studied as a typical example of flows around blunt body since a long time ago because of its significance in fundamental flow physics. Many studies have been accomplished, some of which are experimental, while the majority of which are numerical and mostly at low Reynolds numbers. Unlike general airfoils, the typical characteristic of an elliptic airfoil is the blunt trailing edge, which can cause flow separation and vortex shedding to form Karman vortex street aft of the airfoil. At small angles of attack, the boundary layer is primarily laminar over the airfoil surface, but as $\alpha$ increases, the laminar separation bubble may form near the leading edge on the suction surface of the airfoil, which will result in laminar–turbulent transition. Fig. 2 shows a schematic diagram of the typical flow field structure of an elliptic airfoil. The flow separation near the blunt trailing edge and the transition inside the boundary layer have a great influence on the flow field and aerodynamic characteristics of elliptic airfoil and also pose huge challenges for computational fluid dynamics (CFD) simulation.

In order to reveal the mechanism of unconventional aerodynamic characteristics exhibited by the elliptic airfoil, a numerical simulation method is established by solving the two-dimensional compressible unsteady Reynolds-averaged Navier–Stokes (URANS) equations. A four-equation transition-sensitive turbulence model is used to close the governing equations. Numerical simulations have been performed on an elliptic airfoil with 16% relative thickness for the Reynolds number of $5 \times 10^5$. Combining with the experimental data from wind tunnels, the unconventional aerodynamic characteristics are investigated.

2. Computation scheme

2.1. Governing equation

In order to simulate the unsteady vortices in the flow field, the two-dimensional compressible URANS equations are chosen as the governing equations. The non-dimensional form of the equations in Cartesian coordinates can be written as follows:

$$\frac{\partial Q}{\partial t} + \frac{\partial (E - E_s)}{\partial x} + \frac{\partial (F - F_s)}{\partial y} = 0$$

where $Q = [\rho, \rho u, \rho v, e]^T$ denotes the conservative variables, $\rho, u, v, e$ denote density, components of velocity vector and total energy per unit volume respectively. $E, F$ denote the convective flux while $E_s, F_s$ denote the viscous flux, whose detailed expressions are

$$E = [\rho u, \rho u^2 + p, \rho u v, (e + p) u]^T$$
$$F = [\rho v, \rho u v, \rho v^2 + p, (e + p) v]^T$$
$$E_s = [0, \tau_{xx}, \tau_{xy}, 0]^T$$
$$F_s = [0, \tau_{xy}, \tau_{yy}, 0]^T$$

The viscous shear stress $\tau$ and the heat fluxes $\theta$ are in the form of

$$\begin{align*}
\tau_{xx} &= 2 \mu_s \frac{\partial}{\partial x} (u_x + v_y) \\
\tau_{xy} &= 2 \mu_s \frac{\partial}{\partial y} (u_x + v_y) \\
\tau_{yy} &= \mu_s \frac{\partial}{\partial y} (u_x + v_y) \\
\theta_x &= \kappa T_x + v \tau_{xy} + \kappa T_y \\
\theta_y &= \kappa T_x + v \tau_{xy} + \kappa T_y
\end{align*}$$

where $\kappa$ is the coefficient of thermal conductivity and the total viscosity $\mu$ is calculated as $\mu = \mu_t + \mu_s$, where $\mu_t$ is molecular viscosity calculated by Sutherland law and $\mu_s$ is eddy viscosity determined by turbulence model.
The $\gamma - Re_{\text{it}}$ transition model\textsuperscript{17–19} based only on local variables is adopted to simulate laminar–turbulent transition inside the boundary layer. The $\gamma - Re_{\text{it}}$ transition model was previously developed based on the $\kappa$–$\omega$ SST turbulence model to resolve the laminar–turbulent transition by solving two additional transport equations for the turbulent intermittency $\gamma$ and the transition onset momentum-thickness Reynolds number $Re_{\text{it}}$ which read in the matrix form as follows:

$$\begin{align*}
\frac{\partial \rho \gamma}{\partial t} + \frac{\partial \rho \gamma u_j}{\partial x_j} &= \frac{\partial}{\partial x_j} \left( \mu_t \frac{\partial \gamma}{\partial x_j} + \frac{\partial \rho u_j u_i}{\partial x_i} \right) - \frac{\partial}{\partial x_j} \left( \sigma_n \left( \mu_t + \mu_s \right) \frac{\partial Re_{\text{it}}}{\partial x_j} \right) = \left[ P_f - E_f \right]_x
\end{align*}$$

(4)

The constants for the equations are $\sigma_t = 1.0$ and $\sigma_n = 2.0$.

There is an effective turbulent intermittency $\gamma_{\text{eff}}$ to adjust the original production and destruction terms $P_k$ and $D_k$ in the $\kappa$ equation for the $\kappa$–$\omega$ SST model, which performs as follows:

$$\begin{align*}
\tilde{P}_k &= \gamma_{\text{eff}} P_k \\
\tilde{D}_k &= \min \left( \max \left( \gamma_{\text{eff}}, 0.1 \right), 1.0 \right) D_k \\
\gamma_{\text{eff}} &= \max \left( \gamma, \gamma_{\text{sep}} \right) \\
\gamma_{\text{sep}} &= \min \left( s_t \max \left( 0, \left( \frac{Re_{\text{it}}}{3.235 Re_{\text{cr}}} \right)^{-1} - 1 \right), 2 \right) F_{\text{reattach}} \\
F_{\text{reattach}} &= \exp \left( - \left( R_t / 20 \right)^{10} \right) \\
s_t &= 2.0
\end{align*}$$

(5)

Here, the term $\gamma$ is local turbulent intermittency and $\gamma_{\text{sep}}$ the turbulent intermittency for considering the separation induced transition; $Re_{\text{cr}}$ is the vorticity Reynolds number and $Re_{\text{it}}$ the critical momentum thickness Reynolds number; $R_t$ is the viscous ratio and $F_{\text{f}}$ the blending function from the $Re_{\text{it}}$ equation.

The flow solver used in this paper is an in-house multi-block RANS solver. In this code, temporal marching method is implicit lower-upper symmetric-Gauss–Seidel (LU-SGS)\textsuperscript{20} with sub-iterations in pseudo time, which has 2nd-order precision in unsteady calculations. The spatial discretization scheme for convection terms is Roe\textsuperscript{21} scheme with Harten’s entropy correction. 3rd-order monotonic upstream-centered scheme for conservation laws (MUSCL) reconstruction method is used to increase the Roe scheme to 2nd-order. The viscous fluxes are discretized with 2nd-order centered schemes. In order to overcome the difficulty in solving compressible equations for low $Ma$ flows, Weiss and Smith preconditioning matrix\textsuperscript{22} is introduced.

For turbulence model equations, implicit LU-SGS method is also operated as time marching scheme, which is similar to the discretization of governing equations. Convection terms therein are discretized by 1st-order upwind schemes, while production terms and destruction terms are treated explicitly and implicitly respectively to increase the numerical robustness.

### 2.2. Computational grid

According to the characteristics of the flow field, the computational domain is partitioned into three subzones: the boundary layer region, the wake region and the outer region. To ensure that the boundary location does not influence the flow, the far field boundaries for upstream, the top and bottom boundaries are placed at a distance of 10 chord lengths $c$ from the elliptic airfoil, while the boundaries for downstream are at a distance of 20 chord lengths. In order to accurately simulate the flow inside the boundary layer, a structured O-type grid is generated in the boundary layer region: a total of 201 grid points are distributed on the airfoil surface and clustered near the leading and trailing edges. The first point in the viscous layer is $1 \times 10^{-5}$ chord unit away from the wall to ensure that the $y^+$ value is less than unity over the entire airfoil surface. The mesh in boundary layer increases uniformly from the wall with a growth rate of 1.1. In order to resolve vortex structures in the wake region, 150 grid points are adopted there. An orthogonal C-type grid is employed in the outer region, which finally generates 70000 cells in total for simulation. A grid independent study is conducted by refining the grid in both boundary layer and wake region. No considerable changes were observed when the grid was refined and all the computations were performed on the grid described above. The grid topology and the computational grid generated are shown in Fig. 3.

### 3. Results and analyses

A non-dimensional time step size, $\Delta t = t / T$, of 0.01 is used in the simulation so as to accurately capture the unsteady flow characteristic, where $T = c / V_{\infty}$ is the characteristic time scale based on the ratio of chord length to the free stream velocity $V_{\infty}$. An independent study of time step size is conducted with time steps smaller than 0.01, and no considerable changes are observed in results. The uniform initial condition is applied to all the unsteady simulations. The unsteady calculations are set

![Fig. 3](image-url)
to a total of 5000 time steps with 20 sub-iterations per time step, and it is obvious that 5000 time steps is enough to eliminate the influence of initial value and capture the features of the unsteady flow.

Since the experimental data given by the wind tunnel test in Ref.\textsuperscript{2} are time-averaged, in order to validate the numerical method and conduct a further analysis in the present study, the time-averaged solution is obtained by taking the average over the last 3 oscillation cycles. In Fig. 4, the time-averaged lift, drag and moment coefficients are plotted in comparison with the experimental data. In order to get a clearer image of the differences between the elliptic airfoil and conventional airfoil with sharp trailing edge, NACA0016 airfoil under the same condition is chosen for comparison. It is clearly seen that the lift curve slope of the NACA0016 airfoil remains constant in linear range, the minimum drag coefficient is obtained at

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**Fig. 4** Experimental data for elliptic airfoil and comparison of aerodynamic force and moment coefficients between elliptic airfoil and NACA0016 airfoil.

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**Fig. 5** Time-averaged surface pressure coefficients distribution in comparison with experimental results.
\( \alpha = 0^\circ \), and the pitching moment coefficient about the quarter chord is nearly constant before the airfoil stall occurs. As previously noted, the aerodynamic characteristics of elliptic airfoil are quite different from those of conventional one.

In general, the numerical results are in good agreement with the experimental data. The numerical method is proved to be reliable since the unconventional aerodynamic characteristics of elliptic airfoil, such as the high lift slope at small angles of attack, the nonlinear segment along \( C_L \) distribution, the large \( C_D \) value at \( \alpha = 0^\circ \) and the irregular variation along \( C_m \) distribution are accurately predicted. However, there is one point that needs attention: the computation result of \( C_L \) at \( \alpha = 0^\circ \) is zero while the experimental result is not zero, which implies that an important aspect of the experimental geometry has not been adequately modeled in the computational technique or that there is error in the experimental value; obviously, the later one is reasonable.

In order to gain a deep insight into the unconventional aerodynamic characteristics of the elliptic airfoil, the analysis is presented in the following three different aspects.

**Fig. 6** Time-averaged skin friction coefficients distribution and turbulent kinetic energy distribution.

**Fig. 7** Components of drag, pressure drag and frictional drag coefficients at small angles of attack.
3.1. Characteristics of lift

The aerodynamic performance of an airfoil can be studied easier by referring to the pressure distribution over itself. In order to find out the reason for the high lift and high lift slope at small angles of attack and why a shift exists in the lift slope between $\alpha = 4^\circ$ and $\alpha = 6^\circ$, the time-averaged pressure coefficient $C_p$ distribution is computed for $\alpha = 2^\circ, 4^\circ, 6^\circ, 8^\circ$. The results are in good agreement with the experimental data (shown in Fig. 5), which further validates the numerical method.

It is very interesting to find out that there is a sudden increase in $C_p$ near the trailing edge for $\alpha < 6^\circ$. This local pressure recovery area makes the adverse pressure gradient mild over the most part of the upper surface. The $C_p$ curve over the upper surface is flat, as a result the elliptic airfoil carries more lift than the conventional ones. In addition, the negative pressure peak rises quickly as $\alpha$ increases, which further enhances the lift, and also brings a high lift slope. When $\alpha$ is up to $6^\circ$, the local pressure recovery area appears in the place near the leading edge after the negative pressure peak, forming a slight pressure plateau region. This characteristic pressure distribution is indicative of the formation of laminar separation bubble. The sudden increase in $C_p$ after the bubble leads to a relatively heavy loss in lift. And therefore the lift growth rate decreases.

Actually, to figure out what must be responsible for these unconventional lift characteristics, we need to figure out what happened in the flow around the elliptic airfoil. The time-averaged skin friction coefficient $C_f$ distribution and turbulent kinetic energy (TKE) distribution around the airfoil for different angles of attack are shown in Fig. 6. It can be observed that at $\alpha < 6^\circ$, the $C_f$ distribution curve declines smoothly and TKE is zero except in the separated flow region near the blunt trailing edge, which means the boundary layer is laminar over most of the airfoil. At $\alpha = 6^\circ$, skin friction coefficient $C_f$ becomes negative along chord wise firstly, indicating that the laminar boundary layer separates, and then $C_f$ increases rapidly to be positive, indicating that the laminar–turbulent transition occurs and the turbulent boundary layer reattaches to the airfoil surface. Higher TKE can be observed in the boundary layer beyond the transition point, and the reattached turbulent boundary layer is much more energetic. In summary, the wide range of laminar flow over the elliptic airfoil is the real root cause of the high lift and high lift slope at small angles of attack. Once the boundary layer transition happens, the unconventional lift characteristic disappears.

Fig. 8  Time-averaged vortex structure with stream line at trailing edge.
3.2. Characteristics of drag

At $\alpha < 6^\circ$, we find that the drag coefficient decreases as the flow angle increases and the minimum value of $C_D$ is obtained at $\alpha = 4^\circ$. It is very unconventional because the research object in this study is a symmetrical airfoil. Fig. 7 shows the components of the drag, pressure drag $C_{DP}$ and frictional drag $C_{DF}$, as functions of angles of attack. Since the pressure drag is the main component, the characteristics of drag have a close relationship with the vortices at the trailing edge. The time-averaged vortex structure with mean stream line at small angles of attack is illustrated in Fig. 8. As the angle of attack increases, the streamlines packed in the recirculation region will be denser, and therefore the cross-sectional areas will be smaller which results in smaller pressure drag. However, after the laminar–turbulent transition occurs inside the boundary layer at $\alpha = 6^\circ$, the frictional drag increases rapidly due to the turbulent boundary layer on the upper surface. Hence, $C_D$ begins to increase beyond the angle of attack for minimum drag, which is around $4^\circ$.

3.3. Characteristics of moment

It is known that in the case of conventional airfoil, the pitching moment coefficient $C_m$ about the quarter chord is nearly constant in the linear lift range. By contrast, the pitching moment coefficient of the elliptic airfoil varies very unconventionally with the angle of attack, which is shown in Fig. 4. At small angles of attack, $C_m$ is negative, and this nose-down moment becomes more extreme as the angle of attack increases. After $\alpha = 6^\circ$, this tendency shows a sign of abating and the first inflection point shows up. The pitching moment begins to increase beyond that angle of attack. When $\alpha$ is up to $10^\circ$, $C_m$ becomes positive, which means it changes from nose-down moment to nose-up moment. But when $\alpha$ exceeds $12^\circ$, the second inflection point shows up and the pitching moment becomes nose-down moment soon after.

In order to better understand the moment characteristics, Tables 1 and 2 give the components of moment at small angles of attack and large attack angles respectively, where $C_{mg}$ refers to the contribution of pressure while $C_{mf}$ refers to the contribution of viscous force on moment. Apparently, the $C_{mg}$ is very small. At small angles of attack where $\alpha < 6^\circ$, as shown in Fig. 5, the elliptic airfoil is a bit aft-loaded so the aft section of the airfoil produces more lift and nose-down pitching moment. At $\alpha = 6^\circ$, laminar–turbulent transition takes place inside the boundary layer near the leading edge; although $C_{mg}$ becomes much larger than before, it is still fairly small. The real reason that the moment begins to increase, is that the aft-loaded phenomenon disappears because of transition near the leading edge. While at large angles of attack where $\alpha > 12^\circ$, the $C_{mf}$ becomes negative, which must be caused by reverse flow over the upper surface. This can be confirmed by Fig. 9, velocity contour of the elliptic airfoil at $\alpha = 14^\circ$. The separated boundary layer fails to reattach to the suction surface and massive flow separation can be observed. It is this massive flow separation that leads to the undesirable change of moment.

4. Conclusions

(1) For the flows at relatively low Reynolds number, laminar–turbulent boundary layer transition and separation flow behind the trailing edge play a predominant role in determining aerodynamic characteristics of the elliptic airfoil. Since the flow is essentially unsteady, the URANS method and the transition turbulent model are indeed necessary to accurately simulate the flow field.

(2) The wide range of laminar flow over the elliptic airfoil is the root cause of the high lift slope at small angles of attack. Once the boundary layer transition takes place near the leading edge, the local pressure recovery area leads to great loss to lift and high lift slope characteristic vanishes.

(3) As $\alpha$ increases from $0^\circ$ to $4^\circ$, the main component of drag is pressure drag, the mean cross-sectional area of the separation vortices decreases which leads to the decreases in pressure drag, the frictional drag shows little change such that the total drag decreases. When laminar–turbulent transition occurs at $\alpha = 6^\circ$, the frictional drag increases dramatically, and the pressure drag increases slightly, so that the total drag increases.

### Table 1 Components of moment coefficients at small angles of attack.

<table>
<thead>
<tr>
<th>$\alpha$ (°)</th>
<th>$C_{mg}$</th>
<th>$C_{mf}$</th>
<th>$C_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>−0.0222</td>
<td>0.000048</td>
<td>−0.0221</td>
</tr>
<tr>
<td>4</td>
<td>−0.0377</td>
<td>0.000085</td>
<td>−0.0376</td>
</tr>
<tr>
<td>6</td>
<td>−0.0227</td>
<td>0.000460</td>
<td>−0.0223</td>
</tr>
</tbody>
</table>

### Table 2 Components of moment coefficients at large angles of attack.

<table>
<thead>
<tr>
<th>$\alpha$ (°)</th>
<th>$C_{mg}$</th>
<th>$C_{mf}$</th>
<th>$C_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>−0.0007</td>
<td>0.00048</td>
<td>−0.0003</td>
</tr>
<tr>
<td>12</td>
<td>0.0145</td>
<td>0.00041</td>
<td>0.0149</td>
</tr>
<tr>
<td>14</td>
<td>−0.0019</td>
<td>−0.00020</td>
<td>−0.0021</td>
</tr>
</tbody>
</table>

Fig. 9 Instantaneous velocity contour at $\alpha = 14^\circ$. 
(4) At small angles of attack where boundary layer is laminar over the elliptic airfoil, the aft-load carries nose-down pitching moment; at $\alpha = 6^\circ$ where the boundary layer transition occurs, the loss of lift at the aft part of airfoil results in increased pitching moment; while at large angles of attack where the airfoil stalls, the massive flow separation over the suction surface of airfoil is responsible for irregular variation of pitching moment.

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References


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