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Slip-flow and heat transfer of chemically reacting micropolar fluid through expanding or contracting walls with Hall and ion slip currents

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Abstract The present article deals with the effects of velocity slip, chemical reaction on heat and mass transfer of micropolar fluid in expanding or contracting walls with Hall and ion slip currents. Assume that there is symmetric suction or injection along the channel walls, which are maintained at nonuniform constant temperatures and concentrations. The governing Navier–Stokes equations are reduced to nonlinear ordinary differential equations by using similarity transformations then solved numerically by quasilinearization technique. The effects of various parameters such as wall expansion ratio, chemical reaction parameter, Prandtl number, Schmidt number, slip parameter, Hall and ion slip parameters on nondimensional velocity components, microrotation, temperature and concentration are discussed in detail through graphs. It is observed that the concentration of the fluid is enhanced with viscosity. Further, the temperature and concentration of the fluid are increased whereas the microrotation is decreased for an expansion or contraction of the walls.

1. Introduction

The flow through porous channels has great applications in engineering and science. Examples of these were found in the boundary layer control, the transport of biological fluids through contracting or expanding vessels, paper manufacturing, ablation cooling, the air circulation in the repository system, the regression of the burning surface in solid rocket motors and MHD power generators, etc. The theory of micropolar fluids was initiated by Eringen [1] and this theory constitutes a subclass of microfluids. Si et al. [2–4] analyzed the problems of flow and heat transfer of micropolar fluids with expanding or contracting walls and velocity slip and obtained analytical solution. Srinivasacharya et al. [5] discussed numerically the flow and heat transfer of couple stress fluid in a porous expanding or contracting channel. Gabriel and Chaudry Masood [6] considered the flow and heat transfer of viscous fluid in a porous expanding or contracting channel using Lie group analysis. An exact solution was obtained for the problem of the flow of a viscous fluid in porous channels by Asghar et al. [7]. Majdalani et al. [8] investigated the two

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Nomenclature

t	time	T	temperature
$a(t)$	distance between the origin and upper/lower wall	T_1	temperature of the lower wall
V_1	suction/injection velocity	T_2	temperature of the upper wall
p	fluid pressure	T^*	dimensionless temperature, $\frac{T-T_1}{T_2-T_1}$
$\bar{q}$	velocity vector	C	concentration
c	specific heat at constant temperature	C_1	concentration of the lower wall
$\bar{l}$	microrotation vector	C_2	concentration of the upper wall
N	microrotation component	C^*	dimensionless concentration, $\frac{C-C_1}{C_2-C_1}$
Ec	Eckert number, $\frac{(\mu+k_1)V_1}{\rho ac(T_2-T_1)}$	D_1	mass diffusivity
k	thermal conductivity	$\dot{n}_A$	mass transfer rate
k_1	micropolar viscosity parameter	k_3	chemical reaction rate
u	velocity component in X-direction	Kr	nondimensional chemical reaction parameter, $\frac{k_3a^2}{D_1}$
v	velocity component in Y-direction	Sc	Schmidt number, $\frac{\upsilon}{D_1}$
Pr	Prandtl number, $\frac{\mu c}{k}$		
Re	Reynolds number, $\frac{\rho V_1 a}{\mu}$	*Greek Letters*	
j	gyration parameter	λ	dimensionless y coordinate, $\frac{y}{a}$
$\bar{J}$	current density	α, β, γ	gyro viscosity parameters
J_1	nondimensional gyration parameter, $\frac{\rho j\upsilon}{\gamma}$	ζ	dimensionless axial variable, $\frac{x}{a}$
$\bar{B}$	total magnetic field	ρ	fluid density
$\bar{b}$	induced magnetic field	μ	fluid viscosity
B_0	magnetic flux density	μ'	magnetic permeability
D	rate of deformation tensor	σ	electric conductivity
$\bar{E}$	electric field	βi	ion slip parameter
Ha	Hartmann number, $B_0 a\sqrt{\frac{\sigma}{\mu}}$	βe	Hall parameter
R	nondimensional viscosity parameter, $\frac{k_1}{\mu}$	η	wall expansion ratio, $\frac{a\dot{a}}{\upsilon}$
s_1	nondimensional micropolar parameter, $\frac{k_1a^2}{\gamma}$		
s_2	nondimensional micropolar parameter, $\frac{\gamma c}{a^2k}$		
Sl	slip parameter, $\frac{\sqrt{k_2}}{a\sigma_1}$		

dimensional unsteady viscous fluid flow between expanding or contracting walls with permeability and the problem was solved both numerically and analytically. The problem of unsteady laminar flow of a viscous fluid in expanding or contracting pipe was examined by Bujurke and Pai [9]. Uchida and Akoi [10] considered an unsteady incompressible laminar flow of a viscous fluid in a semi infinite expanding or contracting pipe. Srinivas et al. [11,12] considered the effects of heat and mass transfer to symmetric and asymmetric flow through porous channels with expanding or contracting walls and a perturbation solution was obtained for the reduced governing equations. Dinarvand et al. [13] studied the problem of an incompressible isothermal viscous fluid flow through expanding or contracting gaps with permeable walls and obtained an analytical approximate solution by Homotopy Analysis Method (HAM). Odelu and Naresh [14,15] investigated numerically the MHD flow and heat transfer of a micropolar fluid in a parallel plate channel with periodic suction and injection by considering chemical reaction and Hall and ion slip currents. The flow of an electrically conducting micropolar fluid with Hall and ion slip by considering thermal diffusivity was examined by Motsa and Shateyi [16]. Ziabakhsh and Domairry [17] obtained an analytical approximate solution for micropolar fluid flow in a porous channel with mass transfer. The simultaneous effects of soret and ohmic heating on free convective heat and mass transfer of an electrically conducting micropolar fluid in a porous medium bounded by an infinite vertical surface were examined by Satyanarayana and Sravanthi [18] and they obtained an analytical solution by perturbation method. Uddin and Kumar [19] discussed the problem of a steady incompressible flow of a micropolar fluid through a wedge with Hall and ion slip effects and the flow field equations are solved numerically using Runge–Kutta method. Eldahab and Aziz [20] investigated numerically the effects of the Hall and ion slip on steady free convective flow with viscous dissipation and Joule heating by considering the power law variation of the wall temperature. The flow and convective heat transfer of viscous fluid through a vertical plate with Hall and ion slip currents was studied by Ferdows et al. [21] and a numerical solution was obtained by Runge–Kutta sixth order method. Nasser [22] considered the effects of chemical reaction on MHD flow of a viscous fluid with Hall and ion slip and the reduced nonlinear differential equations are solved numerically by using the Chebyshev pseudospectral method. The effects of thermal radiation and chemical reaction on incompressible flow of an electrically conducting micropolar fluid over an inclined plate have been studied by Das [23]. Pal et al. [24] discussed the problem of oscillatory mixed convection-radiation of a micropolar fluid in a rotating system with Hall and chemical reaction effects and obtained an analytical solution. Bakr [25] investigated the steady as well as unsteady MHD micropolar fluid with constant heat source and chemical reaction effect in a rotating frame of reference. The optimization of energy and cost of heat exchanger for fins over a plate using genetic algorithm have been studied by Najafi et al. [26]. Hajmohammadi et al. [27] considered the

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analysis of constructal design of fins with convective heat transfer. The laminar incompressible flow of nano fluids with heat transfer characteristics by considering the convective boundary conditions was analyzed numerically by Hajmohammadi et al. [28]. The literature states that the velocity slip could not be ruled out as a significant element in the understanding of certain flow characteristics under the porous boundary conditions (Beavers and Joseph [29]). Ramos [30] investigated an asymptotic analytical solution of two dimensional flows of incompressible fluids with a velocity slip length that depended on the axial pressure gradient. However, very few reports were found in the literature for micropolar fluids with expanding or contracting walls and slip boundary condition. The three dimensional flow of nano fluid with heat transfer was considered in the presence of velocity slip and thermal radiation by Hayat et al. [31] and obtained an analytical approximate solution by HAM. Zheng et al. [32] have studied the effects of temperature jump on MHD slip-flow and heat transfer of a viscous fluid over a porous shrinking surface. Hajmohammadi and Nourazar [33] have investigated the flow and heat transfer of a thin gas layer and power law liquid in a cylinder by considering the slip and the stability analysis is discussed by using gradient energy method. Recently, Hajmohammadi et al. [34] have obtained an analytical solution for the two phase flow of power law liquid and gas between two cylinders with stability analysis. Zhang et al. [35] studied the steady Navier–Stokes equations with first and second order accurate slip boundary conditions for describing the two dimensional gaseous laminar flow between two plates. Bhatnagar et al. [36] examined the steady incompressible laminar flow of viscoelastic fluid through a porous cylindrical annulus and the reduced flow field equations are solved numerically using the quasilinearization method. Hymavathi and Shanker [37] applied quasilinearization technique to MHD flow of a visco-elastic fluid in a porous stretching sheet. A quasilinearization method was used to solve the convective flow of a viscous fluid in a vertical channel by Huang [38]. Further, the nonlinear ordinary differential equations can be solved by semi analytical methods. Many authors (Hajmohammadi et al. [39,44], Hajmohammadi and Nourazar [40,43] and Khan et al. [41,42]) used the semi analytical technique to solve the nonlinear ordinary differential equations of the forced convective flow and heat transfer over a plate by considering the thermal conductivity as a function of temperature.

Motivated by the above work, we considered the effects of chemical reaction, Hall and ion slip on two dimensional MHD flow and heat transfer of micropolar fluid with expanding or contracting walls. The flow field equations are reduced to nonlinear ordinary differential equations by similarity transformations and the solution is obtained using the quasilinearization method. The effects of various fluid and geometric parameters on the velocity components, microrotation, temperature distribution and concentration are studied and shown graphically.

2. Formulation of the problem

The two dimensional laminar incompressible micropolar fluid flow through an elongated rectangular channel exhibiting a sufficiently large aspect ratio of height 'a' is considered. The upper and lower walls are assumed to have equal permeability and expand or contract uniformly at a time-dependent rate in the transverse direction only. Hence, their separation is a function of time, $a(t)$. Let the fluid be injected or aspirated uniformly and orthogonally through the channel walls at an absolute velocity V_1. The lower and upper walls are maintained at constant temperatures T_1, T_2 and concentrations C_1, C_2 respectively. The region inside the parallel walls is subjected to a constant external magnetic field of strength B_0 perpendicular to the XY-plane.

The governing equations of the two dimensional micropolar fluid flow, heat and mass transfer in the presence of Hall and ion slip currents and in the absence of body forces and body couples are [46,47] given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial P}{\partial x}+(\mu+k_1)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+k_1\frac{\partial N}{\partial y}-\sigma B_0^2\left(\frac{(1+\beta i\ \beta e)u-\beta e v}{(1+\beta i\ \beta e)^2+\beta e^2}\right) \tag{2}$$

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial P}{\partial y}+(\mu+k_1)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-k_1\frac{\partial N}{\partial x}-\sigma B_0^2\left(\frac{(1+\beta i\ \beta e)v+\beta e u}{(1+\beta i\ \beta e)^2+\beta e^2}\right) \tag{3}$$

$$\rho\left[\frac{\partial N}{\partial t}+u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right]=-2k_1N+k_1\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\gamma\left(\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial y^2}\right) \tag{4}$$

$$\rho c\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right]=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\left(2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right)+\frac{k_1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}-2N\right)^2+\gamma\left(\left(\frac{\partial N}{\partial x}\right)^2+\left(\frac{\partial N}{\partial y}\right)^2\right)+\frac{\sigma B_0^2(u^2+v^2)}{(1+\beta i\ \beta e)^2+\beta e^2} \tag{5}$$

$$\left[\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right]=D_1\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)-k_3(C-C_1) \tag{6}$$

The coefficients μ, k_1, α, β, γ in the above equations are related by the inequalities

$$2\mu+k_1\geqslant 0,\ k_1\geqslant 0,\ 3\alpha+\beta+\gamma\geqslant 0,\ \gamma\geqslant|\beta| \tag{7}$$

Neglecting the displacement currents, the Maxwell equations and the generalized Ohm's law are

$$\nabla\cdot\overline{B}=0,\quad \nabla\times\overline{B}=\mu'\overline{J},\quad \nabla\times\overline{E}=\frac{\partial\overline{B}}{\partial t},\quad \overline{J}=\sigma(\overline{E}+\overline{q}\times\overline{B})-\frac{\beta e}{B_0}(\overline{J}\times\overline{B})+\frac{\beta e\beta i}{B_0^2}(\overline{J}\times\overline{B})\times\overline{B} \tag{8}$$

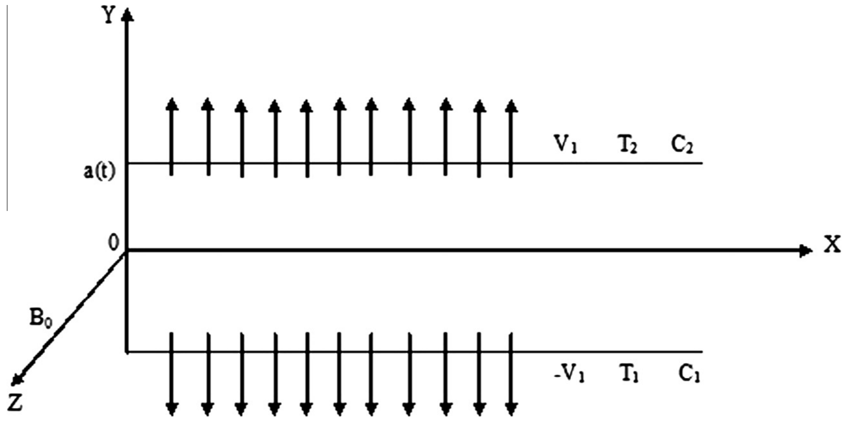

Fig. 1 The geometry of the porous channel with expanding or contracting walls.

where $\overline{B} = B_0\hat{k} + \overline{b}$, $\overline{b}$ is induced magnetic field, βe is the Hall parameter, βi is the ion slip parameter and μ' is magnetic permeability. Assume that the induced magnetic field is negligible compared to the applied magnetic field so that magnetic Reynolds number is small, the electric field is zero and magnetic permeability is constant throughout the flow field.

Following Si et al. [2], we take the velocity and microrotation components as,

$$u(x,\lambda,t) = -\frac{\upsilon x}{a^2}F'(\lambda,t), \quad v(x,\lambda,t) = \frac{\upsilon}{a}F(\lambda,t),$$
$$N(x,\lambda,t) = \frac{\upsilon x}{a^3}G(\lambda,t)$$

Following Srinivas et al. [11], the temperature distribution and concentration are considered as

$$T(x,\lambda,t) = T_1 + \frac{(\mu+k_1)V_1}{\rho ac}\left(\phi_1(\lambda) + \frac{x^2}{a^2}\phi_2(\lambda)\right) \text{ and}$$

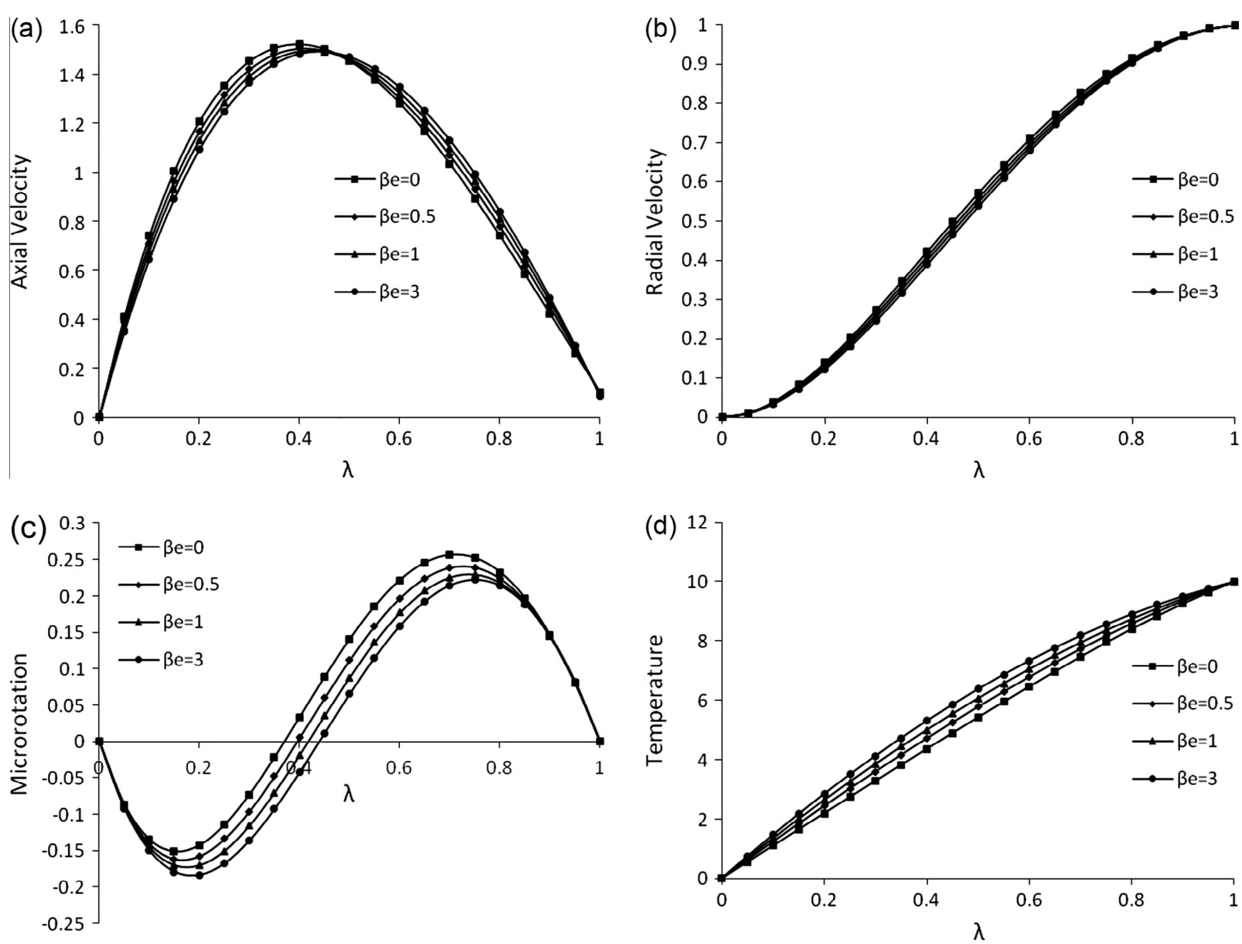

Fig. 2 Effect of βe on (a) axial velocity, (b) radial velocity, (c) microrotation and (d) temperature for Kr = 2, J_1 = 0.02, βi = 0.2, Sl = 0.2, Sc = 0.2, Pr = 0.2, Re = −2, $\eta = 2$, R = 2, s_1 = 2, s_2 = 2, D^{-1} = 0.0, Ha = 4.

$$C(x,\lambda,t) = C_1 + \frac{\dot{n}_A}{av}\left(G_1(\lambda) + \frac{x^2}{a^2}G_2(\lambda)\right) \tag{9}$$

where $\lambda = \frac{y}{a(t)}$ and $F(\lambda,t)$, $G(\lambda,t)$, $\phi_1(\lambda)$, $\phi_2(\lambda)$, $G_1(\lambda)$ and $G_2(\lambda)$ are to be determined.

The boundary conditions on the velocity, microrotation, temperature and concentration are

$$\frac{\partial u}{\partial y} = 0, \quad v = 0, \quad N = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0 \text{ at } \lambda = 0$$

$$u = -\frac{\sqrt{k_2}}{\sigma_1}\frac{\partial u}{\partial y}, \quad v = V_1, \quad N = 0, \quad T = T_2, \quad C = C_2 \text{ at } \lambda = 1 \tag{10}$$

The slip boundary condition (10) is the well-known Beavers and Joseph slip condition (Beavers and Joseph [25]), where σ_1 is a dimensionless constant which depends on the pore size of the permeable material and k_2 is the specific permeability of the porous medium.

Substituting Eq. (9) in Eq. (2)–(6), we have

$$\eta\lambda f''' + 3\eta f'' - Re(ff''' - f'f'') = Rg'' - (1+R)f'^{V} + \frac{Ha^2(1+\beta e\beta i)}{(1+\beta e\beta i)^2 + \beta e^2}f'' \tag{11}$$

$$J_1(-\lambda\eta g' - 3\eta g + Re(fg' - f'g)) = s_1(-2g + f'') + g'' \tag{12}$$

$$\phi_1'' + 2\phi_2 + RePr\left(\frac{4}{1+R}f'^2 + \frac{s_2}{(1+R)Pr}g^2 + \frac{Ha^2}{(1+R)((1+\beta e\beta i)^2 + \beta e^2)}f^2 + \frac{\eta\lambda\phi_1'}{Re} + \frac{\eta\phi_1}{Re} - f\phi_1'\right) = 0 \tag{13}$$

$$\phi_2'' + RePr\left(2f'\phi_2 - f\phi_2' + \frac{f''^2}{1+R} + \frac{R}{2(1+R)}(f'' - 2g)^2 + \frac{Ha^2}{(1+R)((1+\beta e\beta i)^2 + \beta e^2)}f'^2\right) + \frac{Re}{1+R}s_2 g'^2 + Pr\eta\lambda\phi_2' + 3Pr\eta\phi_2 = 0 \tag{14}$$

$$G_1'' = -2G_2 + \mathrm{Kr}ScG_1 + ReSc\left(fG_1' - \frac{\eta}{Re}(G_1 + \lambda G_1')\right) \tag{15}$$

$$G_2'' = ReSc\left(-2f'G_2 + fG_2' - \frac{\eta}{Re}(3G_2 + \lambda G_2')\right) + \mathrm{Kr}ScG_2 \tag{16}$$

where prime denotes the differentiation with respect to λ and $f(\lambda) = \frac{F(\lambda,t)}{Re}$, $g(\lambda) = \frac{G(\lambda,t)}{Re}$.

The dimensionless form of temperature from Eq. (9) can be written as

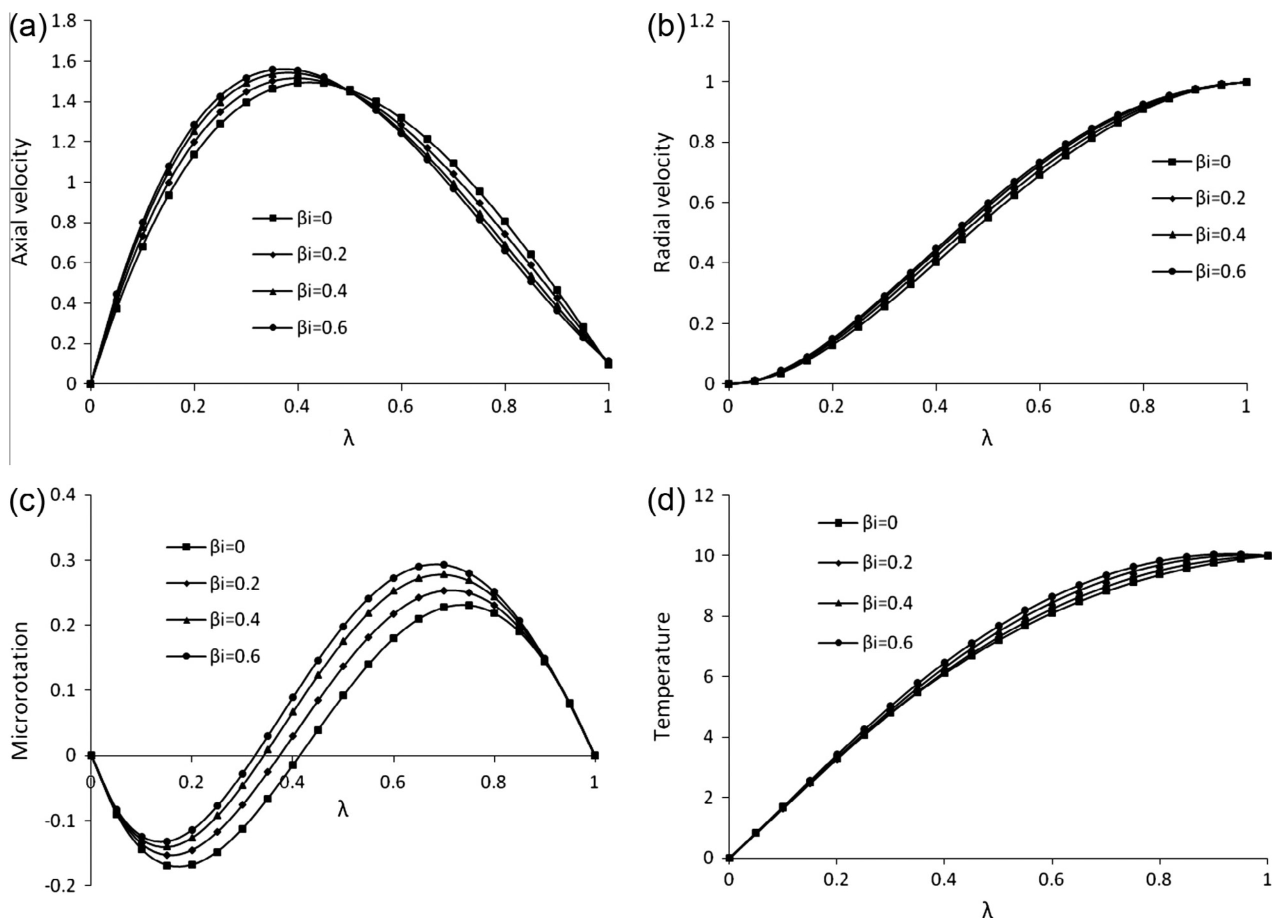

Fig. 3 Effect of βi on (a) axial velocity, (b) radial velocity, (c) microrotation and (d) temperature for Kr = 2, $J_1 = 0.02$, βe = 0.2, Sl = 0.2, $Sc = 0.4$, $Pr = 0.5$, $Re = -2$, $\eta = 2$, $R = 2$, $s_1 = 2$, $s_2 = 2$, $D^{-1} = 0$, $Ha = 15$.

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$$T^* = \frac{T - T_1}{T_2 - T_1} = Ec(\phi_1 + \zeta^2\phi_2) \tag{17}$$

$$C^* = \frac{C - C_1}{C_2 - C_1} = Sh(G_1 + \zeta^2 G_2) \tag{18}$$

The boundary conditions Eq. (10) in terms of $f, g, \phi_1, \phi_2, G_1$ *and* G_2 are

$$\begin{aligned}
&f(0) = 0, \quad f(1) = 1,\\
&f''(0) = 0, \quad f'(1) = -\text{Sl}f''(1),\\
&g(0) = 0, \quad g(1) = 0,\\
&\phi_1'(0) = 0, \quad \phi_1(1) = 1/Ec,\\
&\phi_2'(0) = 0, \quad \phi_2(1) = 0,\\
&G_1'(0) = 0, \quad G_1(1) = 1/Sh,\\
&G_2'(0) = 0, \quad G_2(1) = 0
\end{aligned} \tag{19}$$

3. Solution of the problem

The nonlinear equations Eqs. (11)–(16) are converted into the following system of first order differential equations by the substitution

$$(f, f', f'', f''', g, g', \phi_1, \phi_1', \phi_2, \phi_2', G_1, G_1', G_2, G_2') = (x_1, x_2, x_3, x_4 x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14})$$

$$\frac{dx_1}{d\lambda} = x_2, \quad \frac{dx_2}{d\lambda} = x_3, \quad \frac{dx_3}{d\lambda} = x_4,$$

$$\begin{aligned}
\frac{dx_4}{d\lambda} = {} & \frac{1}{1+R}(Re(x_1x_4 - x_2x_3) + R(s_1(x_3 - 2x_5)\\
& + J_1(-\lambda\eta x_6 - 3\eta x_5 + x_1x_6 - x_2x_5))\\
& + \frac{Ha^2(1+\beta e\beta i)}{(1+\beta e\beta i)^2 + \beta e^2}x_3) - \frac{\eta\lambda x_4}{1+R} - \frac{3\eta x_3}{1+R},
\end{aligned}$$

$$\frac{dx_5}{d\lambda} = x_6,$$

$$\frac{dx_6}{d\lambda} = s_1(x_3 - 2x_5) + J_1(-\lambda\eta x_6 - 3\eta x_5 + (x_1x_6 - x_2x_5))$$

$$\frac{dx_7}{d\lambda} = x_8,$$

$$\begin{aligned}
\frac{dx_8}{d\lambda} = {} & -2x_9 - RePr\Bigg(\frac{4}{1+R}x_2^2 + \frac{s_2}{Pr(1+R)}x_5^2\\
& + \frac{Ha^2}{(1+R)((1+\beta e\beta i)^2 + \beta e^2)}x_1^2 + \frac{\eta x_7}{Re} + \frac{\eta x_8\lambda}{Re} - x_1x_8\Bigg)
\end{aligned}$$

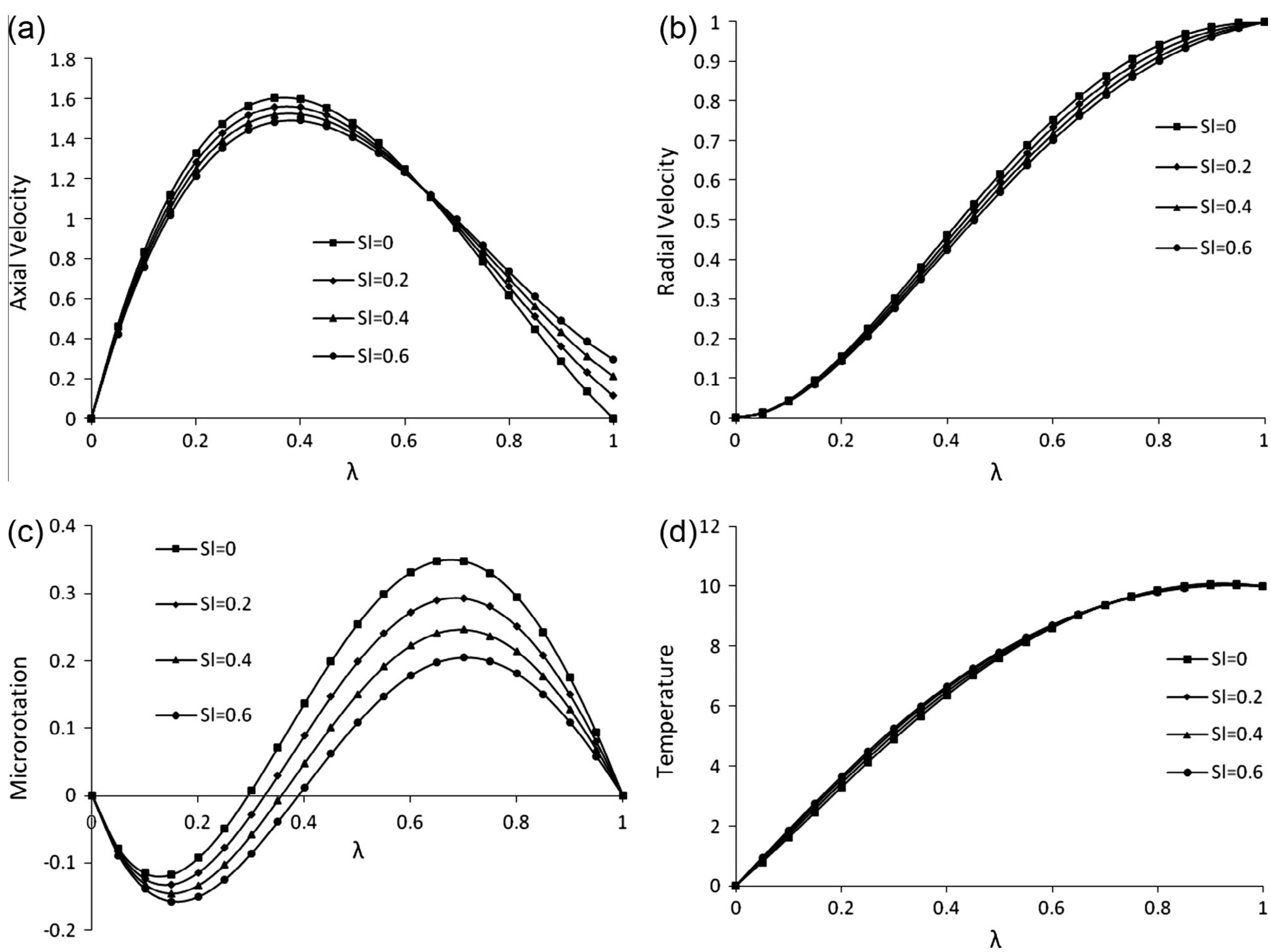

Fig. 4 Effect of Sl on (a) axial velocity, (b) radial velocity, (c) microrotation and (d) temperature For Kr = 2, $J_1 = 0.02$, βe = 5, βi = 0.6, $Sc = 0.4$, $Pr = 0.5$, $Re = -2$, $\eta = 2$, $R = 2$, $s_1 = 2$, $s_2 = 2$, $D^{-1} = 0$, $Ha = 15$.

$$\frac{dx_9}{d\lambda} = x_{10},$$

$$\frac{dx_{10}}{d\lambda} = -RePr\Big(2x_2x_9 - x_1x_{10} + \frac{1}{1+R}x_3^2 + \frac{R}{2(1+R)}(x_3^2 + 4x_5^2 - 4x_3x_5) + \frac{Ha^2}{(1+R)((1+\beta e\beta i)^2 + \beta e^2)}x_2^2\Big) - \frac{Re}{1+R}s_2x_6^2 - Pr\lambda\eta x_{10} - 3Pr\eta x_9$$

$$\frac{dx_{11}}{d\lambda} = x_{12},$$

$$\frac{dx_{12}}{d\lambda} = -2x_{13} + \mathrm{Kr}Scx_{11} + ReSc\Big(x_1x_{12} - \frac{\eta}{Re}(x_{11} + \lambda x_{12})\Big)$$

$$\frac{dx_{13}}{d\lambda} = x_{14}$$

$$\frac{dx_{14}}{d\lambda} = ReSc\Big(-2x_2x_{13} + x_1x_{14} - \frac{\eta}{Re}(3x_{13} + \lambda x_{14})\Big) + \mathrm{Kr}Scx_{13} \quad (20)$$

The boundary conditions in terms of $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}$ are

$$x_1(0) = 0, \quad x_3(0) = 0, \quad x_5(0) = 0, \quad x_8(0) = 0,$$
$$x_{10}(0) = 0, \quad x_{12}(0) = 0, \quad x_{14}(0) = 0,$$

$$x_1(1) = 1, \quad x_2(1) = -\mathrm{Sl}x_3(1), \quad x_5(1) = 0, \quad x_7(1) = 1/Ec,$$
$$x_9(1) = 0, \quad x_{11}(1) = 1/Sh, \quad x_{13}(1) = 0 \quad (21)$$

The system of equations Eq. (20) is solved numerically subject to the boundary conditions Eq. (21) using the quasilinearization method given by Bellman and Kalaba [45].

Let (x_i^n, $i = 1, 2, \ldots, 14$) be an approximate current solution and (x_i^{n+1}, $i = 1, 2, \ldots, 14$) be an improved solution of (20). Using Taylor's series expansion about the current solution by neglecting the second and higher order derivative terms, the coupled first order system (20) is linearized as

$$\frac{dx_1^{n+1}}{d\lambda} = x_2^{n+1}, \quad \frac{dx_2^{n+1}}{d\lambda} = x_3^{n+1}, \quad \frac{dx_3^{n+1}}{d\lambda} = x_4^{n+1},$$

$$\frac{dx_4^{n+1}}{d\lambda} = \frac{1}{1+R}\big(Re(x_1^{n+1}x_4^n + x_4^{n+1}x_1^n - x_2^{n+1}x_3^n - x_2^nx_3^{n+1}) + R(s_1(x_3^{n+1} - 2x_5^{n+1}) + J_1(x_1^{n+1}x_6^n + x_1^nx_6^{n+1} - x_2^{n+1}x_5^n - x_2^nx_5^{n+1} - \lambda\eta x_6^{n+1} - 3\eta x_5^{n+1}))\big) - \frac{1}{1+R}\big(Re(x_1^nx_4^n - x_2^nx_3^n) - RJ_1(x_1^nx_6^n - x_2^nx_5^n)\big) + \frac{Ha^2(1+\beta e\beta i)}{(1+R)((1+\beta e\beta i)^2 + \beta e^2)}x_3^{n+1} - \frac{\eta\lambda x_4^{n+1}}{1+R} - \frac{3\eta x_3^{n+1}}{1+R},$$

$$\frac{dx_5^{n+1}}{d\lambda} = x_6^{n+1},$$

$$\frac{dx_6^{n+1}}{d\lambda} = s_1(x_3^{n+1} - 2x_5^{n+1}) + J_1(x_1^{n+1}x_6^n + x_1^nx_6^{n+1} - x_2^nx_5^{n+1} - x_2^{n+1}x_5^n - \lambda\eta x_6^{n+1} - 3\eta x_5^{n+1}) - J_1(x_1^nx_6^n - x_2^nx_5^n),$$

$$\frac{dx_7^{n+1}}{d\lambda} = x_8^{n+1},$$

$$\frac{dx_8^{n+1}}{d\lambda} = -RePr\Big(\frac{8x_2^nx_2^{n+1}}{1+R} + \frac{2s_2x_5^nx_5^{n+1}}{Pr(1+R)} + \frac{2Ha^2x_1^nx_1^{n+1}}{(1+R)((1+\beta e\beta i)^2 + \beta e^2)} - x_1^nx_8^{n+1} - x_8^nx_1^{n+1} + \frac{\eta x_7^{n+1}}{Re} + \frac{\eta\lambda x_8^{n+1}}{Re}\Big) + RePr\Big(\frac{4x_2^nx_2^n}{1+R} + \frac{s_2x_5^nx_5^n}{Pr(1+R)} + \frac{Ha^2x_1^nx_1^n}{(1+R)((1+\beta e\beta i)^2 + \beta e^2)} - x_1^nx_8^n\Big) - 2x_9^{n+1},$$

$$\frac{dx_9^{n+1}}{d\lambda} = x_{10}^{n+1},$$

$$\frac{dx_{10}^{n+1}}{d\lambda} = -RePr\Big(2x_2^nx_9^{n+1} + 2x_2^{n+1}x_9^n - x_1^nx_{10}^{n+1} - x_1^{n+1}x_{10}^n + \frac{2x_3^nx_3^{n+1}}{1+R} + \frac{R}{2(1+R)}(2x_3^nx_3^{n+1} + 8x_5^nx_5^{n+1} - 4x_3^nx_5^{n+1} - 4x_3^{n+1}x_5^n) + \frac{2Ha^2x_2^nx_2^{n+1}}{(1+R)((1+\beta e\beta i)^2 + \beta e^2)}\Big) - \frac{2Re}{1+R}s_2x_6^nx_6^{n+1} - Pr\lambda\eta x_{10}^{n+1} - 3Pr\eta x_9^{n+1} + RePr\Big(2x_2^nx_9^n - x_1^nx_{10}^n + \frac{x_3^nx_3^n}{1+R} + \frac{R}{2(1+R)}(x_3^nx_3^n + 4x_5^nx_5^n - 4x_3^nx_5^n) + \frac{Ha^2x_2^nx_2^n}{(1+R)((1+\beta e\beta i)^2 + \beta e^2)}\Big) + \frac{Re}{1+R}s_2x_6^nx_6^n$$

$$\frac{dx_{11}^{n+1}}{d\lambda} = x_{12}^{n+1},$$

$$\frac{dx_{12}^{n+1}}{d\lambda} = -2x_{13}^{n+1} + \mathrm{Kr}Scx_{11}^{n+1} + ScRe\Big(x_1^{n+1}x_{12}^n + x_1^nx_{12}^{n+1} - x_1^nx_{12}^n - \frac{\eta}{Re}(x_{11}^{n+1} + \lambda x_{12}^{n+1})\Big),$$

$$\frac{dx_{13}^{n+1}}{d\lambda} = x_{14}^{n+1},$$

$$\frac{dx_{14}^{n+1}}{d\lambda} = \mathrm{Kr}Scx_{13}^{n+1} + ScRe\big(x_1^{n+1}x_{14}^n + x_1^nx_{14}^{n+1} - 2x_2^{n+1}x_{13}^n - 2x_2^nx_{13}^{n+1} - x_1^nx_{14}^n + 2x_2^nx_{13}^n\big) - Sc\big(3\eta x_{13}^{n+1} + \lambda\eta x_{14}^{n+1}\big) \quad (22)$$

To solve for (x_i^{n+1}, $i = 1, 2, \ldots, 14$), the solution to seven separate initial value problems, denoted by $x_i^{h1}(\lambda)$, $x_i^{h2}(\lambda)$, $x_i^{h3}(\lambda)$, $x_i^{h4}(\lambda)$, $x_i^{h5}(\lambda)$, $x_i^{h6}(\lambda)$, $x_i^{h7}(\lambda)$ (which are the solutions of the homogeneous system corresponding to (22)) and $x_i^{p1}(\lambda)$ (which is the particular solution of (22)), with the following initial conditions is obtained by using the 4th order Runge–Kutta method:

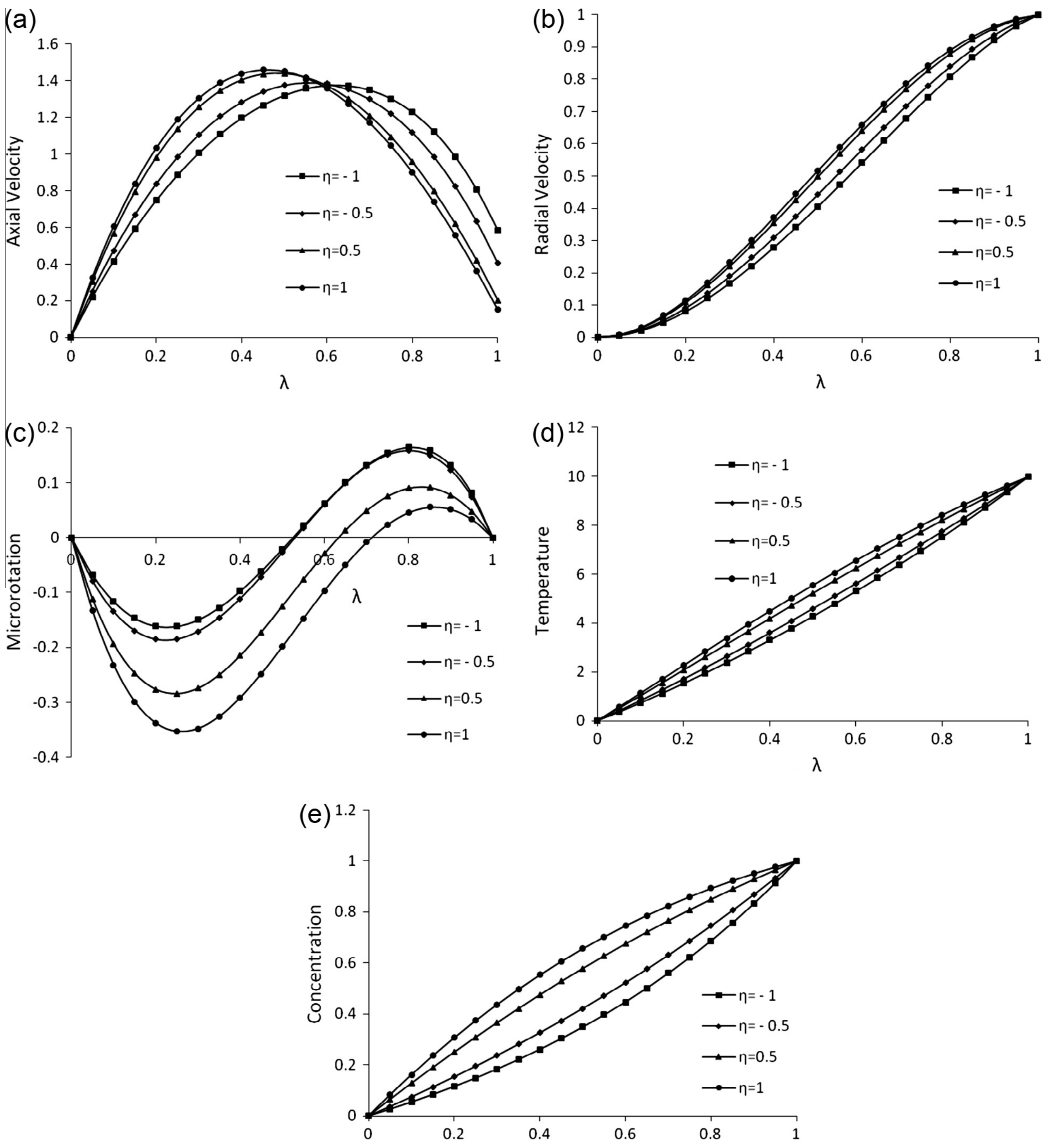

Fig. 5 Effect of η on (a) axial velocity, (b) radial velocity, (c) microrotation, (d) temperature and (e) concentration for Kr = 2, $J_1 = 0.2$, βe = 0.2, βi = 0.2, $Sc = 0.2$, $Pr = 0.2$, $Re = -2$, $D^{-1} = 0$, $R = 2$, $s_1 = 2$, $s_2 = 2$, Sl = 0.2, $Ha = 2$.

$$x_3^{h1}(0) = 1, \quad x_i^{h1}(0) = 0 \text{ for } i \neq 3,$$

$$x_4^{h2}(0) = 1, \quad x_i^{h2}(0) = 0 \text{ for } i \neq 4,$$

$$x_6^{h3}(0) = 1, \quad x_i^{h3}(0) = 0 \text{ for } i \neq 6,$$

$$x_8^{h4}(0) = 1, \quad x_i^{h4}(0) = 0 \text{ for } i \neq 8$$

$$x_{10}^{h5}(0) = 1, \quad x_i^{h5}(0) = 0 \text{ for } i \neq 10$$

$$x_{12}^{h6}(0) = 1, \quad x_i^{h6}(0) = 0 \text{ for } i \neq 12$$

$$x_{14}^{h7}(0) = 1, \quad x_i^{h7}(0) = 0 \text{ for } i \neq 14$$

$$x_1^{p1}(0) = 0,$$

$$x_2^{p1}(0) = x_3^{p1}(0) = x_4^{p1}(0) = x_5^{p1}(0) = 0$$

$$x_6^{p1}(0) = x_7^{p1}(0) = x_8^{p1}(0) = x_9^{p1}(0) = x_{10}^{p1}(0) = x_{11}^{p1}(0) = x_{12}^{p1}(0) = x_{13}^{p1}(0) = x_{14}^{p1}(0) = 0 \quad (23)$$

By using the principle of superposition, the general solution can be written as

$$x_i^{n+1}(\lambda) = C_1 x_i^{h1}(\lambda) + C_2 x_i^{h2}(\lambda) + C_3 x_i^{h3}(\lambda) + C_4 x_i^{h4}(\lambda) + C_5 x_i^{h5}(\lambda) + C_6 x_i^{h6}(\lambda) + C_7 x_i^{h7}(\lambda) + x_i^{p1}(\lambda) \quad (24)$$

where $C_1, C_2, C_3, C_4, C_5, C_6$ *and* C_7 are the unknown constants and are determined by considering the boundary conditions at $\lambda = 1$. This solution (x_i^{n+1}, $i = 1, 2, \ldots, 14$) is then compared with solution at the previous step (x_i^n, $i = 1, 2, \ldots, 14$) and further iteration is performed if the convergence has not been achieved.

4. Results and discussions

The numerical results for the nondimensional velocity components, microrotation, temperature distribution and concentration are calculated correct to six places of decimal for various parameters in the domain [0, 1] (see Fig. 1).

The Hall and ion slip parameters took place due to the strong magnetic field which is applied in the Z-direction. The effect of Hall parameter βe on velocity components, microrotation and temperature distribution is presented in the Fig. 2. From this it is noticed that as βe increases the axial velocity decreases for $0 < \lambda < 0.5$ then increases and the radial velocity and microrotation are decreasing whereas the temperature distribution is increased towards the upper wall. This causes the decrease in the effective conductivity which reduces the damping force on the flow field. Fig. 3 displays the effect of ion slip parameter βi on velocity components, microrotation and temperature. As βi increases the profiles of velocity components, microrotation and temperature follow the opposite trend of βe. The effect of velocity slip parameter 'Sl' on velocity components, microrotation and temperature is presented in the Fig. 4. It is observed that as 'Sl' increases the axial velocity decreases near the upper wall. However, the radial velocity, microrotation and temperature have followed the similar trend of βe. It is due to the fact that the specific permeability decreases the velocity components and microrotation of the fluid. The effect of wall expansion ratio η on velocity components, microrotation, temperature and concentration is presented in the Fig. 5. As η increases the axial velocity also increases between $\lambda = 0$ and $\lambda = 0.5$ and then decreases.

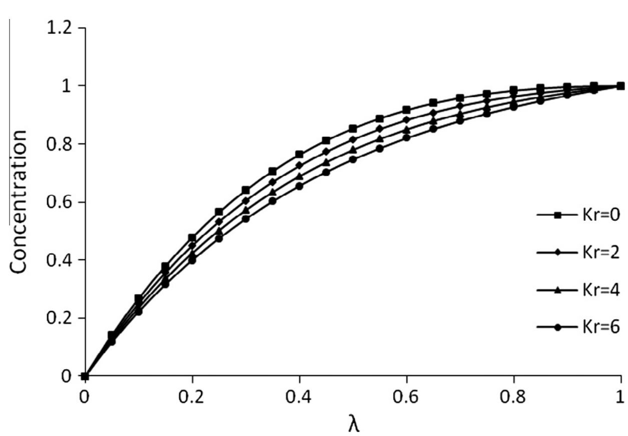

Fig. 6 Effect of Kr on Concentration for $\eta = 2$, $J_1 = 0.2$, βe = 0.2, βi = 0.2, $Sc = 0.22$, $Pr = 0.2$, $Re = -2$, $D^{-1} = 0$, $R = 2$, $s_1 = 2$, $s_2 = 2$, Sl = 0.2, $Ha = 2$.

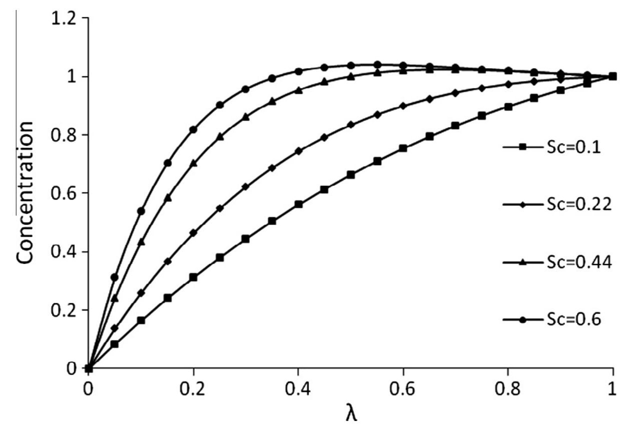

Fig. 7 Effect of Sc on concentration for $\eta = 2$, $J_1 = 0.2$, βe = 0.2, βi = 0.2, Kr = 1, $Pr = 0.2$, $Re = -2$, $D^{-1} = 0$, $R = 2$, $s_1 = 2$, $s_2 = 2$, Sl = 0.2, $Ha = 2$.

However, the radial velocity, temperature and concentration are increased towards the upper wall whereas the microrotation is decreased. Since η is directly proportional to the distance between the walls which is the function of time, as the distance increases the velocity in the Y-direction is also increased. The influence of chemical reaction rate Kr and Schmidt number Sc on concentration is shown in Figs. 6 and 7 respectively. It is observed that the concentration increases as Sc increases, whereas it decreases as Kr increases. This is because the decrease in mass diffusion rate increases the chemical reaction rate and consequently the concentration of the fluid decreases. Since Sc is proportional to the kinematic viscosity, the concentration of the fluid is enhanced with viscosity.

5. Conclusions

The effects of chemical reaction, Hall and ion slip currents on MHD flow and heat transfer of micropolar fluid with expanding or contracting walls and velocity slip are considered. The reduced governing equations are solved numerically by the quasilinearization method. The results are analyzed through graphs for various values of fluid and geometric parameters and from these we conclude that:

- The velocity slip and ion slip effects are similar for velocity components, microrotation and temperature distribution.
- When the walls are expanding or contracting the radial velocity, temperature and concentration of the fluid are increased whereas microrotation is decreased.
- The concentration of the fluid increases with viscosity of the fluid.
- The chemical reaction rate reduces the concentration of the fluid.

References

[1] Eringen AC. Theory of micropolar fluids. J Math Mech 1966;16:1–18.
[2] Si Xinhui, Zheng Liancun, Chen Xuehui, Zhang Xinxin, Cao Limei, Li Min. The effects of slip velocity on a micropolar fluid through a porous channel with expanding or contracting walls. Comput Meth Biomech Biomed Eng 2012;17(4):423–32.

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[3] Si Xinhui, Zheng Liancun, Zhang Xinxin, Chao Ying. The flow of a micropolar fluid through a porous channel with expanding or contracting walls. Cent Eur J Phys 2011;9(3):825–34.
[4] Si Xinhui, Zheng Liancun, Lin Ping, Zhang Xinxin, Zhang Yan. Flow and heat transfer of a micropolar fluid in a porous channel with expanding or contracting walls. Int J Heat Mass Transfer 2013;67:885–95.
[5] Srinivasacharya D, Srinivasacharyulu N, Odelu Ojjela. Flow and heat transfer of couple stress fluid in a porous channel with expanding and contracting walls. Int Commun Heat Mass Transfer 2009;36:180–5.
[6] Gabriel Magalakwe, Chaudry Masood Khalique. Symmetry methods of flow and heat transfer between slowly expanding or contracting walls. Math Prob Eng 2013;2013:1–10.
[7] Asghar S, Mushtaq M, Kara AH. Exact solutions using symmetry methods and conservation laws for the viscous flow through expanding–contracting channels. Appl Math Model 2008;32:2936–40.
[8] Joseph Majdalani, Chong Zhou, Dawson Christopher A. Two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability. J Bio Mech 2002;35:1399–403.
[9] Bujurke NM, Pai NP. Computer extended series solution for unsteady flow in a contracting or expanding pipe. IMA J Appl Math 1998;60(2):151–65.
[10] Uchida, Akoi H. Unsteady flow in a semi infinite contracting or expanding pipe. J Fluid Mech 1977;82(2):371–87.
[11] Srinivas S, Subramanyam Reddy A, Ramamohan TR. A study on thermal-diffusion and diffusion-thermo effects in a two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability. Int J Heat Mass Transfer 2012;55:3008–20.
[12] Subramanyam Reddy A, Srinivas S, Ramamohan TR. Analysis of heat and chemical reaction on an asymmetric laminar flow between slowly expanding or contracting walls. Heat Transfer—Asian Res 2012;42(5):422–43.
[13] Saeed Dinarvand, Mohammad Mehdi Rashidi, Ahmad Doosthoseini. Analytical approximate solutions for two-dimensional viscous flow through expanding or contracting gaps with permeable walls. Cent Eur J Phys 2009;7(4):791–9.
[14] Odelu Ojjela, Naresh Kumar N. Unsteady MHD flow and heat transfer of micropolar fluid in a porous medium between parallel plates. Can J Phys 2015;93(8):880–7.
[15] Odelu Ojjela, Naresh Kumar N. Unsteady heat and mass transfer of chemically reacting micropolar fluid in a porous channel with Hall and ion slip currents. Int Scholarly Res Notices 2014;2014:1–11.
[16] Motsa SS, Shateyi S. The effects of chemical reaction, Hall, and ion-slip currents on MHD micropolar fluid flow with thermal diffusivity using a novel numerical technique. J Appl Math 2012;2012:1–30.
[17] Ziabakhsh Z, Domairry G. Homotopy analysis solution of micropolar flow in a porous channel with high mass transfer. Adv Theor Appl Mech 2008;1(2):79–94.
[18] Satya Narayana PV, Sravanthi CS. Simultaneous effects of soret and ohmic heating on MHD free convective heat and mass transfer flow of micropolar fluid with porous medium. Elixir J Appl Math 2012;48:9379–86.
[19] Uddin Z, Kumar M. Hall and ion-slip effect on MHD boundary layer flow of a micropolar fluid past a wedge. Sci Iran B 2013;20(3):467–76.
[20] Abo-Eldahab Emad M, El Aziz Mohamed A. Viscous dissipation and Joule heating effects on MHD-free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ion-slip currents. Appl Math Model 2005;29:579–95.
[21] Ferdows M, Esrat Jahan, Hamad MM, Masahiro OTA. Effects of Hall and ion-slip currents on free convective heat transfer flow past a vertical plate considering slip conditions. Can J Sci Eng Math 2011;2(2):70–6.
[22] Elgazery Nasser S. The effects of chemical reaction, Hall and ion-slip currents on MHD flow with temperature dependent viscosity and thermal diffusivity. Commun Nonlinear Sci Numer Simul 2009;14(4):1267–83.
[23] Das K. Slip effects on heat and mass transfer in MHD micropolar fluid flow over an inclined plate with thermal radiation and chemical reaction. Int J Numer Meth Fluid Flow 2012;70(1):96–113.
[24] Dulal Pal, Babulal Talukdar, Shivskumara IS, Vajravelu K. Effects of Hall current and chemical reaction on oscillatory mixed convection-radiation of a micropolar fluid in a rotating system. Chem Eng Commun 2012;199(8):943–65.
[25] Bakr AA. Effects of chemical reaction on MHD free convection and mass transfer flow of a micropolar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference. Commun Nonlinear Sci Numer Simul 2011;16(2):698–710.
[26] Hamidreza Najafi, Behzad Najafi, Pooya Hoseinpoori. Energy and cost optimization of a plate and fin heat exchanger using genetic algorithm. Appl Therm Eng 2011;31:1839–47.
[27] Hajmohammadi Mohammad Reza, Sadegh Poozesh, Reza Hosseini. Radiation effect on constructal design analysis of a T-Y-shaped assembly of fins. J Therm Sci Technol 2012;7:677–92.
[28] Hajmohammadi MR, Maleki H, Lorenzini G, Nourazar SS. Effects of Cu and Ag nano-particles on flow and heat transfer from permeable surfaces. Adv Powder Technol 2015;26:193–9.
[29] Beavers GS, Joseph DD. Boundary conditions at a naturally permeable wall. J Fluid Mech 1967;30:197–207.
[30] Ramos JI. Asymptotic analysis of channel flows with slip lengths that depend on the pressure. Appl Math Comput 2007;188:1310–8.
[31] Tasawar Hayat, Maria Imtiaz, Ahmed Alsaedi, Kutbi Marwan A. MHD three-dimensional flow of nanofluid with velocity slip and nonlinear thermal radiation. J Magn Magn Mater 2015;396:31–7.
[32] Zheng Liancun, Niu Jiajia, Zhang Xinxin, Gao Yingtao. MHD flow and heat transfer over a porous shrinking surface with velocity slip and temperature jump. Math Comput Model 2012;56:133–44.
[33] Hajmohammadi MR, Nourazar SS. On the insertion of a thin gas layer in micro cylindrical Couette flows involving power-law liquids. Int J Heat Mass Transfer 2014;75:97–108.
[34] Hajmohammadi Mohammad Reza, Salman Nourazar S, Salman A. Analytical solution for two-phase flow between two rotating cylinders filled with power law liquid and a micro layer of gas. J Mech Sci Technol 2014;28(5):1849–54.
[35] Zhang TT, Jia L, Wang ZC, Li X. The application of homotopy analysis method for 2-dimensional steady slip flow in microchannels. Phys Lett A 2008;372:3223–7.
[36] Bhatnagar R, Vayo HW, Okunbor D. Application of quasilinearization to viscoelastic flow through a porous annulus. Int J Non-Linear Mech 1994;29(1):13–22.
[37] Hymavathi T, Shanker B. A quasilinearization approach to heat transfer in MHD visco-elastic fluid flow. Appl Math Comput 2009;215:2045–54.
[38] Huang CL. Application of quasilinearization technique to the vertical channel flow and heat convection. Int J Non-Linear Mech 1978;13:55–60.
[39] Hajmohammadi Mohammad Reza, Nourazar Seyed Salman, Ali Habibi Manesh. Semi-analytical treatments of conjugate heat transfer. J Mech Eng Sci 2012;227:492–503.
[40] Hajmohammadi MR, Nourazar SS. On the solution of characteristic value problems arising in linear stability analysis; semi analytical approach. Appl Math Comput 2014;239:126–32.
[41] Khan Zafar H, Gul Rahim, Khan Waqar A. Effect of variable thermal conductivity on heat transfer from a hollow sphere with heat generation using homotopy perturbation method. In: ASME 2008 heat transfer summer conference; 2008. p. 301–9.
[42] Gul Rahim, Khan Zafar H, Khan Waqar A. Heat transfer from solids with variable thermal conductivity and uniform internal

heat generation using homotopy perturbation method. In: ASME 2008 heat transfer summer conference; 2008. p. 1–9.

[43] Hajmohammadi Mohammad R, Nourazar SS. Conjugate forced convection heat transfer from a heated flat plate of finite thickness and temperature-dependent thermal conductivity. Heat Transfer Eng 2014;35:863–74.

[44] Hajmohammadi MR, Moulod M, Joneydi Shariatzadeh O, Campo A. Effect of a thick plate on the excess temperature of iso-heat flux heat sources cooled by laminar forced convection flow; conjugate analysis. Numer Heat Transfer Part A 2014;66:205–16.

[45] Bellman RE, Kalaba RE. Quasilinearization and boundary-value problems. New York: Elsevier publishing Co Inc; 1965.

[46] Lukaszewiczn G. Micropolar fluids – theory and applications. Boston: Burkhauser; 1999.

[47] Sutton GW, Sherman A. Engineering magneto hydrodynamics. NewYork: McGrawhill; 1965.

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