Undergraduate Students’ Difficulties in Conceptual Understanding of Derivation

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Abstract

Derivation is one of the fundamental concepts in the learning of university mathematics. It is a prerequisite for other concepts in undergraduate level and its traces are visible in the majority of mathematical courses at university level. Students have difficulties in the learning of this concept which mostly come back to lack of conceptual understanding. The purpose of this study is to investigate the reasons of difficulties, which faced students in conceptual understanding of derivation. The design of this study is qualitative analysis of open-ended questions, and its subjects consisted of 63 undergraduate students. The findings showed students have serious difficulties in understanding conceptually of derivation. The students’ responds indicated that main reasons of difficulties in conceptual understanding of derivation come back to focusing on symbolic aspect more than embodied aspect, lack of making logical connection between these aspects, and weakness of dealing with generalized question. Findings of this study provide information to undergraduate instructors and students to overcome learning difficulties of derivation.

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1. Introduction

Derivation is an important concept to further study of university mathematics that is necessary in order to

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learn other concepts in the different fields of studies (Tall, 1992, 1993, 1997, 2004, 2011). Many university courses depend on calculus specifically derivation and its application in the real world (Tall, 2004, 2010, 2011; Metaxas, 2007; Pepper et al., 2012). In other words, tracts of derivation appear in the advanced mathematics and even other subjects. Therefore, learning derivations can be helpful and useful for students in order for them to learn other mathematical courses at university level (Tall, 2010, 2011; Tarmizi, 2010). However, there are some obstacles in the learning of calculus and its concepts such as derivations (Tarmizi, 2010; Tall, 2010, 2012; Pepper et al., 2012).

Derivation is a difficult concept of mathematics for most of undergraduate students (Tall, 1993, 1997, 2011; Willcox and Bounova, 2004; Metaxas, 2007; Tarmizi, 2010; Pepper et al., 2012). Students’ difficulties in the learning of derivation are caused by their lack of conceptual understanding (Tall, 2011). Many researchers (Tall, 1992, 1997, 2012; Stacey, 2006; Metaxas, 2007) noted that students’ conceptual understanding is not sufficient in the learning of derivation.

There are some methods being introduced to support students to overcome their difficulties in the learning of derivation. Researchers endeavor to support students in the learning of this concept by promoting mathematical thinking with or without computer. There is quite an extensive study on promoting mathematical thinking to helps students’ understanding of calculus, especially derivation (Dubinsky, 1991; Schoenfeld, 1992; Tall, 1986, 1995, 2002; Watson and Mason, 1998; Tall and Yudariah, 2009; Gray and Tall, 2001; Mason, 2002; Roselainay, 2008; Mason, Burton and Stacey, 2010; Kashefi et al., 2012). Although some new methods have been invented to support students’ learning in derivations, but according to Yazdanfar (2006), Roknabadi (2007), Aghaee (2007), Ghanbari (2010, 2012) and Azarang (2008; 2012) undergraduate students have serious difficulties such for learning derivations in undergraduate level. However, most of their difficulties come back to conceptual understanding, but the reasons of these difficulties need to be determined for overcoming them.

This research seems to indicate that derivation is generally difficult for students who either have difficulties grasping the calculus concepts. This paper reports on a study of undergraduate students’ errors in calculus, in particular, their errors in performing derivation. Also, the main objective of this study is to know what kinds of difficulties faced students in the learning of derivation, and what reasons make these difficulties.

2. Method

For this study, 63 students were selected from Islamic Azad university of Shiraz based on their availability without using any sampling method. Those 63 students had taken calculus course in the first year of undergraduate level which the course contained derivation and integral. The aims of study were to know which difficulties faced students in the learning of derivation, and what reasons make these difficulties.

To obtain the goals, the selected students were given open ended question through learning derivation. The questions were about conceptual understanding. In other hand, the given questions had potential to indicate students’ difficulties in conceptual understanding. However, researchers tried to see if students had difficulties in the learning of this concept or not, then they attempted to find reasons of difficulties.

In this research, there were 4 questions for conceptual understanding of derivation. The questions of conceptual understanding have been designed based on the definition of conceptual understanding (NAEP, 2003; Haber and Abboud, 2006) and ideas and theories from researchers such as Orton (1983a, 1983b), Tall (2004, 2008) and Mason (2010, 2012) about understanding of derivation conceptually. Also, three experts who were familiar in this case have verified the questions of conceptual understanding in this research.

The researchers wanted to know if there are difficulties such as conceptual understanding among selected students in the learning of derivation. Then, the finding the reasons out which make these difficulties was important due to students’ respond to open ended questions. It means that if there were difficulties, the researchers would try to find the reasons of the difficulties based on students’ responds to open ended questions. The questions are presented in Table 1.

Table 1. Questions of conceptual understanding in derivation

<table>
<thead>
<tr>
<th>Number</th>
<th>Content of Question</th>
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<tbody>
<tr>
<td>1</td>
<td>What are the definitions of relative and absolute maximum and minimum? Show them graphically via figures.</td>
</tr>
<tr>
<td>2</td>
<td>Show that how the turning point can be found by using graph and figure, and assert that how it can be symbolically.</td>
</tr>
<tr>
<td>3</td>
<td>In the equation of motion, what is difference between average of velocity (ΔV) and velocity moment? Can velocity moment be</td>
</tr>
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shown by using average of velocity (ΔV)? Show them via shape and figure.

4 Based on graph and shape of function, how can say that a function is increasing or decreasing?

The questions of derivation were given to students at first. The score of each student has been calculated for derivation. The marks were scored from 20, because in Iran the scores should be from 0 to 20. However, the researcher found out the mark for responding open ended questions of conceptual understanding separately for each student. A quantitative result has been given descriptively to indicate the mean of students’ marks and rates of students who could answer the questions to see if they actually had difficulties or not. Students’ respond to the questions have been analyzed qualitatively to see the quality of reasons which have made difficulties.

3. Findings

The researchers wanted to know if there any difficulty faced students in answering the questions or not. Therefore, they decided to measure students’ marks in conceptual understanding. The mean of students’ scores for conceptual understanding was 8.7 (from 20). It should be noted that the mean of conceptual understanding was calculated by considering all of the related questions in derivation.

The reasons of these lack and weakness should be figured out by going to details and postures of students’ respond. It needs to use qualitative analysis to check and analyze students’ respond question by question for conceptual understanding in derivation to see what reasons made these difficulties. However, some researchers (Hirst, 2002; Kiat, 2005; Yee and Lam, 2008; Cline et al, 2012) applied qualitative analysis to find the reasons of students’ difficulties. So, qualitative analysis is logical method to know the reasons of students’ difficulties in the learning of derivation.

In this section, students’ responds to open ended questions in both conceptual understanding have been perused for derivation. The related responds to questions of conceptual understanding of derivation have been analyzed to see what kind of difficulties prevented students to answer the questions. In addition, to know the roots and natures of difficulties are goals of doing qualitative analysis in this investigation.

For derivation, there were 4 questions in conceptual understanding. Each question has been analyzed for all of the students who took part in this study.

For question 1, 42 students could not answer it at all. 7 students have written the meaning of the words maximum and minimum. They defined that maximum means the highest range and minimum means lowest range. They did not pay attention to the meaning of these expressions in the calculus. In other words, they were not able to make connection between the meaning of these expressions and their interpretations in the calculus. In addition, they did not use figure to show what maximum and minimum can be. However, 4 students have given right definition of maximum and minimum in calculus. They defined these terms without using figures and graphs. Therefore, 11 students defined the mentioned terms so that 7 persons of them only defined lexical without connecting to calculus. While, 4 of them defined the terms truly in calculus. It should be noted that all of these 11 students have not used figures and graphs to show maximum and minimum. An example of these students’ responds has been shown in Figure 1. It should be noted that the translations of figures which written in Persian have been given in appendix in English.

Fig. 1. A student defined absolute maximum as highest and absolute minimum as lowest point and relative maximum and minimum should be selected based on ordering bigger to smaller
6 students responded this question by using only figure and graph. They could not connect graph of maximum and minimum to their algebraic aspect. In other words, those students were able to interpret these terms based on their shapes and figures. The Figure 2 presents an example of this kind of responds.

![Figure 2](image)

**Fig. 2.** The maximum and minimum have been shown by graph, and the absolute and relative extremes have been remarked in an interval

Only 4 students gave complete answer to this question. They could use both algebraic and graphical aspects to answer the question. They have shown maximum and minimum via graphs. Also, they were able to interpret the properties of maximum and minimum of graphs symbolically. Their answers were the best among 63 students. In figure 3, a model of this kind of responds is available to see what happen in the complete answering of this question.

To answer question 2, 37 students did not write anything about it. 7 students tried to show the meaning of turning point with figure, but they answered in wrong way. It means that their answers were incorrect to show this term via figure and graph. Meanwhile, 14 students were able to respond this question with giving definition only without using any figure. They just wrote the answer in sentences. In contrast, 5 students have responded this question with giving definition, figure and algebraic aspect of turning point. In fact, this kind of answer is the best and the most complete answer to this question. In the Figure 4, all of three kinds of responds have been demonstrated to show what happen in the answering of this question.

![Figure 3](image)

**Fig. 3.** (a) There is a presentation of turning point, but in wrong way. The posture of graph is not clear and it seems that there was no confidence in sketching it. (b) A student explained that to reach turning point the roots of second derivation should be obtained, they can be the turning point if the sign of function are changed around these roots. (c) There is a complete explanation of how can be turning point, and how it should be obtained algebraically. Also, the posture of turning point is shown graphically.

29 students did not answer question 3 at all. They left this question without answering it. 18 students of 63 have responded about average of velocity. They did not mention to velocity moment. Those 18 students asserted the
formula of average velocity which appeared as \( \overline{v} = \frac{v_2 - v_1}{t_2 - t_1} \). However, there was not any indication to velocity moment. 

There was no difference between average of velocity and velocity moment in the responds of 10 students for this question. They answered to the question similar to 18 students who have been mentioned earlier in the previous paragraph, but they believed that these two kinds of velocity are same as each other. Eventually, 6 students have been written the meaning of both average and moment velocity. They could demonstrate the differences between these two types of velocity. Also, they mentioned to the role of derivation in finding moment velocity from average velocity. Briefly, nobody mentioned the utilization of figures and shapes to show the posture of two kinds of velocities.

In the answering question 4, there was not any student who did not respond this question. All the 63 students tried to answer it. About 24 students mentioned to the meaning of increasing and decreasing. They knew the increasing is opposite of deceasing, and their interpreting of these terms was lexical. Those students (24 students) did not display any attempt to show how the increasing and decreasing can be in calculus especially derivation.

21 students tried to show the quality of distinguishing the expressions of increasing and decreasing in calculus. They illustrated that how can diagnose these terms by using derivation. It seems that they knew the way of using derivation to find the mentioned expressions in calculus algebraically. However, they were careless to answer the question based on graphs and shape of a function. In fact, they did not pay attention to the first words of this question. In the Figure 5, an example of these students’ answers is offered to see how the answers looked like for the students.

![Fig. 4. The student wrote that if the sign of derivative function is negative the function is decreasing and if the sign is positive the function is increasing. Based on it answer, it can be founded by using second derivation formula.](image1.png)

In contrast to the students who used algebraic aspect merely, only 11 students have used only figure and shape to show the increasing and decreasing of functions. There was not any trace of algebraic aspect in the answers of those students. They did not display using both graphical and algebraic aspects by making relationship between them.

Finally, 7 students have answered this question by mentioning to graphical and algebraic aspects and made a logical connection between these aspects. They remarked that how the increasing and decreasing could be in the calculus, and how derivation can be used to show where a function is increasing or decreasing. It can be told that their answers were the ideal for this question. In the Figure 6, a model of answers is displayed to see the circumstance of this kind of responds among those 6 students.

![Fig. 5. Based on given answer, the function is decreasing if first and second derivations are positive and function is increasing if the sign are positive.](image2.png)

4. Discussion

The results of calculating students’ marks indicated that students have difficulties in the learning of derivation. The weakness of responding the questions has been found out for conceptual understanding in derivation. The means of students’ marks in conceptual understanding (8.7) verified that there are serious difficulties which faced students in the learning of derivation.

It means that there was a lack of conceptual understanding among undergraduate students who took part in this
Several reasons have been remarked through this analysis for derivation such as not using both graphical and algebraic aspects in a same time, weakness of making relationship between these aspects and focusing on algebraic aspect more than graphical aspect. It seems that most of students like to attempt on the algebraic aspect of derivation to understand its notions conceptually. In addition, students have difficulties to deal with general postures of derivation’s concepts based on the results of qualitative analysis.

Finally, the difficulties of conceptual understanding of derivation can be summarized in two main reasons. First, students do not pay attention to importance of both embodied and symbolic aspects of derivation in the learning of this concept. They focus on symbolic aspect more than embodied or graphical aspect. For instance, they don’t like to use geometrical interpretation in the learning of this concept and even in their responds to the open ended questions. Second, there is a lack of making connection and relationship between embodied and symbolic aspect. Although some students in this investigation tried to make connection between these aspects, they could not able enough to make the connection in right way.

5. Conclusion

The qualitative analysis remarked some reasons for these difficulties. In conceptual understanding, the reasons involved focusing in symbolic aspect rather than graphical aspect, and the weakness of making connection between graphical and symbolic aspects of the concepts.

It would appear that there is a strong necessity to find suitable strategy to cover most of these difficulties for improving students’ conceptual understanding and problem solving abilities within the learning of derivation and integral. The results of this analysis will help future researchers to select and design effectiveness learning techniques to overcome the mentioned difficulties.

It seems that there is a strong necessity to find suitable strategy to cover these difficulties for improving students’ conceptual understanding abilities in the learning of derivation. The results of this analysis can help to select and design effectiveness learning strategy to rectify the mentioned difficulties.

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References


The figures were given in Persian language because the participants were from Iran. In following, the translation of each figure has been demonstrated based on students’ responses.

Appendix

The appendix involved translation of students’ responses which have been presented through figures of the study. The figures were given in Persian language because the participants were from Iran. In following, the translation of each figure has been demonstrated based on students’ responses.
Figure 1:

The biggest number is absolute maximum and the lowest number is absolute minimum. Relative maximum and minimum are founded based on ordering obtained numbers between max and min from bigger to smaller.

Figure 2:

Figure 3:
We take derivative from given function, and find the roots of derivative function. We replaced the roots and boundaries of interval in main function. We compare the answers to know which one is absolute max or min and which one is relative.

\[ f'(x) = 0 \rightarrow x = \text{roots} \]
\[ f(a) = \ldots \]
\[ f(b) = \ldots \]
\[ f(\text{roots}) = \ldots \]

Figure 4:

a)

b)
For turning point, the roots of second derivative function should be obtained. By replacing a root in the main function, if the sign of function changed around the root, this root is length of turning point of function.

c)
The function is increasing from a to b, and it is decreasing from b to c. In the turning points, the direction of function should be changed. Therefore, the points such as b and c are turning points. \[ f''(x) = 0 \rightarrow x = a \Rightarrow (a, f(a)) \] is turning point if the sign of function changed around it.

Figure 5:

If the sign of derivative function is positive, it is increasing. The function is decreasing if the sign is negative. Let
$y = f(x), f'(x) \Rightarrow \begin{cases} \text{if } f' < 0, \text{ is decreasing} \\ \text{if } f' > 0, \text{ is increasing} \end{cases}$, however, by using $f''$ it is recognizable.

**Figure 6:**
If the concavity of function is going down, the function is decreasing. It is increasing if its concavity going up. $f'$ and $f'' < 0$ decreasing, $f'$ and $f'' > 0$, increasing.