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ORIGINAL ARTICLE

Effects of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over an exponentially stretching surface with suction in the presence of internal heat generation/absorption

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Thermal radiation;
Internal heat generation/absorption;
Suction

Abstract In this paper, the problem of unsteady laminar two-dimensional boundary layer flow and heat transfer of an incompressible viscous fluid in the presence of thermal radiation, internal heat generation or absorption, and magnetic field over an exponentially stretching surface subjected to suction with an exponential temperature distribution is discussed numerically. The governing boundary layer equations are reduced to a system of ordinary differential equations. New numerical method using *Mathematica* has been used to solve such system after obtaining the missed initial conditions. Comparison of obtained numerical results is made with previously published results in some special cases, and found to be in a good agreement.

1. Introduction

The problem of mixed convection flow and heat transfer over a continuously moving surface has interesting numerous industrial applications such as hot rolling, paper production, wire drawing, glass fiber production, aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film, condensation process of metallic plate in a cooling bath and glass, and also in polymer industries.

Sakiadis [1] was the first to study the boundary layer flow on continuous solid surfaces. He derived the basic differential and integral momentum equations for such surfaces. Ali [2] obtained the similarity solutions of the laminar boundary layer equations describing heat and flow in quiescent fluid driven by a stretched surface subject to suction or injection. The surface is moving with a power-law velocity distribution and its temperature has a power-law variation. On the other hand, Gupta and Gupta [3] have analyzed the stretching problem with a constant surface temperature.

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Nomenclature

A	unsteadiness parameter ($=\alpha/U_0$) (–)
B_0	magnetic field strength (A m^{-1})
C_f	local skin friction coefficient $\left(=2\tau_w/\rho U_\infty^2\right)$ (–)
c_p	specific heat due to constant pressure (m^2 s^{-2} K^{-1})
f	dimensionless stress functions (–)
f_0	suction parameter (–)
g	gravitational acceleration (m s^{-2})
k	mean absorption coefficient (m^{-1})
L	reference length (m)
M	magnetic parameter $\left(=\sigma B_0^2(1-\alpha t)/U_0\rho\right)$ (–)
N_R	thermal radiation parameter $\left(=16\sigma_s T_\infty^3/3\kappa k\right)$ (kg m^{-2})
Nu_x	Nusselt number (–)
Pr	Prandtl number ($=\rho c_P v/\kappa$) (–)
q_r	radiation heat flux (kg m^{-2})
Q_0	heat source or sink (kg m^{-1} s^{-3} K^{-1})
Re_x	local Reynolds number ($=L\ U_w/v$) (–)
T	temperature of fluid (K)
t	time (s)
T_w	temperature distribution of the stretching surface (K)
T_0	temperature of fluid adjacent to the stretching surface (K)
T_∞	temperature of the free stream (K)
u	velocity of fluid along the x-axis (m s^{-1})
U_w	velocity of stretching surface (m s^{-1})
U_0	reference velocity (m s^{-1})
v	velocity of fluid along the y-axis (m s^{-1})
V_w	velocity of suction (m s^{-1})
X	dimensionless coordinate ($=x/L$) (–)
x	Cartesian coordinate along the surface (m)
y	Cartesian coordinate along the normal to the surface (m)

Greek letters

α	positive constant (s^{-1})
β	volumetric coefficient of thermal expansion (K^{-1})
δ	heat generation or absorption parameter ($=Q_0(1-\alpha t)/U_0\rho c_p$) (–)
η	similarity variable (-)
θ	similarity temperature function (-)
κ	thermal conductivity (kg m s^{-3} K^{-1})
λ	permeability parameter $\left(=g\beta T_0/U_0^2\right)$ (-)
μ	dynamic viscosity of fluid ($=\kappa/\rho c_p$) (kg m^{-1} s^{-1})
v	kinematic viscosity of the fluid ($=\mu/\rho$) (m^2 s^{-1})
ρ	density of fluid (kg m^{-3})
σ	electrical conductivity (Ω^{-1} m^{-1})
σ_s	Stefan–Boltzman constant (kg m^{-2} K^{-4})
τ_w	skin friction (kg m^{-1} s^{-2})
ψ	stream function (m^2 s^{-1})

Superscript

$'$	differentiation with respect to η

Subscripts

w	stretching surface conditions
0	fluid conditions adjacent to the stretching surface
∞	fluid conditions far away from the stretching surface

Andersson et al. [4] analysed the momentum and heat transfer in a laminar liquid film on a horizontal stretching sheet where the governing time dependent boundary layer equations are reduced to a set of ordinary differential equations by means of an exact similarity transformation. Ishak et al. [5] studied the unsteady laminar boundary layer flow over a continuously stretching surface in a viscous and incompressible quiescent fluid where the unsteadiness in the flow and temperature fields is caused by the time dependence of the stretching velocity and the surface heat flux. Heat transfer over an unsteady stretching surface with internal heat generation or absorption was studied by Elbashbeshy and Bazid [6] where the governing time dependent boundary layer equations are solved numerically. Elbashbeshy and Emam [7] studied the effects of thermal radiation and heat transfer over an unsteady stretching surface embedded in a porous medium in the presence of heat source or sink.

Sharma and Singh [8] investigated the unsteady two-dimensional flow of viscous incompressible fluid about a stagnation point on a stretching sheet in the presence of time dependent free stream. The effects of thermal radiation and magnetic field on unsteady boundary layer mixed convection flow and heat transfer problem from a vertical porous stretching surface was investigated by Elbashbeshy and Aldawody [9]. The authors in Refs. [8,9] transformed the governing equations of motion and energy into non-linear ordinary differential equations which were solved numerically using shooting method.

Magyari and Keller [10] investigated numerically the similarity solutions of the steady thermal boundary layer on an exponentially stretching continuous surface with an exponentially temperature distribution. Elbashbeshy [11] examined numerically the similarity solutions of the laminar boundary layer equations describing heat and flow in a quiescent fluid driven by an exponentially stretching surface subject to suction. The problem of steady laminar two-dimensional boundary layer flow and heat transfer of an incompressible viscous fluid with a presence of thermal radiation over an exponentially stretching sheet is investigated numerically using the Keller-box method by Bidin and Nazar [12]. The thermal boundary layer on an exponentially stretching continuous surface with an exponential temperature distribution in the presence of the magnetic field effect is investigated numerically by Al-odat *et al*. [13]. The effect of radiation on the boundary layer flow and heat transfer of a viscous fluid over an exponentially stretching sheet is studied by Sajid and Hayat [14].

The steady magneto-hydrodynamic (MHD) flow of a second grade fluid in the presence of radiation is analyzed by Hayat et al. [15]. The problem of magneto-hydrodynamic mixed convective flow and heat transfer of an electrically conducting, power-law fluid past a stretching surface in the presence of heat generation/absorption and thermal radiation has been analyzed by Chen [16]. An analytic technique, namely, the homotopy analysis method, is applied by Liao [17] to give a series solution of the unsteady boundary-layer flow over an impermeable stretching plate. Ahmad et al. [18] provides an analytic solution for the problem of unsteady axisymmetric flow of a second-grade fluid over a radially stretching sheet. The problem of unsteady boundary layer flow of a second grade over a stretching sheet is investigated by Sajid et al. [19].

In this paper, we discuss numerically the effect of thermal radiation and magnetic field on the unsteady laminar two-dimensional boundary layer flow and heat transfer in the presence of internal heat generation or absorption over an exponentially stretching surface subjected to suction with an exponential temperature distribution. The present study may be regarded as an extension of Elbashbeshy [11].

2. Mathematical formulation

Consider an unsteady two-dimensional mixed convection boundary layer flow of an incompressible viscous fluid along a stretching surface. The x-axis is taken along the stretching surface in the direction of motion and the y-axis is perpendicular to it as shown in Fig. 1. The stretching surface has the velocity $U_w(x,t) = U_0\, e^{x/L}/(1-\alpha t)$ and the temperature distribution $T_w(x,t) = T_\infty + [T_0 e^{x/2L}/(1-\alpha t)^2]$ where U_0 is the reference velocity, α is a positive constant with dimension reciprocal time, L is the reference length, t is the time, T_∞ is the fluid temperature far away from the stretching surface and T_0 is the fluid temperature adjacent to the stretching surface. A uniform magnetic field of strength B_0 is applied normally to the stretching surface which produces magnetic effect in the x-axis. If the effect of the induced magnetic field is neglected by taking a small magnetic Reynolds number, the continuity, momentum and energy equations governing such type of flow will be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\left[\kappa\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+Q_0(T-T_\infty)\right], \tag{3}$$

subjected to the boundary conditions

$$\begin{aligned}&u=U_w(x,t),\quad v=-V_w(x,t),\quad T=T_w(x,t)\quad \text{at } y=0,\\ &u\to 0,\quad T\to T_\infty\quad \text{as } y\to\infty,\end{aligned} \tag{4}$$

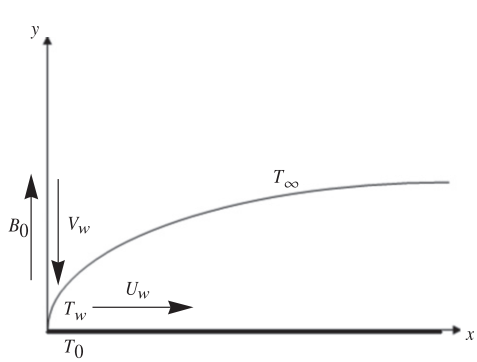

Figure 1 Physical model.

where u and v are the fluid velocity components along x and y axes, respectively, v is the kinematic viscosity, g is the gravity field, β is the volumetric coefficient of thermal expansion, T is the fluid temperature, σ is the electrical conductivity, ρ is the fluid density, κ is the thermal conductivity, c_p is the specific heat at constant pressure, q_r is the radiation heat flux, $V_w(x,t) = f_0(U_0 v/2L(1-\alpha t))^{1/2} e^{x/2L}$ is the velocity of suction ($V_w > 0$), $f_0 \geqslant 0$ is the suction parameter and Q_0 is the uniform volumetric heat generation or absorption.

The radiation heat flux is evaluated by using Rosseland approximation [13] to be

$$q_r=-\frac{4\sigma_s}{3k}\frac{\partial T^4}{\partial y}, \tag{5}$$

where σ_s is the Stefan–Boltzman constant and k is the absorption coefficient. T^4 may be linearly expanded in a Taylor's series about T_∞ to get

$$T^4=4T_\infty^3 T-3T_\infty^4. \tag{6}$$

Substituting Eqs. (5) and (6) into Eq. (3) to get

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\left[\left(\kappa+\frac{16\sigma_s T_\infty^3}{3k}\right)\frac{\partial^2 T}{\partial y^2}+Q_0(T-T_\infty)\right]. \tag{7}$$

The equation of continuity is satisfied if we choose a stream function $\psi(x,y)$ such that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Also, the following similarity transformations

$$\eta=\sqrt{\frac{U_0}{2vL(1-\alpha t)}}e^{x/2L}y, \tag{8}$$

$$\psi(x,y)=\sqrt{\frac{2U_0 vL}{(1-\alpha t)}}e^{x/2L}f(\eta), \tag{9}$$

$$T=T_\infty+\frac{T_0}{(1-\alpha t)^2}e^{x/2L}\theta(\eta), \tag{10}$$

will be substituted into Eqs. (2) and (7) to obtain the following set of ordinary differential equations:

$$f'''+ff''-2f'^2-Le^{-X}(A(2f'+\eta f'')+2Mf'-2\lambda e^{-X/2}\theta)=0, \tag{11}$$

$$\theta''+\frac{\Pr}{1+N_R}(f\theta'-f'\theta-Le^{-X}(A(4\theta+\eta\theta')-\delta\theta))=0, \tag{12}$$

with the boundary conditions

$$\begin{aligned}&\eta=0:\ f=f_0,\quad f'=1,\quad \theta'=1,\\ &\eta\to\infty:\ f'\to 0,\quad \theta\to 0,\end{aligned} \tag{13}$$

where the primes denote the differentiation with respect to η, $X = x/L$ is a dimensionless coordinate, $A = \alpha/U_0$ is the unsteadiness parameter, $\lambda = g\beta T_0/U_0^2$ is the permeability parameter, $M = \sigma B_0^2(1-\alpha t)/U_0\rho$ is the magnetic parameter, $\Pr = \rho c_P v/\kappa$ is the Prandtl number, $N_R = 16\sigma_s T_\infty^3/3\kappa k$ is the thermal radiation parameter and $\delta = Q_0(1-\alpha t)/U_0\rho c_p$ is the heat generation ($\delta > 0$) and absorption ($\delta < 0$) parameter.

The physical quantities of interest in this problem is the skin friction coefficient C_f and the local Nusselt number Nu_x which are defined as

Table 1 Comparison of $-\theta'(0)$ for $X = A = \lambda = \delta = f_0 = 0$.

N_R	M	Pr	Elbashbeshy [11]	Bidin and Nazar [12]	Ishak [20]	Present results
0	0	0.72	0.76778			0.76728
		1	0.95478	0.9548	0.9548	0.95478
		2			1.4715	1.47146
		3	1.86907	1.8691	1.8691	1.86907
		5			2.5001	2.50013
		10	3.66037		3.6604	3.66037
0	0.5	1			0.8611	0.86109
2/3		1		0.6765		0.67650
		3		1.3807		1.38075

Table 2 The values of $-f''(0)$ and $-\theta'(0)$ for various values of X, N_R, δ, A, Pr, f_0, λ and M.

Parameters (fixed values)	Parameter (different values)		$-f''(0)$	$-\theta'(0)$
$A = \lambda = \delta = M = 0.1$, $f_0 = 0$, Pr = 0.72, $N_R = 5$	X	0.5	1.29553	0.26039
		1	1.29598	0.24053
		1.5	1.29341	0.22545
		2	1.29042	0.21404
$A = \lambda = \delta = M = 0.1$, $X = f_0 = 0$, Pr = 0.72	N_R	0	1.32269	0.84787
		1	1.30617	0.55593
		3	1.29254	0.3652
		5	1.28608	0.29667
		10	1.27832	0.20109
$A = \lambda = M = 0.1$, $X = f_0 = 0$, Pr = 0.72, $N_R = 5$	δ	−1	1.31253	0.61151
		0.5	1.30457	0.49573
		0	1.29077	0.33348
		0.1	1.28609	0.28667
$\lambda = \delta = M = 0.1$, $X = f_0 = 0$, Pr = 7, $N_R = 3$	A	0.05	1.31607	1.32037
		0.2	1.39486	1.62685
		0.5	1.54084	2.09365
		0.7	1.63066	2.35025
$A = \lambda = \delta = M = 0.1$, $X = f_0 = 0$, $N_R = 5$	Pr	0.72	1.28603	0.28641
		1	1.29121	0.34852
		3	1.31371	0.67922
$A = \lambda = \delta = M = 0.1$, $X = 0$, Pr = 0.72, $N_R = 5$	f_0	0	1.28608	0.28667
		1	1.77622	0.33381
		2	2.43359	0.39598
$A = M = 0.1$, $\delta = -0.5$, $X = f_0 = 0$, Pr = 0.72, $N_R = 5$	λ	0	1.41266	0.48752
		0.1	1.30457	0.49573
		0.2	1.20258	0.50249
		0.3	1.10472	0.50839
$A = 0.3$, $\lambda = 0.1$, $\delta = -0.5$, $X = f_0 = 0$, Pr = 0.72, $N_R = 5$	M	0	1.35149	0.579
		0.2	1.48958	0.57398
		0.4	1.61636	0.56984
		0.6	1.73421	0.5664
		0.8	1.84476	0.56346
		1	1.94919	0.56091

$$C_f = \frac{2\mu(\partial u/\partial y)_{y=0}}{\sqrt{Re_x}}, \quad Nu_x = -\frac{x(\partial T/\partial y)_{y=0}}{T_w - T_\infty}, \quad \frac{1}{2}C_f\sqrt{Re_x} = f''(0), \quad Nu_x/\sqrt{Re_x} = -\theta'(0), \tag{14}$$

where $\mu = \kappa/\rho c_p$ is the dynamic viscosity of the fluid and $Re_x = x\ U_w/v$ is Reynolds number.

3. Numerical solution and discussions

Eqs. (11) and (12) subjected to the boundary condition (13) are converted into the following simultaneous system of first order differential equations as follows:

$$W_1' = W_2, \tag{15}$$

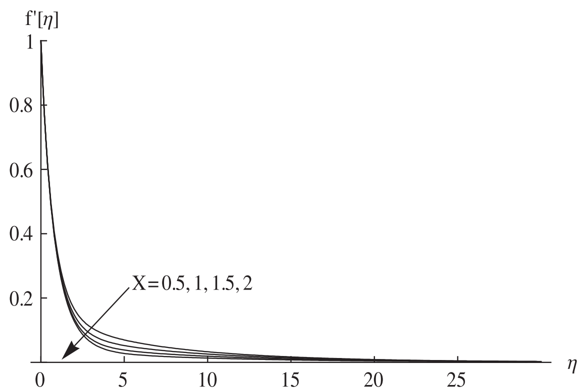

Fig. 2 The velocity profiles for various values of X with $A = \lambda = \delta = M = 0.1, f_0 = 0$, Pr = 0.72 and $N_R = 5$.

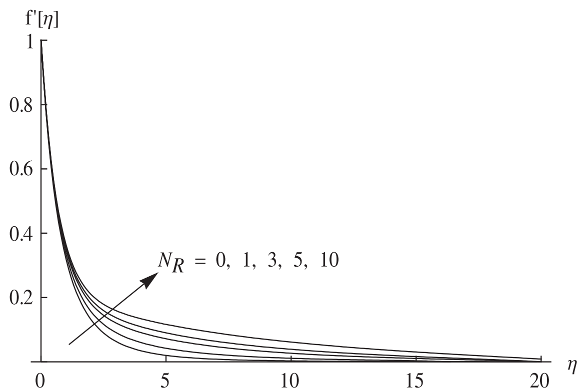

Fig. 3 The velocity profiles for various values of N_R with $A = \lambda = \delta = M = 0.1$, $X = f_0 = 0$ and Pr = 0.72.

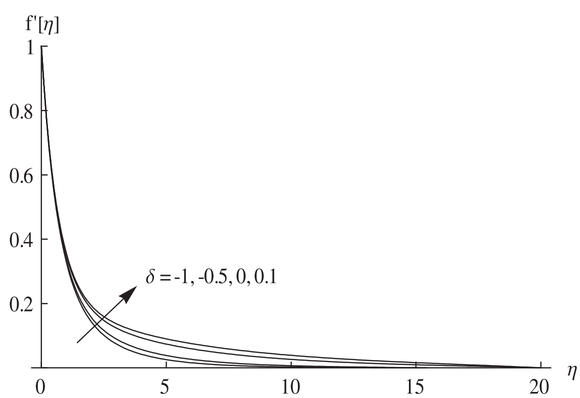

Fig. 4 The velocity profiles for various values of δ with $A = \lambda = M = 0.1$, $X = f_0 = 0$, Pr = 0.72 and $N_R = 5$.

$$W_2' = W_3, \tag{16}$$

$$W_3' = -W_1W_3 + 2W_2^2 + Le^{-X}(A(2W_2 + \eta W_3) + 2MW_2 - \lambda e^{-X/2}W_4), \tag{17}$$

$$W_4' = W_5, \tag{18}$$

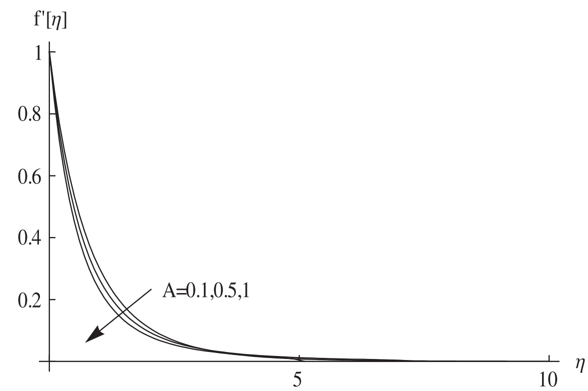

Fig. 5 The velocity profiles for various values of A with $\lambda = \delta = M = 0.1$, $X = f_0 = 0$, Pr = 7 and $N_R = 3$.

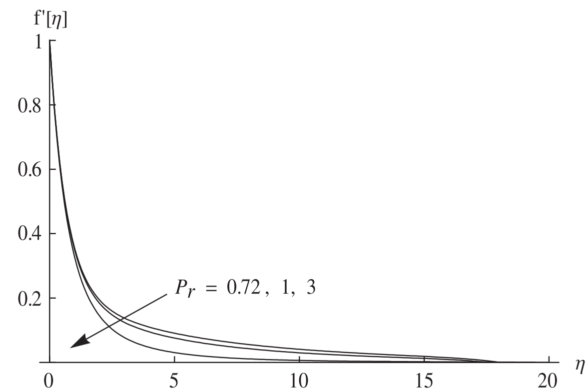

Fig. 6 The velocity profiles for various values of Pr with $A = \lambda = \delta = M = 0.1$, $X = f_0 = 0$ and $N_R = 5$.

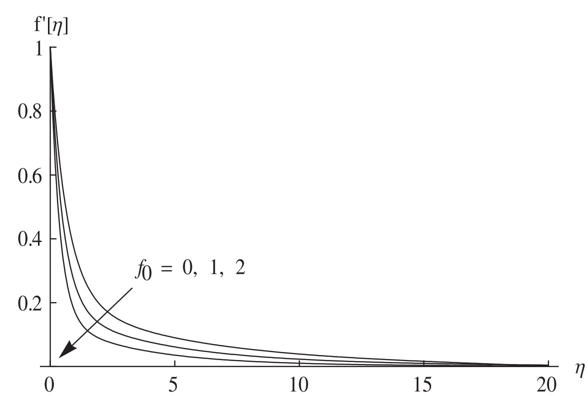

Fig. 7 The velocity profiles for various values of f_0 with $A = \lambda = \delta = M = 0.1$, $X = 0$, Pr = 0.72 and $N_R = 5$.

$$W_5' = \frac{\mathrm{Pr}}{1 + N_R}(-W_1W_5 + W_2W_4 + Le^{-X}(A(4W_4 + \eta W_5) - \delta W_4)), \tag{19}$$

where $W_1 = f$, $W_2 = f'$, $W_3 = f''$, $W_4 = \theta$ and $W_5 = \theta'$.

The initial conditions are

$$W_1(0) = f_0, W_2(0) = 1, W_3(0) = s_1, W_4(0) = 1, W_5(0) = s_2, \tag{20}$$

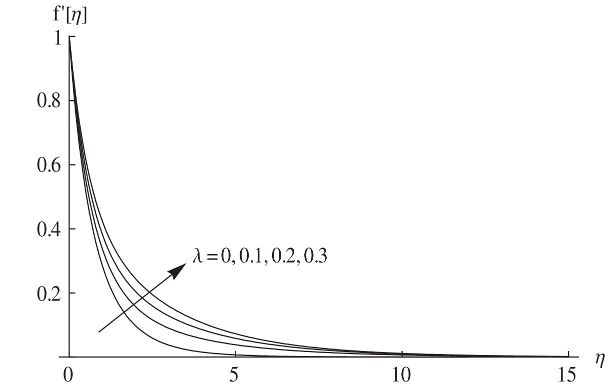

Fig. 8 The velocity profiles for various values of λ with $A = M = 0.1$, $\delta = -0.5$, $X = f_0 = 0$, $\Pr = 0.72$ and $N_R = 5$.

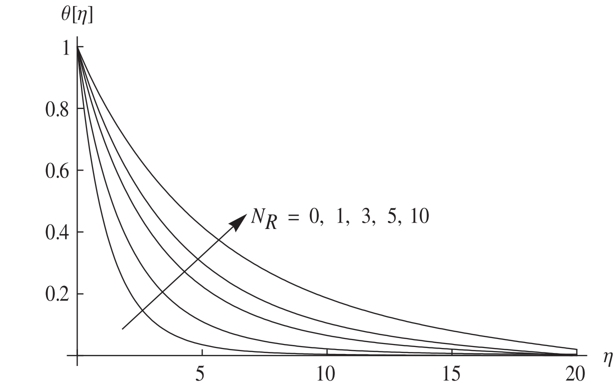

Fig. 11 The temperature profiles for various values of N_R with $X = f_0 = 0$, $A = \lambda = \delta = M = 0.1$ and $\Pr = 0.72$.

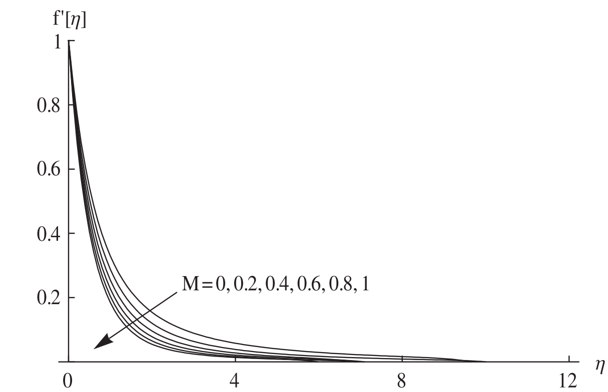

Fig. 9 The velocity profiles for various values of M with $A = 0.3$, $\lambda = 0.1$, $\delta = -0.5$, $X = f_0 = 0$, $\Pr = 0.72$ and $N_R = 5$.

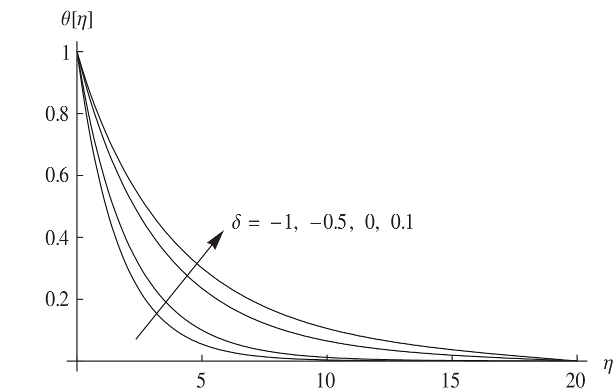

Fig. 12 The temperature profiles for various values of δ with $A = \lambda = M = 0.1$, $X = f_0 = 0$, $\Pr = 0.72$ and $N_R = 5$.

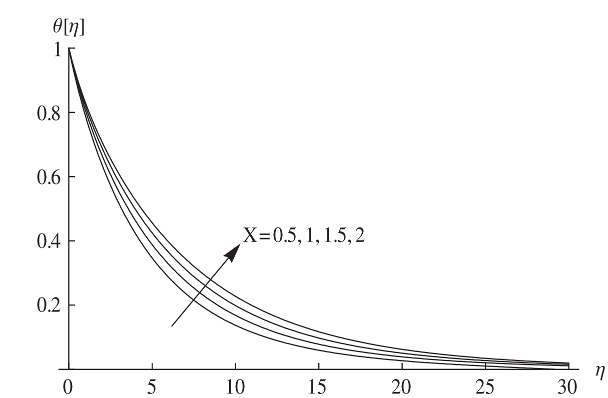

Fig. 10 The temperature profiles for various values of X with $f_0 = 0$, $A = \lambda = \delta = M = 0.1$, $\Pr = 0.72$ and $N_R = 5$.

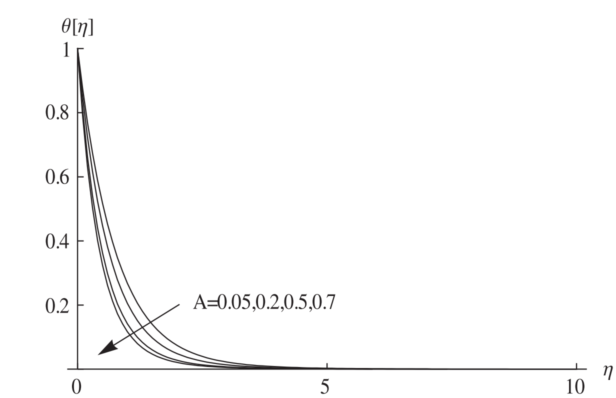

Fig. 13 The temperature profiles for various values of A with $\lambda = \delta = M = 0.1$, $X = f_0 = 0$, $\Pr = 7$ and $N_R = 3$.

where s_1 and s_2 are priori unknowns to be determined as a part of the solution.

By using NDSolve subroutine in Mathematica, we can get a solution for the system Eqs. (15)–(20). The values of s_1 and s_2 are determined upon solving the boundary conditions $W_2(\eta_{\max}) = 0$ and $W_4(\eta_{\max}) = 0$. Once s_1 and s_2 are determined, the system will be closed and can be solved numerically again by NDSolve subroutine to get the final results. Consequently, only one integration path is enough to solve the problem instead of consuming the time with iteration techniques like the shooting method.

The computations have been carried out for various values of the previously defined parameters X, A,λ, M, $\Pr$, N_R, δ and f_0. The accuracy of the numerical scheme is checked out by performing various comparisons at different conditions with previously published papers. The results for the local Nusselt number, $-\theta'(0)$, are compared with those reported in Refs. [11,12] for $X = A = \lambda = \delta = M = f_0 = 0$ with different values for Pr and N_R. The quantitative comparison is shown in Table 1 and found to be in a good agreement. The results for steady exponential surface without suction can be recovered from the present study and found to be in a good agreement with Ishak [20] as shown in Table 1.

It is obvious that the value of skin friction coefficient $f''(0)$ is negative for all values of the different parameters as shown in Table 2. Physically, the negative value of $f''(0)$ means the surface exerts a drag force on the fluid which is suitable for our present problem because the stretching surface will induce the flow. The values of skin friction coefficient $f''(0)$ are increased by increasing the dimensionless coordinate X, the thermal radiation parameter N_R, the heat generation ($\delta > 0$) and absorption ($\delta < 0$) parameter or the permeability parameter λ as shown in Rable 2. On the other hand, these values are decreased by increasing the unsteadiness parameter A, the Prandtl number Pr, the suction parameter f_0 or the magnetic parameter M as shown in Table 2.

The values of the Nusselt number $-\theta'(0)$ are increased by increasing the dimensionless coordinate X, the thermal radiation parameter N_R, or the heat generation ($\delta > 0$) and absorption ($\delta < 0$) parameter as shown in Table 2. On the other hand, these values are decreased by increasing the unsteadiness parameter A, the Prandtl number Pr or the suction parameter f_0, as shown in Table 2. Finally, the values of the local Nusselt number are slightly decreased by increasing the permeability parameter λ and slightly increased by increasing the parameter magnetic M as shown in Table 2.

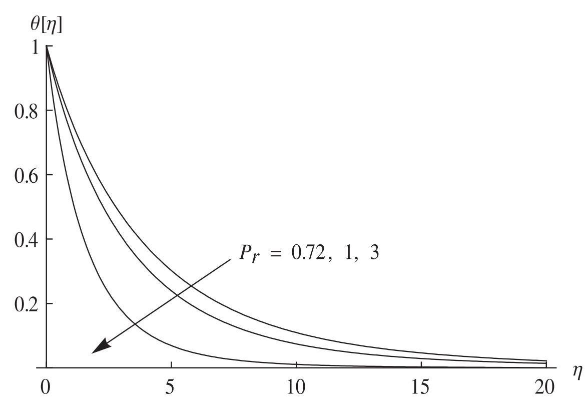

Fig. 14 The temperature profiles for various values of Pr with $X = f_0 = 0$, $A = \lambda = \delta = M = 0.1$ and $N_R = 5$.

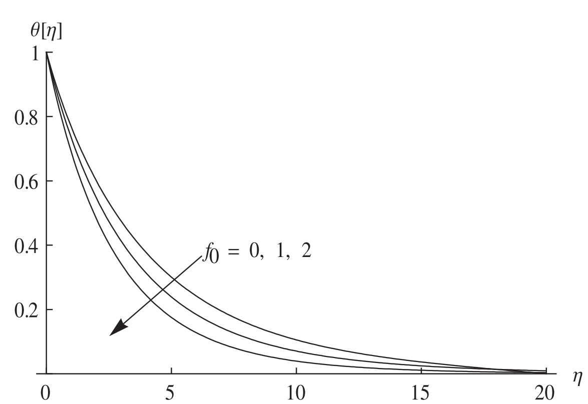

Fig. 15 The temperature profiles for various values of f_0 with Pr = 0.72, $N_R = 5$, $X = 0$ and $A = \lambda = \delta = M = 0.1$.

The velocity profiles for various values of X, N_R, δ, A, Pr, f_0, λ and M are presented in Figs. 2–9, respectively. Also, the temperature profiles for various values of X, N_R, δ, A, Pr and f_0 are presented in Figs. 10–15, respectively. Figs. 5 and 13 show the effect of unsteadiness parameter A on the velocity and temperature profiles, respectively. One can note that the increase of A leads to the thinning of the velocity and temperature boundary layers.

The effect of the thermal radiation is shown in Fig. 3, it is observed that increasing the values of N_R has the tendency to increase the velocity and temperature boundary layers. Also, The effect of the thermal radiation parameter N_R increase is to increase the temperature boundary layer as shown in Fig. 11 which explained by the fact that if the thermal radiation parameter increases, the mean absorption coefficient k will be decreased which in turn increases the divergence of the radiative heat flux. Hence, the rate of radiative heat transferred to the fluid will be increased so that the fluid temperature will be increased.

The increase of heat generation (absorption) coefficient δ tends to increase both the fluid velocity and temperature as shown in Figs. 4 and 12. The effect of the dimensionless coordinate X on the velocity and the temperature of the fluid is shown in Figs. 2 and 10, respectively. The increase of X results in increasing the fluid temperature while the fluid velocity is decreased.

The increase of the magnetic parameter M tends to decrease the fluid velocity as shown in Fig. 9. The increase of the Prandtl number Pr has the effect of decreasing the velocity as well as the temperature of the fluid as shown in Figs. 6 and 14, respectively. Finally, Fig. 7 shows that the fluid velocity decreases with the increase of the suction parameter f_0 while the increase of f_0 decreases the fluid temperature as shown in Fig. 15.

4. Conclusion

The present study may be regarded as an extension of Elbashbeshy [11] . New numerical method using *Mathematica* has been used to solve such system after obtaining the missed initial conditions. Numerical solutions have been obtained for the effects of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over an exponentially stretching surface with suction in the presence of internal heat generation/absorption. An appropriate similarity transforms were used to transform the system of time-dependent partial differential equations to a set of ordinary differential equations which are solved by using Mathematica. Numerical computations show that the present values of the rate of heat transfer are in a great agreement with those obtained by previous investigations. The following results are obtained:

1. The skin friction increases with an increase in the dimensionless coordinate, the thermal radiation, the heat generation/absorption and the permeability parameters while it decreases with an increase in the Prandtl number, the unsteadiness, the suction and the magnetic parameters.

2. The Nusselt number increases with an increase in the dimensionless coordinate, the thermal radiation and the heat generation/absorption parameters while it decreases with an increase in the Prandtl number, the unsteadiness and the suction parameters.
3. The effect of the thermal radiation becomes more significant as the thermal radiation parameter increased because the radiative heat transferred to the fluid will be increased.
4. The effect of the magnetic field becomes more significant as the magnetic parameter increased because the fluid will slow down.

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