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Phenomenological relation between distribution and fragmentation functions

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Abstract

We study the relation between the quark distribution function $q(x)$ and the fragmentation function $D_q(z)$ based on a general form $D_q(x) = C(z)z^{\alpha}q(z)$ for valence and sea quarks. By adopting two known parametrizations of quark distributions for the proton, we find three simple options for the fragmentation functions that can provide a good description of the available experimental data on proton production in e^+e^- inelastic annihilation. These three options support the revised Gribov–Lipatov relation $D_q(z) = zq(z)$ at $z \to 1$, as an approximate relation for the connection between distribution and fragmentation functions. The three options differ in the sea contributions and lead to distinct predictions for antiproton production in the reaction $p + p \to \bar{p} + X$, thus they are distinguishable in future experiments at RHIC-BNL.

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The quark distribution function $q(x)$ and the quark fragmentation function $D_q(z)$ are two basic quantities on the structure of hadrons. It would be very useful if there exist simple connections between them, so that one can predict the poorly known $D_q(z)$ from the rather well-known $q(x)$. Conversely, one could predict the quark distribution functions of a hadron that cannot be used as a target from the quark fragmentation to the same hadron. By simple crossing, it is possible to relate the two functions by the Drell–Levy–Yan relation [1]

$$D_q(z) = zq\left(\frac{1}{z}\right). \tag{1}$$

Such relation is known to be valid in the scaling parton model [2], but a rigorous non-perturbative crossing is not possible to all orders of QCD [2,3]. Also it does not apply for both sides in their physical regions [4]. There is a similar relation, the so-called Gribov–Lipatov "reciprocity" relation [5], which connects the distribution and fragmentation functions in a form

$$D_q(z) \propto q(x), \tag{2}$$

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with both sides in their physical regions. Such relation has been used as a useful "ansatz" to model the quark fragmentation functions based on predictions of quark distributions functions [5,6]. Recently, based on theoretical arguments with some assumptions [4], a revised form of the Gribov–Lipatov relation, i.e.,

$$D_q(z) = zq(z), \tag{3}$$

has been suggested as an approximate relation at $z \to 1$. Thus it is necessary to check the validity of Eq. (2) and/or Eq. (3) by means of careful phenomenological studies.

The nucleon is a satisfactory laboratory to check the relation between fragmentation and distribution functions, since we have data both on the quark distributions of the nucleon from deep inelastic scattering (DIS) [7,8] and on the fragmentation functions of quark to proton from e^+e^- inelastic annihilation (IA). Various parametrizations of parton distributions have been obtained from the DIS experimental data on the nucleon, with rather high precision [9–12]. We can start with any set of parton distributions to parametrize the fragmentation functions. In this Letter, we will adopt the CTEQ parametrization (CTEQ6 set 1) [9] of the parton distributions and for comparison, we will also use another recent parametrization obtained in the statistical physical picture, by Bourrely, Soffer and Buccella (BSB) [10]. These parametrizations provide reliable information on the valence and sea quarks and their respective roles can be studied separately. On the other hand, some experimental data on proton production in e^+e^- IA are available [13] and can be used to constrain the shape of the fragmentation functions of quark to proton. By parametrizing the fragmentation functions of quark to proton based on a reliable set of parton distributions and confronting them with the available experimental data on proton production, we thus have an effective way to learn about the relation between fragmentation and distribution functions. For this analysis, we adopt a general form to relate fragmentation and distribution functions and we make a distinction between valence and sea quarks, as follows:

$$D_v(z) = C_v(z) z^\alpha q_v(z),$$
$$D_s(z) = C_s(z) z^\alpha q_s(z). \tag{4}$$

The above forms are always correct, since $C_v(z)$ and $C_s(z)$ are in principle arbitrary. Strictly speaking, there is no way to discriminate between "valence" and "sea" fragmentation, because both "valence" and "sea" quarks q (not $\bar{q}$) can fragment at a same z. Since it is not possible to distinguish whether the fragmenting quark is "valence" or "sea", we should consider Eq. (4) as a phenomenological parametrization for the fragmentation functions of quarks and antiquarks, as follows:

$$D_q(z) = D_v(z) + D_s(z),$$
$$D_{\bar{q}}(z) = D_s(z). \tag{5}$$

It will be shown in this Letter that, in order to fit the experimental data of proton production in IA, with fragmentation functions based on the above parametrizations, we can have very simple forms of $C_v(z)$ and $C_s(z)$ together with simple values for α. Three options are found to be very good: (1) $C_v = 1$ and $C_s = 0$ for $\alpha = 0$, (2) $C_v = C_s = 1$ for $\alpha = 0.5$, and (3) $C_v = 1$ and $C_s = 3$ for $\alpha = 1$. Since the sea quark contributions are negligible at large x, all of the three options support the revised Gribov–Lipatov relation $D_q(z) = zq(z)$ at $z \to 1$, as an approximate relation for the connection between distribution and fragmentation functions. Option 1 corresponds to the case of a suppressed sea, whereas option 3 corresponds to the case of an enhanced sea, so they provide two extreme situations. Thus these three options lead to different fragmentation pictures related to the respective roles of valence and sea quarks. We will show that they provide different predictions for proton/antiproton productions in the reactions $p + p \to p(\bar{p}) + X$, thus they are distinguishable in future experiments at RHIC-BNL.

The revised Gribov–Lipatov relation Eq. (3) is only known to be valid near $z \to 1$ and on a certain energy scale Q_0^2, in the leading approximation [2]. Since distribution and fragmentation functions have different evolution behaviors, it is appropriate to apply the relation at a starting energy scale Q_0^2, and then to evolve the fragmentation functions to the experimental scale. Our goal is to get a good description for the cross section of proton production in e^+e^- IA [13]. In our analysis, we adopt the initial energy scale Q_0^2 of the quark distributions, i.e., it is $Q_0 = 1.3$ GeV for the CTEQ parametrization [9] and $Q_0 = 2$ GeV for the BSB parametrization [10]. In addition, in the evolution the gluon fragmentation functions are also needed,

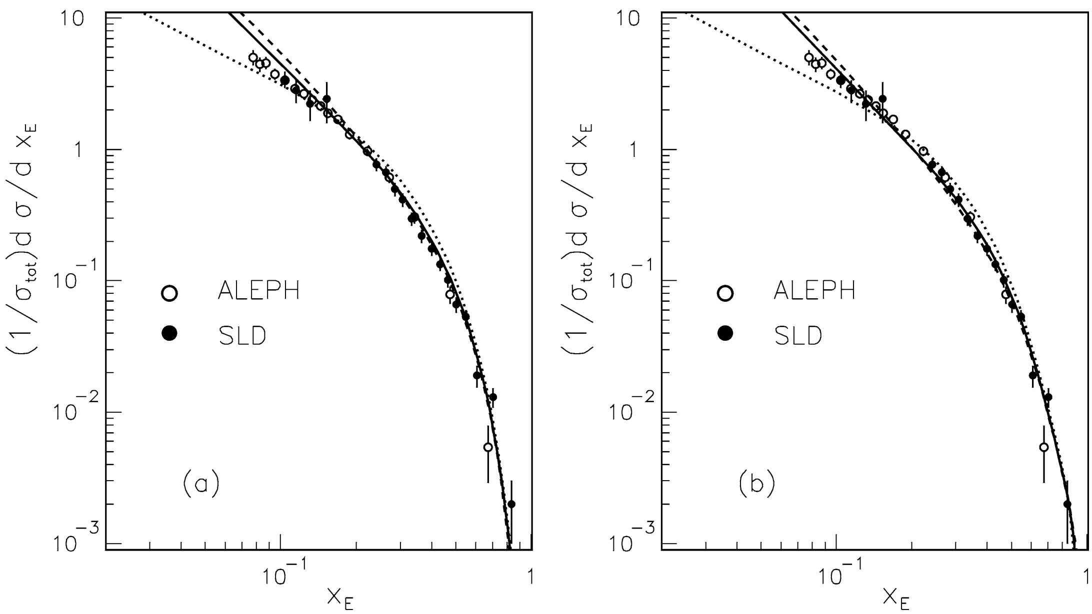

Fig. 1. Predictions for the cross section of proton production in e^+e^- annihilation. Dotted, solid, dashed curves correspond to the three options of 1, 2 and 3, respectively, with (a) CTQE and (b) BSB parametrizations of the parton distributions. The experimental data are taken from [13,14].

and we assume that they have the same relation with the corresponding gluon distribution functions as the sea quarks. For the evolution we adopt the evolution package of Ref. [10], modified for the fragmentation functions.

In the quark–parton model, the differential cross section for the semi-inclusive proton production process $e^+e^- \to p + X$ can be expressed to leading order as

$$\frac{1}{\sigma_{\rm tot}} \frac{d\sigma}{dx_E} = \frac{\sum_q \widehat{C}_q [D_q(x_E, Q^2) + D_{\bar{q}}(x_E, Q^2)]}{\sum_q \widehat{C}_q}, \tag{6}$$

where $x_E = 2E_p/\sqrt{s}$, s is the total center-of-mass (c.m.) energy squared, E_p is the energy of the produced proton in the e^+e^- c.m. frame, and $\sigma_{\rm tot}$ is the total cross section for the process. In Eq. (6), $\widehat{C}_q$ reads

$$\widehat{C}_q = e_q^2 - 2\chi_1 v_e v_q e_q + \chi_2 (a_e^2 + v_e^2)(a_q^2 + v_q^2), \tag{7}$$

with

$$\chi_1 = \frac{1}{16 \sin^2\theta_W \cos^2\theta_W} \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}, \tag{8}$$

$$\chi_2 = \frac{1}{256 \sin^4\theta_W \cos^4\theta_W} \frac{s^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}, \tag{9}$$

$$a_e = -1, \tag{10}$$

$$v_e = -1 + 4\sin^2\theta_W, \tag{11}$$

$$a_q = 2T_{3q}, \tag{12}$$

and

$$v_q = 2T_{3q} - 4e_q \sin^2\theta_W, \tag{13}$$

where $T_{3q} = 1/2$ for u, while $T_{3q} = -1/2$ for d, s quarks. Moreover $N_c = 3$ is the color number, e_q is the charge of the quark in units of the proton charge, θ is the angle between the outgoing quark and the incoming electron, θ_W is the Weinberg angle, and M_Z and Γ_Z are the mass and width of Z^0.

In Fig. 1 we present our results of the proton production in e^+e^- IA with the three simple options for the relation between distribution and fragmentation functions Eq. (4). Option 1: $C_v = 1$ and $C_s = 0$ for $\alpha = 0$; option 2: $C_v = C_s = 1$ for $\alpha = 0.5$; and option 3: $C_v = 1$ and $C_s = 3$ for $\alpha = 1$. We find that all of these three options fit rather well the experimental data in a large z range, though there is some discrepancy at small z. From Fig. 1(a) and (b), we find that there are only small differences in the predictions when we use the two different choices of quark distributions parametrizations. Thus our analysis of the relation between

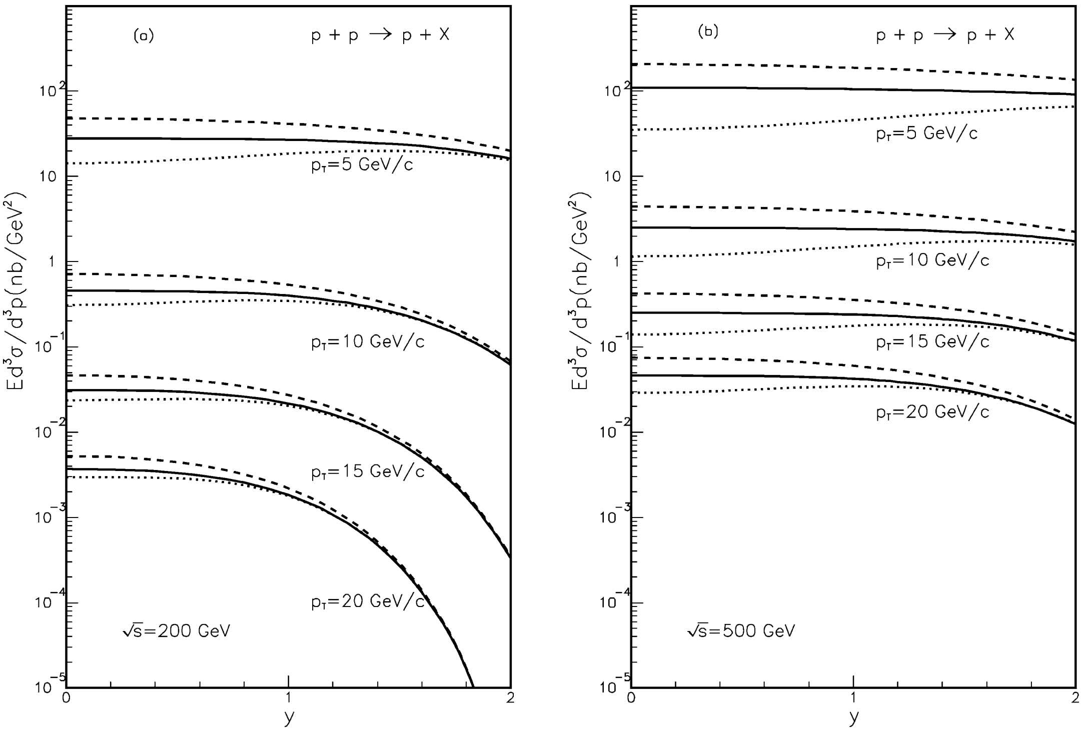

Fig. 2. Predictions of the rapidity distribution of the cross section of proton production in pp collision. Dotted, solid, and dashed curves correspond to the predictions from the three options 1, 2 and 3, respectively, for c.m. energy $\sqrt{s} = 200$ GeV (a) and $\sqrt{s} = 500$ GeV (b).

distribution and fragmentation functions is not sensitive to the available parametrizations of quark distributions which are well constrained by a vast number of experimental data. All of the three options support the validity of the revised Gribov–Lipatov relation Eq. (3) at $z \to 1$.

As indicated above, the three options differ in the roles played by the valence and sea quark contributions in the fragmentation functions. Option 2 corresponds to a situation with equal contributions from valence and sea quarks, whereas option 1 and option 3 correspond to the situations with sea suppressed and enhanced, respectively. Though such differences do not show up significantly in $e^+e^- \to p + X$, they will show up in the proton/antiproton productions in $p + p \to p(\bar{p}) + X$, as will be shown in the following. Thus we can test these three different options by examining predictions in these processes, where the roles played by valence and sea quarks are different in comparison with e^+e^- IA. For the sake of simplicity, we only adopt the fragmentation functions based on the CTEQ parametrization of the parton distributions. In fact, the new facility RHIC at BNL is expected to be running at some high c.m. energy such as $\sqrt{s} = 200$ GeV and $\sqrt{s} = 500$ GeV. We thus use these fragmentation functions to predict the cross section of the proton/antiproton productions via

$$p + p \to H + X, \tag{14}$$

where the produced hadron H, is a proton or an antiproton, respectively. The invariant cross section of the above process can be expressed as

$$E_H \frac{d^3\sigma}{d^3 p_H} = \sum_{abcd} \int_{\bar{x}_a}^{1} dx_a \int_{\bar{x}_b}^{1} dx_b\, f_a(x_a, Q^2) f_b(x_b, Q^2) \times D_c^H(z, Q^2) \frac{1}{\pi z} \frac{d\hat{\sigma}}{d\hat{t}}(ab \to cd). \tag{15}$$

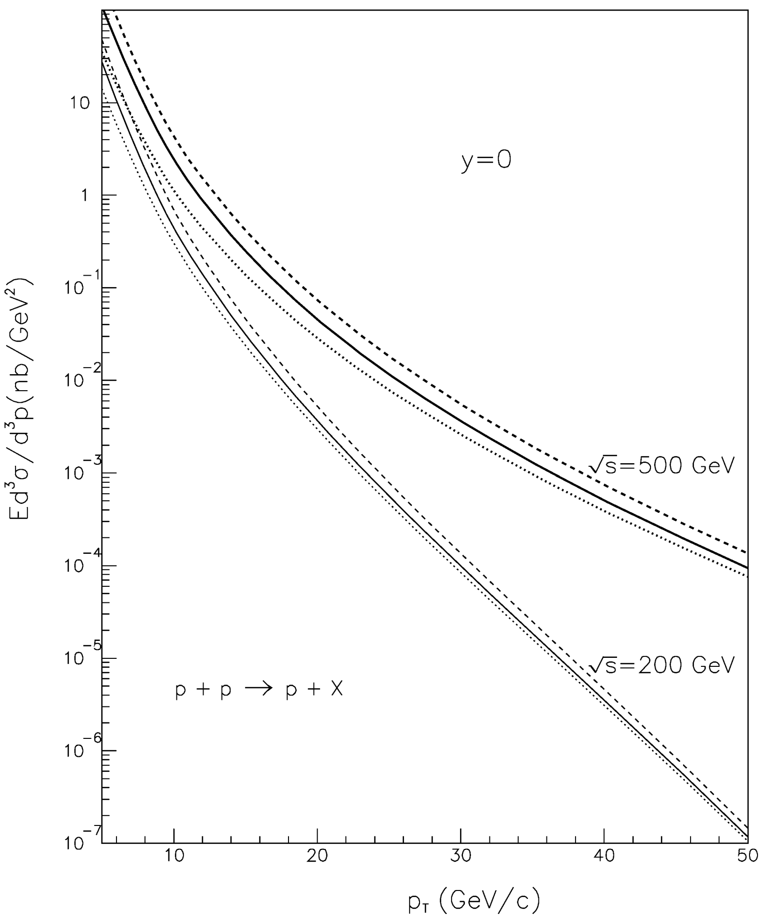

Fig. 3. Predictions of the transverse momentum distribution of the cross section of proton production in pp collision. Dotted, solid, and dashed curves correspond to the predictions from the three options 1, 2 and 3, respectively, for c.m. energy $\sqrt{s} = 200$ GeV (a) and $\sqrt{s} = 500$ GeV (b).

In Fig. 2, we present the rapidity distribution of the cross section for proton production at the c.m. energy $\sqrt{s} = 200$ GeV (Fig. 2(a)) and $\sqrt{s} = 500$ GeV (Fig. 2(b)), for various values of the transverse momentum p_T of the produced hadron. In Fig. 3, the p_T distribution of the cross sections is shown at the above mentioned two values of the c.m. energy. We see that the predictions do not show large differences for the three options. In Figs. 4 and 5, the rapidity and the transverse momentum distributions of the cross sections are shown for antiproton production and now we observe significant differences for the predictions from the three options. This can be easily understood since antiproton production is sensitive to the sea quark fragmentation. We thus conclude that antiproton production in the reaction $p + p \to \bar{p} + X$ can discriminate clearly between the three options of quark fragmentation functions.

In summary, we have studied the relation between fragmentation and distribution functions based on known parametrizations of parton distributions and the available experimental data on proton production in e^+e^- IA. With general relations between fragmentation and distribution functions, we find three simple options that can provide a good description of the proton production data in e^+e^- IA. All of the three options support the revised Gribov–Lipatov relation $D_q(z) = zq(z)$ at $z \to 1$ as an approximate relation, and they differ in the roles played by valence and sea quarks in the fragmentation. Such difference can show

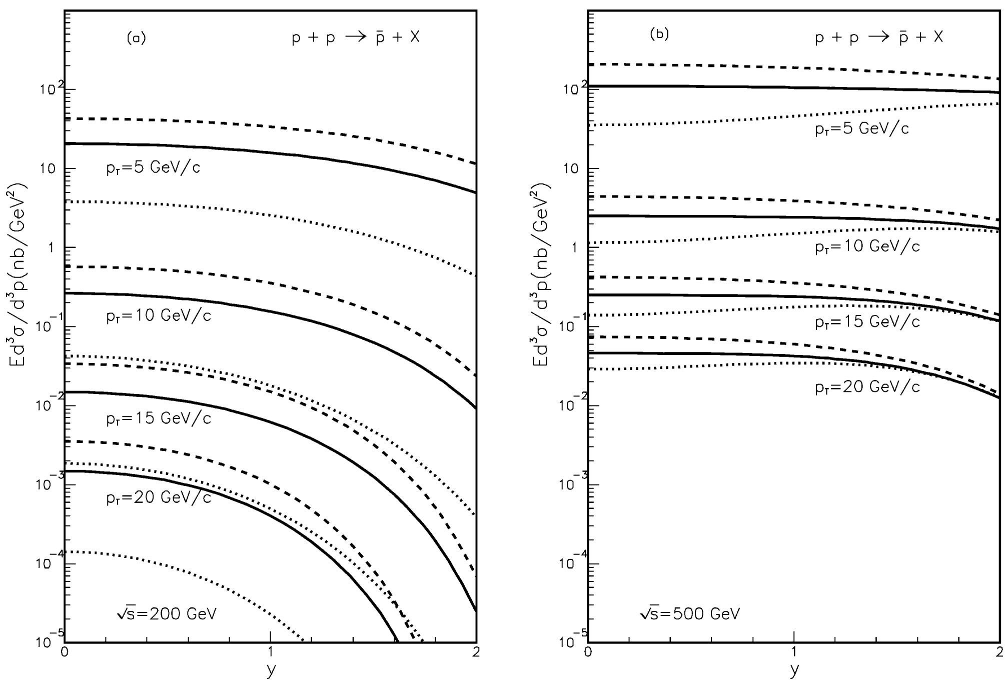

Fig. 4. The same as Fig. 2, but for antiproton production.

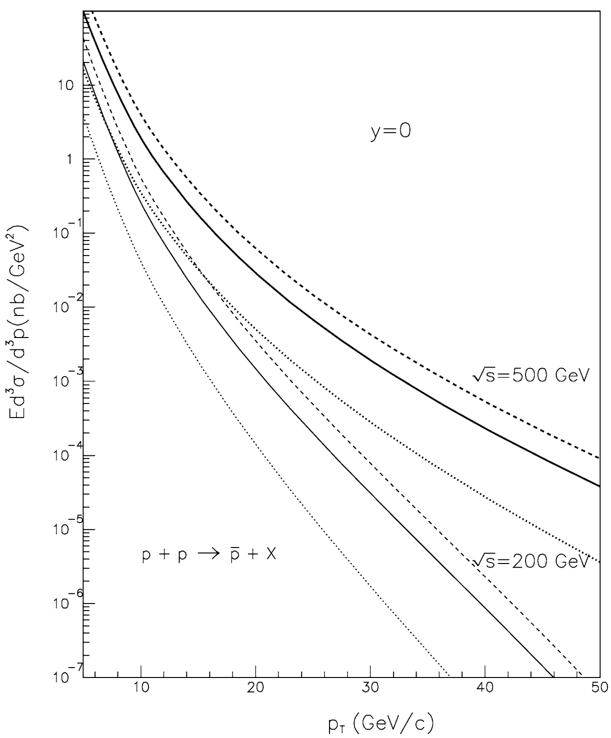

Fig. 5. The same as Fig. 3, but for antiproton production.

up especially from antiproton production in the reaction $p + p \to \bar{p} + X$, thus they can be tested in future experiments at RHIC-BNL.

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References

[1] S.D. Drell, D.J. Levy, T.-M. Yan, Phys. Rev. 187 (1969) 2159; S.D. Drell, D.J. Levy, T.-M. Yan, Phys. Rev. D 1 (1970) 1617.

[2] J. Blümlein, V. Radindran, W.L. van Neerven, Nucl. Phys. B 589 (2000) 349.

[3] For some relevant theoretical works, see, e.g., J. Pestieau, P. Roy, Phys. Lett. B 30 (1969) 483;
P.M. Fishbane, J.D. Sullivan, Phys. Lett. B 37 (1971) 68;
A. Suri, Phys. Rev. D 4 (1971) 510;
R. Gatto, G. Preparata, Nucl. Phys. B 47 (1972) 313;
H.D. Dahmen, F. Steiner, Phys. Lett. B 43 (1973) 217;
R. Gatto, P. Menotti, Nuovo Cimento A 2 (1971) 881;
R. Gatto, P. Menotti, Nuovo Cimento A 7 (1972) 118;
R. Gatto, P. Menotti, I. Vendramin, Lett. Nuovo Cimento 4 (1972) 79;
R. Gatto, P. Menotti, I. Vendramin, Ann. Phys. (N.Y.) 79 (1973) 1;
P.V. Landshoff, J.C. Polkinghorne, R.D. Short, Nucl. Phys. B 28 (1971) 225;
P.V. Landshoff, J.C. Polkinghorne, Phys. Rev. D 6 (1972) 3708;
P.V. Landshoff, J.C. Polkinghorne, Nucl. Phys. B 53 (1973) 473;
G. Altarelli, L. Maiani, Phys. Lett. B 41 (1972) 480;
G. Altarelli, L. Maiani, Nucl. Phys. B 51 (1973) 509;
S. Ferrara, R. Gatto, G. Parisi, Phys. Lett. B 44 (1973) 381.

[4] V. Barone, A. Drago, B.-Q. Ma, Phys. Rev. C 62 (2000) 062201(R).

[5] V.N. Gribov, L.N. Lipatov, Phys. Lett. B 37 (1971) 78;
V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 675;
S.J. Brodsky, B.-Q. Ma, Phys. Lett. B 392 (1997) 452.

[6] B.-Q. Ma, I. Schmidt, J.-J. Yang, Phys. Rev. D 61 (2000) 034017;
B.-Q. Ma, I. Schmidt, J.-J. Yang, Phys. Lett. B 477 (2000) 107;
B.-Q. Ma, I. Schmidt, J. Soffer, J.-J. Yang, Eur. Phys. J. C 16 (2000) 657;
B.-Q. Ma, I. Schmidt, J. Soffer, J.-J. Yang, Phys. Rev. D 62 (2000) 114009;
B.-Q. Ma, I. Schmidt, J. Soffer, J.-J. Yang, Phys. Rev. D 64 (2001) 014007;
B.-Q. Ma, I. Schmidt, J. Soffer, J.-J. Yang, Phys. Rev. D 65 (2002) 034004.

[7] For a review, see, e.g., J.F. Owens, W.-K. Tung, Annu. Rev. Nucl. Part. Sci. 42 (1992) 291.

[8] For a review, see, e.g., P.L. McGaughey, J.M. Moss, J.C. Peng, Annu. Rev. Nucl. Part. Sci. 49 (1999) 217.

[9] CTEQ Collaboration, H.L. Lai, et al., Eur. Phys. J. C 12 (2000) 375.

[10] C. Bourrely, J. Soffer, F. Buccella, Eur. Phys. J. C 23 (2002) 487.

[11] M. Glück, E. Reya, A. Vogt, Z. Phys. C 67 (1995) 433.

[12] A.D. Martin, R.G. Roberts, W.J. Stiring, R.S. Thorne, Eur. Phys. J. C 14 (2000) 133.

[13] ALEPH Collaboration, R. Barate, et al., Phys. Rep. 294 (1998) 1.

[14] SLD Collaboration, K. Abe, et al., Phys. Rev. D 59 (1999) 052001.