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Closed string Ramond–Ramond proposed higher derivative interactions on fermionic amplitudes in IIB

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Abstract

The complete form of the amplitude of one closed string Ramond–Ramond (RR), two fermionic strings and one scalar field in IIB superstring theory has been computed in detail. Deriving $\langle V_C V_{\bar{\psi}} V_{\psi} V_{\phi}\rangle$ by using suitable gauge fixing, we discover some new vertices and their higher derivative corrections. We investigate both infinite gauge and scalar u-channel poles of this amplitude. In particular, by using the fact that the kinetic term of fermion fields has no correction, employing Born–Infeld action, the Wess–Zumino terms and their higher derivative corrections, we discover all infinite t, s-channel fermion poles. The couplings between one RR and two fermions and all their infinite higher derivative corrections have been explored. In order to look for all infinite $(s+t+u)$-channel scalar/gauge poles for $p+2=n$, $p=n$ cases, we obtain the couplings between two fermions–two scalars and two fermions, one scalar and one gauge field as well as all their infinite higher derivative corrections in type IIB. Specifically we make various comments based on arXiv:1205.5079 in favor of universality conjecture for all order higher derivative corrections (with or without low energy expansion) and the relation of open/closed string that is responsible for all superstring scattering amplitudes in IIA, IIB.

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1. Introduction

D-branes [1–3] have been clarifying essential tools in most of the progresses in theoretical high energy physics as well as in superstring theories. As an instance of dynamical aspects of D-branes, one may consider the transition between both open and closed string [4]. D-brane physics has diverse dual descriptions (see string dualities [5]). Some more examples like the $D0/D4$ system with their explanations have been realized in the introduction of [6]. In particular in [7] we have shown how the world volume must appear from super gravity point of view, namely we set new kind of ADM reduction to IIB which was reduced to hyperboloidal H^5 space in 5D and showed that A(ds) brane world might be understood.

In IIA (IIB) string theories, D_p-branes with even (odd) p (which is the spatial dimension of a D_p-brane) are related to BPS branes in which there is no instability and supersymmetry is not broken. Apart from these properties, BPS branes carry Ramond–Ramond (RR) charge. To describe dynamics of branes, one has to work with some proper effective actions, namely one has to deal with bosonic effective actions and also should distinguish it from its supersymmetric version. Bosonic actions in the presence of various D_p-brane configurations were considered in [8].

The supersymmetrized version of bosonic effective action that appeared in [8] has not been completely found yet, however it is worth mentioning [9] as an important reference. In this paper, in order to avoid some details we address some of the main references. One might start reading [10] to explore the effective action for a bosonic D_p-brane. To follow the supersymmetric action for a D_p-brane Ref. [11] should be considered.

For a comprehensive review of Myers terms, the Chern–Simons action, the Wess–Zumino actions and more significantly for Born–Infeld action [12,13] and all references therein are suggested. The three standard effective field theory methods namely pull-back, Taylor expansion and Myers terms have been addressed in [14]. Some methods for looking for all the higher derivative corrections of Myers, Chern–Simons and Born–Infeld actions, have been expressed in [14].

Let us point out an issue in favor of scattering theory in string theory. We may hint to a conjecture which appeared in [15] where BPS open strings quantum effects might indicate the host branes curvature. Given some attempts [16–18], it would be nice to find the complete form of Wess–Zumino (WZ) and DBI actions. In this paper we provide some more data and our S-matrix will be useful for all order α' determination of DBI and WZ actions. We just refer to [19] out of so many works that involved either the scattering from stable branes or dealt with intriguing applications of the branes. Note that some of our higher derivative corrections come from the couplings of lower dimensional branes with closed string RR, meanwhile lower dimensional branes should be realized as some soliton objects. To be more specific, in [6] we explored dissolving lower dimensional branes inside higher dimension branes [20]. Another example is related to $D(-1)/D3$ configuration, where this system has N^2 entropy behavior and this result can be interpreted by taking into account higher order α' Myers terms. Some applications of new couplings (including their corrections) in M-theory are recently addressed in [21,20].

Having set some of the past works on Myers terms and WZ effective actions [12,22–26], we would like to explore all the infinite two fermions–two scalars and two fermions, one scalar and one gauge field couplings as well as their all order α' higher derivative corrections. Basically we want to find out all the infinite effective couplings between two fermions two scalars and two fermions, a scalar and a gauge field by matching field theory vertices with an infinite number of scalar/gauge $(t+s+u)$-channel poles of the string amplitude of $\langle V_C V_\phi V_{\bar{\psi}} V_\psi \rangle$ for $p+2=n$,

$p = n$ cases (n is the rank of RR field strength H) accordingly. These new couplings which we intend to derive might have some applications to F-theory [27] as well as M-theory [6,20,21].

This work illustrates the fact that super Yang–Mills vertices such as two fermion–two scalar couplings and in particular two fermion–one gauge–one scalar couplings give rise precisely to the same scalar/gauge poles as those poles which appeared in the string theory amplitude of $\langle V_C V_\phi V_{\bar{\psi}} V_\psi \rangle$.

This paper is organized as follows.

In the next section, we explain superstring scattering computations of a closed string RR field, two fermion fields and one scalar field, to actually obtain the complete and closed form of the correlators of $\langle V_C V_\phi V_{\bar{\psi}} V_\psi \rangle$. Our computations make sense in IIB superstring theory as both fermions here carry the same chirality.

Then by expanding the amplitude at low energy limit and by finding the desired vertices such as the vertex of one RR, two fermion fields and its extensions (to all orders in α') as well as using WZ terms, we produce both infinite scalar and gauge u-channel poles for $p + 2 = n$ and $p = n$ cases. We move on to produce all infinite t, s-channel fermion poles by obtaining the infinite extensions of the vertex of one RR and two fermions. Finally we summarize our results and talk about the deep relation that exists between a closed string and an open string and we find that indeed it is responsible for matching superstring amplitudes with their effective field theories. In order to avoid introducing some more details and notations, we refer the reader to Appendices A and B of [14,22].

This paper provides some more information on the universal behavior of the higher derivative corrections [23]. In particular the calculations of this paper clarify that the universal conjecture for higher order corrections which appeared in [23] works even for fermionic amplitudes, including two fermion–two scalar couplings. Thus our S-matrix serves one more test of our understanding of the full DBI, WZ effective actions. This universal conjecture might also be useful for deriving all the singularities of the higher point functions of BPS amplitudes without the need for applying direct conformal field theory computations. This universal behavior should have origins coming from the deep relation of a closed and an open string. This relation may clarify closed string's composite nature only in terms of open strings. We describe it further in the conclusion section.

2. Notations and analysis of $\langle V_{\bar{\psi}} V_\psi V_\phi \rangle$

In this section we use conformal field theory methods to actually find out the entire amplitude of a closed string RR (C-field), two fermions (with the same chirality) and one scalar field in the world volume of IIB superstring theory.

Given the vast recent research works on scattering amplitudes, it is indeed impossible to address all attempts on this subject, however we would like to address some of the works that carried out at tree level computations [28–30,12,22,23]. The needed vertex operators for our purpose are[1]

$$\begin{aligned}
V_\phi^{(0)}(x) &= \xi_i \big(\partial X^i(x) + \alpha' ik \cdot \psi \psi^i(x)\big) e^{\alpha' ik \cdot X(x)},\\
V_{\bar{\Psi}}^{(-1/2)}(x) &= \bar{u}^A e^{-\phi(x)/2} S_A(x) e^{\alpha' iq \cdot X(x)},\\
V_{\Psi}^{(-1/2)}(x) &= u^B e^{-\phi(x)/2} S_B(x) e^{\alpha' iq \cdot X(x)},
\end{aligned}$$

[1] We clearly consider α' in this paper.

$$V_C^{(-\frac{1}{2},-\frac{1}{2})}(z,\bar{z})=(P_-\not{H}_{(n)}M_p)^{\alpha\beta}e^{-\phi(z)/2}S_\alpha(z)e^{i\frac{\alpha'}{2}p\cdot X(z)}e^{-\phi(\bar{z})/2}S_\beta(\bar{z})e^{i\frac{\alpha'}{2}p\cdot D\cdot X(\bar{z})}, \tag{1}$$

(p,q,k) become the momenta of the RR, fermion and scalar field accordingly. Their on-shell condition is $k^2=q^2=p^2=0$. Note that u^A is the fermion's wave function of Majorana–Weyl in ten dimensions of space–time. Spin indices have been raised by $C^{\alpha\beta}$ (charge conjugation matrix),

$$(P_-\not{H}_{(n)})^{\alpha\beta}=C^{\alpha\delta}(P_-\not{H}_{(n)})_\delta{}^\beta, \tag{2}$$

where the definitions of the traces are

$$\begin{aligned}&\mathrm{Tr}\big(P_-\not{H}_{(n)}M_p\gamma^k\big)\equiv(P_-\not{H}_{(n)}M_p)^{\alpha\beta}\big(\gamma^kC^{-1}\big)_{\alpha\beta},\\&\mathrm{Tr}\big(P_-\not{H}_{(n)}M_p\Gamma^{iab}\big)\equiv(P_-\not{H}_{(n)}M_p)^{\alpha\beta}\big(\Gamma^{iab}C^{-1}\big)_{\alpha\beta},\end{aligned} \tag{3}$$

where P_- is a projection operator, $P_-=\frac{1}{2}(1-\gamma^{11})$ and the field strength of RR is

$$\not{H}_{(n)}=\frac{a_n}{n!}H_{\mu_1\cdots\mu_n}\gamma^{\mu_1}\cdots\gamma^{\mu_n},$$

where for type IIB theory, $n=1,3,5$ and $a_n=1$. We use doubling trick to deal with just holomorphic functions (for further details, see Appendix A of [14]). The amplitude of two fermions and one gauge field has been found out in [31]; however, to get familiar with the notations let us start working with tree level amplitude of two fermions and one scalar field. This three point function $\langle V_{\bar{\psi}}V_\psi V_\phi\rangle$ at disk level is given by

$$\mathcal{A}^{\bar{\Psi},\Psi,\phi}\sim\sum_{\text{non-cyclic}}\int dx_1\,dx_2\,dx_3\,\mathrm{Tr}\big\langle V_{\bar{\Psi}}^{(-1/2)}(x_1)V_\Psi^{(-1/2)}(x_2)V_\phi^{(-1)}(x_3)\big\rangle. \tag{4}$$

Substituting the vertex operators and taking into account the following holomorphic correlators

$$\begin{aligned}&\big\langle X^\mu(z)X^\nu(w)\big\rangle=-\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-w),\\&\big\langle\psi^\mu(z)\psi^\nu(w)\big\rangle=-\frac{\alpha'}{2}\eta^{\mu\nu}(z-w)^{-1},\\&\big\langle\phi(z)\phi(w)\big\rangle=-\log(z-w).\end{aligned} \tag{5}$$

Eq. (4) can be written down as

$$\begin{aligned}\mathcal{A}^{\bar{\Psi},\psi,\phi}={}&iT_p2^{1/2}\pi\alpha'\int dx_1\,dx_2\,dx_3\,\xi_{1i}x_{12}^{-1/4}(x_{13}x_{23})^{-1/2}\\&\times|x_{12}|^{\alpha'^2k_1.k_2}|x_{13}|^{\alpha'^2k_1.k_3}|x_{23}|^{\alpha'^2k_2.k_3}\\&\times\big\langle:S_A(x_1):S_B(x_2):\psi^i(x_3):\big\rangle\bar{u}_1^Au_2^B,\end{aligned} \tag{6}$$

Note that we have normalized the amplitude by a coefficient of $(iT_p2^{1/2}\pi\alpha')$. Using

$$\big\langle:S_A(x_1):S_B(x_2):\psi^i(x_3):\big\rangle=2^{-1/2}x_{12}^{-3/4}(x_{31}x_{32})^{-1/2}\big(\gamma^i\big)_{AB},$$

one can show that (6) is now $SL(2,R)$ invariant. We do gauge fixing as $(x_1,x_2,x_3)=(0,1,\infty)$. Setting this gauge fixing into (6), the amplitude becomes

$$\mathcal{A}^{\bar{\Psi},\psi,\phi}=iT_p\pi\alpha'\bar{u}_1^A\gamma_{AB}^iu_2^B\xi_i\big(\mathrm{Tr}(\lambda_1\lambda_2\lambda_3)-\mathrm{Tr}(\lambda_1\lambda_3\lambda_2)\big). \tag{7}$$

The final result of the string theory can be reproduced in field theory by extracting the kinetic term of the fermion fields $(2\pi\alpha' T_p)\,\mathrm{Tr}(\bar\psi\gamma^\mu D_\mu\psi)$. One has to extract the covariant derivative of fermion field and take into account the commutator in its definition as $(D^i\psi = \partial^i\psi - i[\phi^i,\psi])$. It is also good to know that the Wick-like rule [32,33] has been extended in [25,26,12,14] to find various correlators including two spin operators and an infinite number of fermions and/or currents.

2.1. The complete and closed form of $\langle V_C V_\phi V_{\bar\psi} V_\psi\rangle$

After providing the necessary details we now compute the complete form (entire result to all orders in α') of the scattering amplitude of one closed string RR (in the bulk), two fermions with the same chirality and a scalar field in the world volume of BPS branes. Regarding the chirality of the fermions our computation makes sense just in IIB superstring theory and the entire result cannot be extended to IIA (because fermions have different chirality in IIA). As a matter of fact neither there are any gauge nor scalar $(t+s+u)$-channel poles for this particular amplitude in IIA. Therefore all order corrections of two fermions–two scalars and two fermions–a gauge–a scalar of this paper cannot be applied to IIA. This $\langle V_C V_\phi V_{\bar\psi} V_\psi\rangle$ amplitude can be looked for as follows

$$\mathcal{A}^{C\phi\bar\psi\psi} \sim \int dx_1\,dx_2\,dx_3\,dz\,d\bar z\,\big\langle V_\phi^{(0)}(x_1)V_{\bar\psi}^{(-1/2)}(x_2)V_\psi^{(-1/2)}(x_3)V_{RR}^{(-\frac12,-\frac12)}(z,\bar z)\big\rangle. \tag{8}$$

For disk amplitudes all three open strings have to be embedded on the boundary and RR should be located inside of the disk. Notice that we just want to keep track of $\mathrm{Tr}(\lambda_1\lambda_2\lambda_3)$ ordering. By replacing all the vertex operators into (8), one might reveal that the amplitude should be divided to two different parts $(\mathcal{A}_1,\mathcal{A}_2)$. The final result is complicated so we decided to carry out each part of the amplitude separately.

First, we look for $(\mathcal{A}_1)$ in which for this part we need to know the correlation function of four spin operators (with the same chirality) in ten dimensions. This correlation function can be found in [34,35], thus we just replace it into the amplitude and the result should be read as

$$\begin{aligned}\mathcal{A}_1^{C\phi\bar\psi\psi} = {}& \frac{\mu_p\pi^{-1/2}}{4}\int dx_1\,dx_2\,dx_3\,dx_4\,dx_5\,(P_-\not{H}_{(n)}M_p)^{\gamma\delta}\xi_{1i}\\ &\times \bar u_1^A u_2^B(x_{23}x_{24}x_{25}x_{34}x_{35}x_{45})^{-1}\\ &\times\frac12\big[(\gamma^\mu C)_{\gamma\delta}(\gamma_\mu C)_{AB}x_{43}x_{52} - (\gamma^\mu C)_{\gamma B}(\gamma_\mu C)_{A\delta}x_{45}x_{23}\big]I_1\,\mathrm{Tr}(\lambda_1\lambda_2\lambda_3),\end{aligned} \tag{9}$$

where we normalized the amplitude by $\frac{\mu_p\pi^{-1/2}}{4}$, $x_{ij}=x_i-x_j$, $x_4=z=x+iy$, $x_5=\bar z = x-iy$, and

$$I_1 = \big\langle :\partial X^i(x_1)e^{\alpha' ik_1\cdot X(x_1)} : e^{\alpha' ik_2\cdot X(x_2)} : e^{\alpha' ik_3\cdot X(x_3)} : e^{i\frac{\alpha'}{2}p\cdot X(x_4)} : e^{i\frac{\alpha'}{2}p\cdot D\cdot X(x_5)} :\big\rangle.$$

Using Wick theorem one obtains

$$\begin{aligned}I_1 = {}&\left(\frac{ip^i x_{54}}{x_{14}x_{15}}\right)|x_{12}|^{\alpha'^2k_1\cdot k_2}|x_{13}|^{\alpha'^2k_1\cdot k_3}|x_{14}x_{15}|^{\frac{\alpha'^2}{2}k_1\cdot p}|x_{23}|^{\alpha'^2k_2\cdot k_3}|x_{24}x_{25}|^{\frac{\alpha'^2}{2}k_2\cdot p}\\ &\times|x_{34}x_{35}|^{\frac{\alpha'^2}{2}k_3 vp}|x_{45}|^{\frac{\alpha'^2}{4}p\cdot D\cdot p}.\end{aligned}$$

By replacing I_1 in the amplitude, we realize that the amplitude is $SL(2,R)$ invariant. Using the following Mandelstam variables

$$s=-\frac{\alpha'}{2}(k_1+k_3)^2,\qquad t=-\frac{\alpha'}{2}(k_1+k_2)^2,\qquad u=-\frac{\alpha'}{2}(k_3+k_2)^2,$$

and in particular carrying out a special gauge fixing as ($x_1=0, x_2=1, x_3=\infty$), one can obtain the first part of the amplitude as

$$\begin{aligned}\mathcal{A}_1^{C\phi\bar\psi\psi}={}&\frac{\mu_p\pi^{-1/2}}{4}(P_-\not{H}_{(n)}M_p)^{\gamma\delta}\xi_{1i}\bar u_1^A u_2^B\left(\frac{-ip^i}{2}\right)\\&\times\int\!\!\int dz\,d\bar z\,|z|^{2t+2s-2}|1-z|^{2t+2u-2}(z-\bar z)^{-2(t+s+u)},\\&\times\left[(\gamma^\mu C)_{\gamma\delta}(\gamma_\mu C)_{AB}(1-\bar z)+(z-\bar z)(\gamma^\mu C)_{\gamma B}(\gamma_\mu C)_{A\delta}\right]\mathrm{Tr}(\lambda_1\lambda_2\lambda_3),\end{aligned}\tag{10}$$

In order to actually get the entire result, these integrals must be done on the closed string position (we propose [36] and Appendix B of [14] for further details). Therefore the complete form of the first part of the amplitude is given as follows

$$\begin{aligned}\mathcal{A}_1^{C\phi\bar\psi\psi}={}&\frac{\mu_p\pi^{-1/2}}{4}(P_-\not{H}_{(n)}M_p)^{\gamma\delta}\xi_{1i}\bar u_1^A u_2^B\left(\frac{-ip^i}{2}\right)\\&\times\left[(\gamma^\mu C)_{\gamma\delta}(\gamma_\mu C)_{AB}\left(stL_1+\frac{1}{2}L_2\right)+(\gamma^\mu C)_{\gamma B}(\gamma_\mu C)_{A\delta}L_2\right]\\&\times\mathrm{Tr}(\lambda_1\lambda_2\lambda_3),\end{aligned}\tag{11}$$

with

$$\begin{aligned}L_1&=(2)^{-2(t+s+u)}\pi\frac{\Gamma(-u)\Gamma(-s)\Gamma(-t)\Gamma(-t-s-u+\frac{1}{2})}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)},\\L_2&=(2)^{-2(t+s+u)+1}\pi\frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u+1)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)}.\end{aligned}\tag{12}$$

This part of the amplitude is not vanished for different cases. For example, for $n=p+2$ the first term in (11) has infinite singularities in u-channel and in particular it involves many contact terms. The expansion is low energy expansion which reflects the fact that all Mandelstam variables must send to zero (for further details on the expansions see [12]). Hence, it becomes obvious that the first term of (11) includes all massless poles; however, we postpone its field theory computations to the next section to see what kinds of open strings, namely gauge/scalar or fermion can be replaced in the propagator.

Let us move to the second part of the amplitude. Having replaced the second part of scalar vertex operator and the other vertices into (8), the second part of the amplitude can be found as follows

$$\begin{aligned}\mathcal{A}_2^{C\phi\bar\psi\psi}={}&\frac{\mu_p\pi^{-1/2}}{4}\int dx_1\,dx_2\,dx_3\,dx_4\,dx_5\,(P_-\not{H}_{(n)}M_p)^{\gamma\delta}\xi_{1i}(2ik_{1a})\\&\times\bar u_1^\alpha u_2^\beta(x_{23}x_{24}x_{25}x_{34}x_{35}x_{45})^{-1/4}\\&\times\langle:\psi^a\psi^i(x_1):S_\alpha(x_2):S_\beta(x_3):S_\gamma(x_4):S_\delta(x_5):\rangle I\,\mathrm{Tr}(\lambda_1\lambda_2\lambda_3),\end{aligned}\tag{13}$$

in which

$$\begin{aligned}I={}&|x_{12}|^{\alpha'^2k_1\cdot k_2}|x_{13}|^{\alpha'^2k_1\cdot k_3}|x_{14}x_{15}|^{\frac{\alpha'^2}{2}k_1\cdot p}|x_{23}|^{\alpha'^2k_2\cdot k_3}|x_{24}x_{25}|^{\frac{\alpha'^2}{2}k_2\cdot p}\\&\times|x_{34}x_{35}|^{\frac{\alpha'^2}{2}k_3\cdot p}|x_{45}|^{\frac{\alpha'^2}{4}p\cdot D\cdot p}.\end{aligned}$$

The only subtlety in the second part of the amplitude is how to derive the correlation function between four spin operators (with the same chirality) and one current. Here we try to summarize the procedure of deriving this correlator. First, we need to take into account the following OPE

$$:\psi^\mu\psi^\nu(x_1):S_\alpha(x_2):\sim -\frac{1}{2}\big(\Gamma^{\mu\nu}\big)_\alpha^\lambda S_\lambda(x_2)x_{12}^{-1}, \tag{14}$$

with the definition of antisymmetric matrix as

$$\Gamma^{\mu\nu}=\frac{1}{2}\big(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu\big).$$

The next step is to replacing this OPE (14) into the following correlator

$$\big\langle:\psi^a\psi^i(x_1):S_\alpha(x_2):S_\beta(x_3):S_\gamma(x_4):S_\delta(x_5):\big\rangle,$$

and make use of the rest of the correlator which is the correlator of four spin operators (it is given in [35] and it has two different terms). Note that we have to apply the same formalism for the other OPEs and finally add them up. Concerning this method we have eight different terms; however, in order to obtain the final answer some extraordinary works are needed. Let us point them out. We need to extract all gamma matrices and make use of the commutator and anticommutator relations $\{\gamma^a,\gamma^b\}=-2\eta^{ab}$, $\{\gamma^a,\gamma^i\}=0$. The next step is to use the world-sheet fermion correlators as below:

$$\big\langle\psi^\mu(z)\psi^\nu(w)\big\rangle=-\frac{\alpha'}{2}\eta^{\mu\nu}(z-w)^{-1},$$
$$\big\langle:S_A(x_1):S_B(x_2):\psi^i(x_3):\big\rangle=2^{-1/2}x_{12}^{-3/4}(x_{31}x_{32})^{-1/2}\big(\gamma^i\big)_{AB}.$$

We also need to add all the terms carrying common coefficients of the gamma matrices. Finally one has to construct different combinations of the gamma matrices. Having taken remarks that appeared in Appendices A.1, A.3, B.3 and Section 6 of [35], one can clarify how the various terms come from. The final answer for $\langle:\psi^a\psi^i(x_1):S_\alpha(x_2):S_\beta(x_3):S_\gamma(x_4):S_\delta(x_5):\rangle$ has rather complicated result, therefore let us just point out several tests in favor of getting the correct result for our correlator.

The first test of our computation is to produce the leading singularities of the amplitude in which our calculation passes this test. The other unusual check after having replaced the final answer for the correlator into the amplitude is in fact the $SL(2,R)$ invariance of the amplitude in which our result satisfies that constraint. We gauge fix the amplitude as before and evaluate the integrals on closed string location. We write down the final answer for the second part of the amplitude:

$$\begin{aligned}\mathcal{A}_2^{C\phi\bar\psi\psi}={}&\frac{\mu_p\pi^{-1/2}}{16}(P_-\not{H}_{(n)}M_p)^{\gamma\delta}\xi_{1i}(2ik_{1a})\bar u_1^\alpha u_2^\beta\\&\times(\mathcal{A}_{21}+\mathcal{A}_{22}+\mathcal{A}_{23}+\mathcal{A}_{24}+\mathcal{A}_{25}+\mathcal{A}_{26})\,\mathrm{Tr}(\lambda_1\lambda_2\lambda_3),\end{aligned}$$

such that

$$\mathcal{A}_{21}=-\big(\Gamma^{ai\mu}C\big)_{\alpha\beta}(\gamma_\mu C)_{\gamma\delta}\left[\frac{1}{2}u(s+t)L_1+L_3(-s-t)\right],$$
$$\mathcal{A}_{22}=-\big(\Gamma^{ai\mu}C\big)_{\alpha\delta}(\gamma_\mu C)_{\gamma\beta}\left[L_1(us)-\frac{1}{2}L_2\right],$$
$$\mathcal{A}_{23}=\big(\Gamma^{ai\mu}C\big)_{\gamma\beta}(\gamma_\mu C)_{\alpha\delta}\left[L_1(ut)-\frac{1}{2}L_2\right],$$

$$\begin{aligned}
\mathcal{A}_{24} &= \left(\Gamma^{ai\mu}C\right)_{\gamma\delta}(\gamma_\mu C)_{\alpha\beta}\left[L_1(st)+\frac{1}{2}L_2\right],\\
\mathcal{A}_{25} &= (P_-H\!\!\!/_{(n)}M_p)^{\gamma\delta}\xi_{1i}(2ik_{1a})\bar{u}_1^\alpha u_2^\beta\left[u(s+t)L_1+L_2\right]\left(\left(\gamma^i C\right)_{\gamma\beta}\left(\gamma^a C\right)_{\alpha\delta}\right.\\
&\quad\left.-\left(\gamma^i C\right)_{\alpha\delta}\left(\gamma^a C\right)_{\gamma\beta}\right),\\
\mathcal{A}_{26} &= \left[-\frac{1}{2}u(s-t)L_1+(-t+s)L_3\right]\left(-\left(\gamma^i C\right)_{\gamma\delta}\left(\gamma^a C\right)_{\alpha\beta}+\left(\gamma^i C\right)_{\alpha\beta}\left(\gamma^a C\right)_{\gamma\delta}\right), \qquad (15)
\end{aligned}$$

where L_1, L_2 appeared in (12) and L_3 is

$$L_3 = (2)^{-2(t+s+u)-1}\pi\frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)}. \qquad (16)$$

The first terms of $\mathcal{A}_{21}$, $\mathcal{A}_{22}$, $\mathcal{A}_{25}$, $\mathcal{A}_{26}$ ($\mathcal{A}_{23}$) have just t-channel (s-channel) fermion poles and in particular all the terms including the coefficients of L_2 are just related to infinite contact interactions of one RR, two fermions and one scalar field in which they do not have any contribution to the singularities. On the other hand the first term of $\mathcal{A}_{24}$ has just either infinite u-channel gauge or scalar poles. Notice that the second terms of $\mathcal{A}_{21}$, $\mathcal{A}_{25}$, $\mathcal{A}_{26}$ involve just s-channel fermion poles. Finally the third and fourth terms of $\mathcal{A}_{21}$, $\mathcal{A}_{26}$ consist of indeed an infinite number of massless $(t+s+u)$-channel poles. In field theory we would clarify what kinds of poles we would have.

Note that these infinite poles have to be produced either by infinite higher derivative corrections to two scalars–two fermions or by two fermions–one scalar–one gauge field corrections in IIB. The amplitude is also antisymmetric with respect to the interchange of the fermions as we expected.

We make some comments on T-duality. The complete form of this S-matrix cannot be obtained by setting T-duality to the previous results (see [22,23]), since our amplitude includes some terms carrying p^i (closed string momentum in transverse direction). These terms cannot derived by applying T-duality, given the fact that winding modes are not embedded in the explicit form of RR vertex operator. Likewise it is shown in [22] that $C\phi AA$ (one RR, a scalar and two gauge fields) amplitude cannot be fully derived from $CAAA$. Thus we employ direct computations to find out some special patterns for superstring amplitudes, including fermion vertex operators.

The expansion is low energy expansion which reflects the fact that all Mandelstam variables should send to zero ($t,s,u\to 0$) such that this relation $t+s+u=-p^a p_a$ holds. The closed form of the expansion of stL_1, L_3 (to be able to produce u-channel poles [37] and $(t+s+u)$-channel poles [12]) can be written down as

$$\begin{aligned}
stL_1 &= -\pi^{3/2}\left[\sum_{n=-1}^{\infty} b_n\left(\frac{1}{u}(t+s)^{n+1}\right)+\sum_{p,n,m=0}^{\infty} e_{p,n,m}u^p(st)^n(s+t)^m\right],\\
L_3 &= -\frac{\pi^{5/2}}{2}\left(\sum_{n=0}^{\infty} c_n(s+t+u)^n+\frac{\sum_{n,m=0}^{\infty} c_{n,m}[s^n t^m+s^m t^n]}{(t+s+u)}\right.\\
&\quad\left.+\sum_{p,n,m=0}^{\infty} f_{p,n,m}(s+t+u)^p\left[(s+t)^n(st)^m\right]\right), \qquad (17)
\end{aligned}$$

where suL_1 and utL_1 can be derived by replacing $t\leftrightarrow u$ and $s\leftrightarrow u$ inside stL_1 expansion and $L_2=-4(t+s+u)L_3$ which has just contact terms. Some of the coefficients are

$$b_{-1}=1,\qquad b_0=0,\qquad b_1=\frac{1}{6}\pi^2,\qquad b_2=2\zeta(3),\qquad c_0=0,$$
$$c_1=-\frac{\pi^2}{6},\qquad e_{0,0,1}=\frac{1}{3}\pi^2,\qquad c_2=-2\zeta(3),\qquad c_{1,1}=\frac{\pi^2}{6},$$
$$e_{0,1,0}=2\zeta(3),\qquad e_{1,0,0}=\frac{1}{6}\pi^2,\qquad e_{1,0,2}=\frac{19}{60}\pi^4,\qquad e_{1,0,1}=6\zeta(3),$$
$$c_{0,0}=\frac{1}{2},\qquad f_{0,1,0}=\frac{\pi^2}{3},\qquad f_{0,0,1}=-2\zeta(3). \tag{18}$$

It is important to mention that the general structure of b_n coefficients of this paper is exactly the same structure of b_n coefficients that appeared in the amplitude of one RR and three scalars and they have quite universal behavior [12]; however, some coefficients of c_n, $f_{p,n,m}$ include differences from the coefficients of non-BPS amplitudes [26]. Let us move to field theory section and produce all u,t,s-channel gauge, scalar/fermion poles. In addition to them, we study different $(u+t+s)$-channel scalar and gauge poles in order to obtain all infinite higher derivative corrections of two fermions–two scalars or two fermions–one gauge–one scalar in the world volume of BPS branes of IIB superstring theory.

3. Infinite u-channel scalar poles for $p+2=n$ case

One can expand all the terms of the closed form of $\langle V_C V_\phi V_{\bar\psi} V_\psi\rangle$ which carry stL_1 coefficient, to actually produce both infinite u-channel scalar and gauge poles. It is shown in [12,14,22] that the kinetic term of gauge and scalar fields do not receive any correction. If one considers the stL_1 expansion then one can collect all the infinite u-channel scalar poles of the string theory amplitude as follows

$$\mathcal{A}_1^{C\phi\bar\psi\psi}=\frac{\mu_p i p^i\pi}{8}\sum_{n=-1}^{\infty}b_n\left(\frac{1}{u}(t+s)^{n+1}\right)(P_-\not{H}_{(n)}M_p)^{\gamma\delta}\xi_{1i}\bar u_1^A u_2^B$$
$$\times\left(\gamma^\mu C\right)_{\gamma\delta}(\gamma_\mu C)_{AB}\operatorname{Tr}(\lambda_1\lambda_2\lambda_3). \tag{19}$$

In the above amplitude μ can be both world volume and transverse direction. First, we set it to transverse direction ($\mu=j$) and extract the trace as

$$(P_-\not{H}_{(n)}M_p)^{\gamma\delta}\left(\gamma^j C\right)_{\gamma\delta}=\frac{32}{2(p+1)!}\left(\varepsilon^v\right)^{a_0\cdots a_p}H^j_{a_0\cdots a_p}. \tag{20}$$

Replacing the trace in the amplitude we obtain

$$\mathcal{A}_1^{C\phi\bar\psi\psi}=\frac{2\mu_p i p^i\pi}{(p+1)!}\sum_{n=-1}^{\infty}b_n\left(\frac{1}{u}(t+s)^{n+1}\right)\xi_{1i}$$
$$\times\bar u_1^A(\gamma_j)_{AB}u_2^B\left(\varepsilon^v\right)^{a_0\cdots a_p}H^j_{a_0\cdots a_p}\operatorname{Tr}(\lambda_1\lambda_2\lambda_3). \tag{21}$$

In below we will show that the kinetic term of fermion fields has to be fixed and it does not receive any correction. The massless poles should be reproduced by the following non-Abelian kinetic terms[2]:

[2] By replacing $2\pi\alpha'\phi^i=X^i$ the kinetic term of the scalar field gets canonically normalized; however, in this paper we keep the standard notation for the kinetic term of the open strings as they appear in (22).

$$-T_p\left(2\pi\alpha'\right)\mathrm{Tr}\left(\frac{(2\pi\alpha')}{2}D_a\phi^i D^a\phi_i-\frac{(2\pi\alpha')}{4}F_{ab}F^{ba}-\bar{\Psi}\gamma^a D_a\Psi\right). \tag{22}$$

To work with the field theory of an amplitude including RR and some massless scalar fields one must consider three different approaches to explore their vertices. Basically these methods are either Wess–Zumino (WZ) terms that introduced by Myers [8], or the needed pull-back methods or the so-called Taylor expansion (they are argued in Section 5 of [14]).

One has to take into account the following field theory amplitude to produce all scalar u-channel poles

$$\mathcal{A}=V^i_\alpha(C_{p+1},\phi_1,\phi)G^{ij}_{\alpha\beta}(\phi)V^j_\beta(\phi,\bar{\Psi}_1,\Psi_2). \tag{23}$$

Here we should employ Taylor expansion to obtain the vertex of two scalars and one RR $(p+1)$-form field as follows:

$$i\frac{\lambda^2\mu_p}{2!(p+1)!}\int d^{p+1}\sigma\left(\varepsilon^v\right)^{a_0\cdots a_p}\mathrm{Tr}\left(\partial_i\partial_j C^{(p+1)}_{a_0\cdots a_p}\phi^i\phi^j\right), \tag{24}$$

where $\lambda=2\pi\alpha'$. The vertex of one RR and two scalars can be constructed from (24)

$$V^i_\alpha(C_{p+1},\phi_1,\phi)=i\frac{\lambda^2\mu_p}{2!(p+1)!}\left(-ip^i\right)H^j_{a_0\cdots a_p}\xi_{1j}\left(\varepsilon^v\right)^{a_0\cdots a_p}\mathrm{Tr}\left(\lambda_1\lambda^\alpha\right). \tag{25}$$

The scalar propagator is derived from scalar fields kinetic term (the first term in (22)). To obtain the vertex of two on-shell fermions and one off-shell scalar field we need to work with the kinetic term of fermion fields (the last term in (22)) where the commutator in the definition of covariant derivative of fermion field has to be considered, such that

$$\begin{aligned}
&V^\beta_j(\bar{\Psi}_1,\Psi_2,\phi)=T_p\left(2\pi\alpha'\right)\bar{u}^A_1\gamma^j_{AB}u^B_2\left(\mathrm{Tr}\left(\lambda_2\lambda_3\lambda^\beta\right)-\mathrm{Tr}\left(\lambda_3\lambda_2\lambda^\beta\right)\right),\\
&G^{ij}_{\alpha\beta}(\phi)=\frac{-i\delta_{\alpha\beta}\delta^{ij}}{T_p(2\pi\alpha')^2k^2}=\frac{-i\delta_{\alpha\beta}\delta^{ij}}{T_p(2\pi\alpha')^2u},
\end{aligned} \tag{26}$$

k is the momentum of off-shell scalar field in the propagator. Now if we replace the above vertices in the field theory amplitude of (23) then the first simple u-channel scalar pole of the string theory amplitude (for $n=-1$ in (21)) can be precisely produced. However, the amplitude has infinite u-channel poles. In order to deal with them a key point has to be made. The kinetic term of fermion fields has no correction (as it is fixed in DBI action) and scalar propagator does not receive any correction either (because it is just simple pole). Therefore the only way to produce all the other massless scalar poles is to devote infinite higher derivative corrections to the vertex of one RR $(p+1)$-form field and two scalar fields as follows:

$$\begin{aligned}
&i\frac{\lambda^2\mu_p}{2!(p+1)!}\int d^{p+1}\sigma\left(\varepsilon^v\right)^{a_0\cdots a_p}\sum_{n=-1}^{\infty}b_n\left(\alpha'\right)^{n+1}\\
&\quad\times\mathrm{Tr}\left(\partial_i\partial_j C^{(p+1)}_{a_0\cdots a_p}D^{a_0}\cdots D^{a_n}\phi^i D_{a_0}\cdots D_{a_n}\phi^j\right).
\end{aligned} \tag{27}$$

Notice that in (27) all the commutators in the covariant derivative of scalar fields should be ignored as we need not have any gauge field. Now by applying standard field theory techniques we can extract the vertex of one RR, one on-shell fermion and an on-shell fermion field from (27) as below:

$$
\begin{aligned}
&V^i_\alpha(C_{p+1},\phi_1,\phi)\\
&\quad = i\frac{\lambda^2\mu_p}{2!(p+1)!}\left(-ip^i\right)H^j_{a_0\cdots a_p}\xi_{1j}\left(\varepsilon^v\right)^{a_0\cdots a_p}\sum_{n=-1}^{\infty}b_n\left(\alpha' k_1\cdot k\right)^{n+1}\operatorname{Tr}\left(\lambda_1\lambda^\alpha\right),
\end{aligned}
\tag{28}
$$

substituting (28) and (26) into (23), we obtain

$$
\frac{\alpha'\pi p^i\mu_p}{(p+1)!}\xi_{1i}\bar{u}^A_1(\gamma_j)_{AB}u^B_2\sum_{n=-1}^{\infty}b_n\frac{(t+s)^{n+1}}{u}\left(\varepsilon^v\right)^{a_0\cdots a_p}H^j_{a_0\cdots a_p}\operatorname{Tr}(\lambda_1\lambda_2\lambda_3),
\tag{29}
$$

which is exactly all the infinite u-channel scalar poles inside the string amplitude of (21), as we expected. Hence to provide all infinite u-channel scalar poles in field theory, we had to generalize the vertex of one RR $(p+1)$-form field, an on-shell and an off-shell scalar field. We observed that the closed string Ramond–Ramond has induced all infinite higher derivative corrections to two scalar fields. This phenomenon seemed to be universal as it so happens for the other BPS and non-BPS amplitudes (see [12,22,23,14]). In the next section we construct all infinite higher derivative corrections of one RR $(p-1)$-form field, one gauge and one scalar field to be able to produce all the infinite u-channel gauge poles of $C\phi\bar{\psi}\psi$ amplitude.

4. Infinite u-channel gauge poles for $p = n$ case

In this section we are going to produce all the singularities that appeared in $\mathcal{A}_{24}$. By extracting the trace, replacing the first part of the expansion of stL_1 (just singularities) and setting μ to be located in world volume direction ($\mu = b$) in $\mathcal{A}_{24}$, one can express all the infinite u-channel gauge poles of the string theory amplitude as follows:

$$
\begin{aligned}
\mathcal{A}_{24} ={}& \frac{4\mu_p\pi\xi_{1i}(ik_{1a})}{p!}\bar{u}^A_1(\gamma_b)_{AB}u^B_2\sum_{n=-1}^{\infty}b_n\frac{1}{u}(t+s)^{n+1}\\
&\times\left(\varepsilon^v\right)^{a_0\cdots a_{p-2}ab}H^i_{a_0\cdots a_{p-2}}\operatorname{Tr}(\lambda_1\lambda_2\lambda_3).
\end{aligned}
\tag{30}
$$

Note that we have ignored the second term of $\mathcal{A}_{24}$ because it is just contact interaction which has no singularity. In [12,14,22] we have explained how to derive all contact terms in the field theory. To produce all the infinite u-channel gauge poles the following Feynman rule in field theory has to be considered

$$
\mathcal{A} = V^a_\alpha(C_{p-1},\phi_1,A)G^{ab}_{\alpha\beta}(A)V^b_\beta(A,\bar{\Psi}_1,\Psi_2),
\tag{31}
$$

where $V^a_\alpha(C_{p-1},\phi_1,A)$ can be derived by making use of the combination of Taylor expansion and WZ terms as follows

$$
i\frac{\lambda^2\mu_p}{p!}\int d^{p+1}\sigma\operatorname{Tr}\left(\partial_i C_{(p-1)}\wedge F\phi^i\right).
\tag{32}
$$

If we apply standard field theory techniques then one can feasibly derive the vertex of one RR-$(p-1)$-form field, one off-shell gauge field and one on-shell scalar field as below

$$
V^a_\alpha(C_{p-1},\phi_1,A) = i\frac{\lambda^2\mu_p}{p!}H^i_{a_0\cdots a_{p-2}}\xi_{1i}k_{a_{p-1}}\left(\varepsilon^v\right)^{a_0\cdots a_{p-1}a}\operatorname{Tr}\left(\lambda_1\lambda^\alpha\right)
\tag{33}
$$

where k is momentum of off-shell gauge field $k = k_2 + k_3$. To derive gauge propagator one should work with the kinetic term of gauge fields. To obtain the vertex of two on-shell fermion fields and one off-shell gauge field ($V^b_\beta(\bar\Psi_1,\Psi_2,A)$), one needs to make use of the kinetic term of fermions and also to extract covariant derivative of the fermion field ($D^a\psi=\partial^a\psi-i[A^a,\psi]$). Essentially one has to take into account all possible orderings of the gauge and fermions to be able to obtain $V^b_\beta(A,\bar\Psi_1,\Psi_2)$ as well as gauge propagator

$$V^b_\beta(A,\bar\Psi_1,\Psi_2)=T_p\big(2\pi\alpha'\big)\bar u^A_1\gamma^b_{AB}u^B_2\big(\mathrm{Tr}\big(\lambda_2\lambda_3\lambda^\beta\big)-\mathrm{Tr}\big(\lambda_3\lambda_2\lambda^\beta\big)\big),$$
$$G^{ab}_{\alpha\beta}(A)=\frac{-i\delta_{\alpha\beta}\delta^{ab}}{T_p(2\pi\alpha')^2k^2}=\frac{-i\delta_{\alpha\beta}\delta^{ab}}{T_p(2\pi\alpha')^2u},\tag{34}$$

with k becomes off-shell gauge field propagator ($k^a=(k_2+k_3)^a=(-p-k_1)^a$). Replacing the above vertices in the field theory amplitude of (31) we can precisely find the first simple u-channel gauge pole of the string theory (for $n=-1$ in the amplitude of (30)). The constraint for Ramond–Ramond $p^a(\varepsilon^v)^{a_0\cdots a_{p-1}a}=0$ has also been used. It is worth trying to produce all infinite u-channel massless gauge poles. We made some points in the previous section and understood that the kinetic term of fermion fields has no correction and simple scalar/gauge propagators also do not receive any correction. Hence, the only way to produce all the other u-channel gauge poles is to propose higher derivative corrections to the vertex of one RR $(p-1)$-form field, one on-shell scalar and an off-shell gauge field as follows

$$i\frac{\lambda^2\mu_p}{p!}\int d^{p+1}\sigma\sum_{n=-1}^{\infty}b_n\big(\alpha'\big)^{n+1}\mathrm{Tr}\big(\partial_iC_{(p-1)}\wedge D^{a_0}\cdots D^{a_n}FD_{a_0}\cdots D_{a_n}\phi^i\big).\tag{35}$$

Now we are ready to extract the higher extension of the vertex of $V^a_\alpha(C_{p-1},\phi_1,A)$ as below

$$V^a_\alpha(C_{p-1},\phi_1,A)=i\frac{\lambda^2\mu_p}{p!}H^i_{a_0\cdots a_{p-2}}\xi_{1i}k_{a_{p-1}}\big(\varepsilon^v\big)^{a_0\cdots a_{p-1}a}\sum_{n=-1}^{\infty}b_n\big(\alpha'k_1\cdot k\big)^{n+1}\mathrm{Tr}\big(\lambda_1\lambda^\alpha\big).\tag{36}$$

Notice that (36) is all order extension of (33). Substituting (36) into (31) we are able to exactly reproduce all infinite u-channel gauge poles of the amplitude that appeared in (30). Hence closed string Ramond–Ramond $(p-1)$-form field has proposed all infinite higher derivative corrections to an on-shell scalar and one off-shell gauge field. We might wonder about this universal behavior of higher derivative corrections that RR has induced to open strings. Just for the completeness we mention that, this phenomenon has also been seen in non-BPS brane systems (see [14]).

5. Infinite t, s-channel fermion poles

For $C\phi\bar\psi\psi$ amplitude, there is no graviton propagator in s,t channels, because the particle exchanged must have non-zero fermion number. Furthermore, the particles exchanged are corresponding to open string excitations thus for these channels there is no coupling between one Ramond vertex operator, one scalar and one graviton as one cannot saturate the total super ghost charge for disk amplitude. Therefore the only propagator for these channels is indeed fermionic propagator. To find all the singularities related to t-channel we need to replace just the first term of usL_1 expansion inside $A_{21}, A_{22}, A_{25}, A_{26}$, extract the related traces and simplify the amplitude more. By applying these points, we are able to write down all $t(s)$-channel fermion poles of the string amplitude as below:

$$\mathcal{A} = \frac{\alpha' \mu_p \pi \xi_{1i}(2ik_{1a})}{(p+1)!} \bar{u}_1^A \big(\gamma^a\big)_{AB} u_2^B \sum_{n=-1}^{\infty} b_n \left[\frac{1}{t}(u+s)^{n+1} - \frac{1}{s}(u+t)^{n+1}\right] \times \big(\varepsilon^v\big)^{a_0\cdots a_p} H^i_{a_0\cdots a_p} \operatorname{Tr}(\lambda_1\lambda_2\lambda_3). \tag{37}$$

The amplitude is totally antisymmetric under interchange of the fermionic strings ($s \leftrightarrow t$), thus there is no need to produce all s-channel poles. We just produce all fermionic t-channel poles and finally by interchanging $2 \leftrightarrow 3$ and $s \leftrightarrow t$ in all kinematic relations, all s-channel fermionic poles can be concluded as well. All the terms that include L_2 coefficients are just contact interactions and have nothing to do with these singularities. Let us write down the rule to derive all the fermionic t-channel poles

$$\mathcal{A} = V_\alpha(C_{p+1}, \Psi_3, \bar{\Psi}) G_{\alpha\beta}(\Psi) V_\beta(\Psi, \bar{\Psi}_2, \phi_1). \tag{38}$$

The fermion propagator is found by making use of the kinetic term of fermion fields (the last term in (22)). If we extract the covariant derivative of fermion inside its kinetic term ($D^i\psi = \partial^i\psi - i[\phi^i, \psi]$) and count all possible orderings of the scalar and fermions, we can derive the vertex of one on-shell, one off-shell fermion and an on-shell scalar field ($V_\beta(\Psi, \bar{\Psi}_2, \phi_1)$) as well as fermion propagator as below

$$\begin{aligned} V_\beta(\Psi, \bar{\Psi}_2, \phi_1) &= T_p\big(2\pi\alpha'\big)\bar{u}_1^A \gamma_A^j \xi_{1j}\big(\operatorname{Tr}\big(\lambda_1\lambda_2\lambda^\beta\big) - \operatorname{Tr}\big(\lambda_2\lambda_1\lambda^\beta\big)\big), \\ G_{\alpha\beta}(\psi) &= \frac{-i\delta_{\alpha\beta}}{T_p(2\pi\alpha')\not{k}} = \frac{-i\delta_{\alpha\beta}\gamma^a(k_1+k_2)_a}{T_p(2\pi\alpha')t}. \end{aligned} \tag{39}$$

In order to find $V_\alpha(C_{p+1}, \bar{\Psi}, \Psi)$, one has to find out new coupling between one RR $(p+1)$-form field, one on-shell and one off-shell fermion field in the world volume of BPS branes as follows

$$i\frac{(2\pi\alpha')\mu_p}{(p+1)!}\int d^{p+1}\sigma \operatorname{Tr}\big(C_{a_0\cdots a_p}\bar{\Psi}\gamma^j\partial_j\Psi\big)\big(\varepsilon^v\big)^{a_0\cdots a_p}, \tag{40}$$

which is in fact supersymmetrized version of known bosonic couplings. Note that the equations of motion for fermions must be taken into account ($\not{k}_{2a}\bar{u} = \not{k}_{3a}u = 0$) as well. Now the vertex of one RR $(p+1)$-form field, one off-shell and one on-shell fermion field can be derived from (40) as follows

$$V_\alpha(C_{p+1}, \bar{\Psi}, \Psi_2) = i\frac{(2\pi\alpha')\mu_p}{(p+1)!} H^i_{a_0\cdots a_p}\gamma^i u_2\big(\varepsilon^v\big)^{a_0\cdots a_p}\operatorname{Tr}\big(\lambda_3\lambda^\alpha\big). \tag{41}$$

If we replace the vertices that appeared in (39) and (41) to the field theory amplitude of (38) then we can find the first simple t-channel fermion pole of the string theory amplitude (for $n=-1$ in (37)). Once more in order to derive all infinite t- or s-channel fermion poles, one has to impose the infinite higher derivative interactions to new coupling (40). Thus the kinetic term of fermion field should be fixed. Now one has to look for the infinite higher derivative corrections to the vertex of $V_\alpha(C_{p+1}, \Psi_3, \bar{\Psi})$ as follows

$$i\frac{(2\pi\alpha')\mu_p}{(p+1)!}\int d^{p+1}\sigma \sum_{n=-1}^{\infty} b_n\big(\alpha'\big)^{n+1} \times \operatorname{Tr}\big(C_{a_0\cdots a_p}D^{a_0}\cdots D^{a_n}\bar{\Psi}\gamma^i D_{a_0}\cdots D_{a_n}\partial_i\Psi\big)\big(\varepsilon^v\big)^{a_0\cdots a_p}. \tag{42}$$

If we consider (42) then the complete form of the $V_\alpha(C_{p+1}, \Psi_3, \bar{\Psi})$ to all orders in α' will become

$$V_\alpha(C_{p+1},\bar{\Psi},\Psi)=i\frac{(2\pi\alpha')\mu_p}{(p+1)!}H^i_{a_0\cdots a_p}\gamma^i u_2\big(\varepsilon^v\big)^{a_0\cdots a_p}\operatorname{Tr}\big(\lambda_3\lambda^\alpha\big)\sum_{n=-1}^{\infty}b_n\big(\alpha' k_3.k\big)^{n+1}. \quad (43)$$

An important point should be emphasized, namely all the commutator terms inside the covariant derivative of fermion fields in (42) must be neglected. Note that in producing all fermion poles in field theory side, we have employed the equations of motion for fermion fields $\not{k}_2\bar{u}=\not{k}_3 u=0$ as well. Replacing (43) into (38), one can show that all fermion $t(s)$-channel fermion poles are exactly reconstructed. Therefore the closed string Ramond–Ramond $(p+1)$-form field has induced an infinite number of higher derivative corrections to two fermions, two scalar fields as well as one scalar–one gauge field. This clearly confirms that, this phenomenon (producing all poles of the string amplitude by postulating infinite higher derivative corrections to the vertex of one RR and some open string vertex operators) is quite universal and might be useful for deriving all the singularities of the higher point functions of either BPS or non-BPS branes without the need for knowing the exact results of the world sheet integrals of the higher point functions.

5.1. Infinite higher derivative corrections to two fermions–two scalars and all scalar poles for $p=n-2$ case

The first goal of this section is to produce poles order by order and to actually observe whether or not the universal conjecture on all order α' higher derivative corrections (which has been made for bosonic amplitude in [23]) can be held for the fermionic amplitudes. Another goal would be determining all the infinite higher derivative corrections of two fermions–two scalars in IIB superstring theory. We first consider the infinite massless scalar $(s+t+u)=-p^a p_a$ channel poles of the string amplitude of $C_{p+1}\bar{\Psi}\Psi\phi$. If we extract the related traces in the second term of A_{26} then one finds out all the singularities as below

$$\mathcal{A}=-\frac{\alpha'\pi^{-1/2}\mu_p}{(p+1)!}\big(\varepsilon^v\big)^{a_0\cdots a_p}H^i_{a_0\cdots a_p}\xi_{1i}(2ik_{1a})\bar{u}_1^A\big(\gamma^a\big)_{AB}u_2^B\operatorname{Tr}(\lambda_1\lambda_2\lambda_3)\big[(-t+s)L_3\big]. \quad (44)$$

The expansion of L_3 has infinite $(t+s+u)$-channel poles so let us see its expansion before the simplification:

$$L_3=-\pi^{5/2}\bigg(\frac{1}{2(t+s+u)}+\frac{\pi^2(t^2+s^2+u^2)}{12(t+s+u)}+\frac{\xi(3)(t^3+s^3+u^3+tsu)}{(t+s+u)}+\cdots\bigg). \quad (45)$$

For the moment we just work with the first $(t+s+u)$-channel pole. Reminding the coupling of one RR and one scalar field is worthwhile for the field theory amplitude

$$\lambda\mu_p\int d^{p+1}\sigma\frac{1}{(p+1)!}\big(\varepsilon^v\big)^{a_0\cdots a_p}\operatorname{Tr}\big(\phi^i\big)H^{(p+2)}_{ia_0\cdots a_p}. \quad (46)$$

In order to produce the first $(t+s+u)$-channel scalar pole, the following Feynman rule has to be taken into account

$$\mathcal{A}=V^i_\alpha(C_{p+1},\phi)G^{ij}_{\alpha\beta}(\phi)V^j_\beta(\phi,\bar{\Psi},\Psi,\phi_1), \quad (47)$$

where the scalar propagator can be readily derived from the kinetic term of the scalar fields and $V^i_\alpha(C_{p+1},\phi)$ is derived from (46) so that

$$G^{ij}_{\alpha\beta}(\phi) = \frac{-i\delta_{\alpha\beta}\delta^{ij}}{T_p(2\pi\alpha')^2k^2} = \frac{-i\delta_{\alpha\beta}\delta^{ij}}{T_p(2\pi\alpha')^2(t+s+u)},$$
$$V^i_\alpha(C_{p+1},\phi) = i\big(2\pi\alpha'\big)\mu_p\frac{1}{(p+1)!}\big(\varepsilon^v\big)^{a_0\cdots a_p}H^i_{a_0\cdots a_p}\,\mathrm{Tr}(\lambda_\alpha). \tag{48}$$

Notice that scalar in $V^i_\alpha(C_{p+1},\phi)$ must be Abelian so $\mathrm{Tr}(\lambda_\alpha)$ makes sense for the Abelian matrix λ_α. If one looks at (44) then one understands that the first simple $(t+s+u)$-channel scalar pole has to be discovered by all the couplings between two scalars and two fermions in such a way that they should carry three momenta. The reason for this sharp result is that in (44) apart from the field strength of RR (which is absorbed in $V^i_\alpha(C_{p+1},\phi)$), the other terms carry three momenta. One can show that in field theory analysis the kinematic factor of $(-t+s)$ can be factorized. This is the key point in favor of a given universal conjecture of all infinite α' higher derivative corrections of the string amplitudes. Further details can be found in [23]. In order to find out two fermion two scalar couplings we need to write down all possible desired couplings carrying three momenta as below

$$\frac{T_p(2\pi\alpha')^3}{4}\big(\bar\Psi\gamma^aD_b\Psi D^a\phi^iD^b\phi_i + D^a\phi^iD^b\phi_i\bar\Psi\gamma^aD_b\Psi\big),$$
$$\frac{T_p(2\pi\alpha')^3}{8}\bar\Psi\gamma^aD_a\Psi D^b\phi^iD_b\phi_i. \tag{49}$$

Based on the prescription for all order higher order corrections given in [23], one needs to consider the multiplication of the kinetic terms of the open strings to end up with their all order α' higher derivative corrections. Thus for this part of the amplitude, we must multiply the kinetic terms of the fermion fields and scalar fields which considered in (49). Note that the coefficients of the couplings in (49) would be fixed in such a way that the first $(t+s+u)$-channel scalar pole in (44) would be resulted. Likewise the last section, all the commutator terms in the definitions of the covariant derivative of fermion/scalar fields must be overlooked. In order to produce the field theory vertex operators for the following coupling

$$\bar\Psi\gamma^aD_b\Psi D^a\phi^iD^b\phi_i,$$

one has to consider two possible orderings

$$\mathrm{Tr}(\lambda_2\lambda_3\lambda_1\lambda_\beta),\quad \mathrm{Tr}(\lambda_2\lambda_3\lambda_\beta\lambda_1),$$

where λ_β is related to Abelian scalar field in the propagator. By Replacing these orderings to the Feynman amplitude (47), $\mathrm{Tr}(\lambda_1\lambda_2\lambda_3)$ is produced. Thus by extracting $\frac{T_p(2\pi\alpha')^3}{4}(\bar\Psi\gamma^aD_b\Psi\times D^a\phi^iD^b\phi_i + D^a\phi^iD^b\phi_i\bar\Psi\gamma^aD_b\Psi)$ couplings of (49) and by considering all their orderings one obtains

$$V^j_\beta(\phi,\bar\Psi,\Psi,\phi_1) = i\frac{T_p(2\pi\alpha')^3}{4}\bar u^A\big(\gamma^a\big)_{AB}u^B\xi_{1j}\left(-k_{1a}\frac{t}{2} - k_{4a}\frac{s}{2}\right)\mathrm{Tr}(\lambda_1\lambda_2\lambda_3\lambda_\beta).$$

Note that in order to derive $V^j_\beta(\phi,\bar\Psi,\Psi,\phi_1)$, we used the momentum conservation $(k_1+k_2+k_3+k_4)^a = 0$ and made use of the equations of motion for fermion fields $(k^a_2\gamma_a\bar u = k^a_3\gamma_a u = 0)$. Setting these remarks, we are able to find the vertex of two on-shell fermions, one on-shell and one off-shell scalar field as follows

$$V^j_\beta(\phi,\bar\Psi,\Psi,\phi_1) = i\frac{T_p(2\pi\alpha')^3}{4}k_{1a}\bar u^A\big(\gamma^a\big)_{AB}u^B\xi_{1j}\left(-\frac{t}{2}+\frac{s}{2}\right)\mathrm{Tr}(\lambda_1\lambda_2\lambda_3\lambda_\beta). \tag{50}$$

Now if we replace (50) into (47) and also consider the first term of (45) to (44) then we are able to exactly produce the first $(t+s+u)$-channel scalar pole of (44).

For $(\bar{\Psi}\gamma^a D_a\Psi D^b\phi^i D_b\phi_i)$ coupling one has to consider two different mentioned orderings. If one extracts this coupling and considers standard field theory techniques then we may obtain

$$V_\beta^j(\phi,\bar{\Psi},\Psi,\phi_1) = -ik_{3a}\bar{u}^A\left(\gamma^a\right)_{AB}u^B\xi_{1j}u\,\mathrm{Tr}(\lambda_1\lambda_2\lambda_3\lambda_\beta),$$

one might apply on-shell condition $(t+s+u=0)$ to the above vertex operator to remove Mandelstam variable of u; however, by applying the equation of motion for fermion field $(\not{k}_3u^B=0)$ to the above vertex, we realize that, this coupling does not have any contribution to field theory amplitude. It is clear from the expansion of L_3 (45) that the string amplitude (44) has infinite massless $(t+s+u)$-channel scalar poles [37]. It is also seen in [14] that the vertex of $V_\alpha^i(C_{p+1},\phi)$ and the simple scalar propagator do not require any corrections, so one expects that all infinite $(t+s+u)$-channel scalar poles are related to all order α' higher derivative corrections of two fermions–two scalar fields of IIB superstring theory.

Let us now generalize our method to find out all order α' higher derivative corrections to two fermion–two scalar field couplings to be able to produce all the infinite $(t+s+u)$-channel scalar poles. Indeed we need to apply some higher derivative operators $(\mathcal{D}_{nm},\mathcal{D}'_{nm})$ to all couplings that have non-zero contributions to the field theory amplitude (the first and second term in (49)). Having taken the following couplings[3]

$$\begin{aligned}\mathcal{L}^{n,m} &= \pi^3\alpha'^{n+m+3}\\ &\quad\times T_p\Big(a_{n,m}\,\mathrm{Tr}\big[\mathcal{D}_{nm}\big(\bar{\Psi}\gamma^a D_b\Psi D^a\phi^i D^b\phi_i\big)+\mathcal{D}_{nm}\big(D^a\phi^i D^b\phi_i\bar{\Psi}\gamma^a D_b\Psi\big)+\text{h.c.}\big]\\ &\quad+ib_{n,m}\,\mathrm{Tr}\big[\mathcal{D}'_{nm}\big(\bar{\Psi}\gamma^a D_b\Psi D^a\phi^i D^b\phi_i\big)+\mathcal{D}'_{nm}\big(D^a\phi^i D^b\phi_i\bar{\Psi}\gamma^a D_b\Psi\big)+\text{h.c.}\big]\Big),\end{aligned} \tag{51}$$

with the following definitions of the higher derivative operators of $\mathcal{D}_{nm},\mathcal{D}'_{nm}$

$$\begin{aligned}\mathcal{D}_{nm}(EFGH) &\equiv D_{b_1}\cdots D_{b_m}D_{a_1}\cdots D_{a_n}EFD^{a_1}\cdots D^{a_n}GD^{b_1}\cdots D^{b_m}H,\\ \mathcal{D}'_{nm}(EFGH) &\equiv D_{b_1}\cdots D_{b_m}D_{a_1}\cdots D_{a_n}ED^{a_1}\cdots D^{a_n}FGD^{b_1}\cdots D^{b_m}H,\end{aligned}$$

we are now able to show that all the infinite $(t+s+u)$-channel scalar poles can be produced. In order to do so, we focus on the terms that carry $a_{n,m}$ coefficients. If we apply standard field theory techniques, consider hermitian conjugate of the first and second couplings in (51), take into account momentum conservation in world volume direction, use the equations of motion for fermion fields and finally take all different possible orderings[4] then one obtains the vertex of two on-shell fermions–one on-shell and one off-shell scalar to all orders of α' as follows:

$$V_\beta^j(\phi,\bar{\Psi},\Psi,\phi_1) = iT_p\frac{(2\pi\alpha')^3}{4}k_{1a}\bar{u}^A\left(\gamma^a\right)_{AB}u^B\xi_{1j}\left(-\frac{t}{2}t^ns^m+\frac{s}{2}t^ms^n\right)\mathrm{Tr}(\lambda_1\lambda_2\lambda_3\lambda_\beta). \tag{52}$$

Now if one replaces (52) into (47) then one is able to produce exactly all the infinite $(t+s+u)$-channel scalar poles in (44). For instance by putting $n,m=0$ inside (52), we showed

[3] Recent computations for all order α' higher derivative corrections of two fermions–two tachyons confirm that the same universal conjecture holds even for non-BPS branes (see [38]).

[4] Notice that BPS branes do not carry Chan–Paton factor so we do not expect to have any $(-\mathcal{D}_{nm})$, $(-\mathcal{D}'_{nm})$ operators in the couplings (51).

(compare (52) with (50)) that the first $(t+s+u)$-channel scalar pole is produced. Let us go on, now by replacing $n=1$, $m=0$ to (52) we obtain

$$V^j_\beta(\phi,\bar{\Psi},\Psi,\phi_1)=iT_p\frac{(2\pi\alpha')^3}{4}k_{1a}\bar{u}^A\left(\gamma^a\right)_{AB}u^B\xi_{1j}\left(-\frac{t^2}{2}+\frac{s^2}{2}\right). \tag{53}$$

Let us replace (53) into the field theory amplitude (47) and also consider the second term of the L_3 expansion of (17) inside (44). To do so, we discover that the over all coefficient of string amplitude $(k_{1a}(s-t))$ can be extracted from (53). The rest of the coefficients in field theory amplitude, namely $\frac{1}{2}(s+t)$ would be canceled if we would compare them with the coefficient of $c_{1,1}(s+t)$ of the string amplitude. Therefore the first simple $(t+s+u)$-channel scalar pole of the string and field theory amplitude is exactly matched. If we concentrate on the terms in (51) carrying $b_{n,m}$ coefficients and extract the vertex we derive

$$V^j_\beta(\phi,\bar{\Psi},\Psi,\phi_1)=iT_p\frac{(2\pi\alpha')^3}{4}k_{1a}\bar{u}^A\left(\gamma^a\right)_{AB}u^B\xi_{1j}\left(-\frac{t}{2}u^ns^m+\frac{s}{2}t^mu^n\right)\mathrm{Tr}(\lambda_1\lambda_2\lambda_3\lambda_\beta). \tag{54}$$

However, one has to apply on-shell condition $(s+t+u)=0$ into the above vertex to be able to produce all the infinite scalar poles. Indeed similar checks for all order α' higher derivative corrections to the other amplitudes in [12,22] have been carried out. This ends our goal of exploring all order α' higher derivative corrections to two fermions–two scalars of the world volume of BPS branes in type IIB superstring theory.

5.2. The higher derivative corrections to two fermions–one gauge and one scalar field and the infinite gauge poles for $p=n$ case

The final singularities in the amplitude of $C_{p-1}\bar{\Psi}\Psi\phi$ are related to all infinite massless $(s+t+u)$-channel gauge poles of the string theory. The goal of this section is to find out non-zero couplings of one gauge–one scalar and two fermions of IIB string theory and to fix their coefficients by producing all infinite gauge poles of the string amplitude. Of course, essentially one wants to derive all the infinite corrections. It would also be nice to see whether or not the universal conjecture about higher derivative corrections to all orders in α', made in [23] holds for fermionic amplitude? Our last goal is to fix the coefficients of all order corrections. In fact these coefficients must be found just by comparing the couplings in field theory with string theory amplitude and not by any other tools such as T-duality. One finds the singular terms in the string amplitude for $p=n$ case as follows:

$$\mathcal{A}=-\frac{\alpha'\pi^{-1/2}\mu_p}{(p)!}\left(\varepsilon^v\right)^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}}\xi_{1i}(2ik_{1a})\bar{u}_1^A\left(\gamma^i\right)_{AB}u_2^B\,\mathrm{Tr}(\lambda_1\lambda_2\lambda_3)[-2tL_3]. \tag{55}$$

A crucial remark is in order. The vertex of RR, looks like to the fermions vertex operators so one may suppose the prescription for $\langle V_C V_A V_A V_A\rangle$ in [12] or $\langle V_C V_\phi V_A V_A\rangle$ in [22][5] can be applied to this section as well but in this section we are looking for two fermions–one gauge–one scalar couplings. Thus we are not allowed to make use of the kinetic term of the gauge fields

[5] The complete form of two gauge–two scalar couplings to all orders in α' is found by comparing field theory amplitude with string theory amplitude of $\langle V_C V_\phi V_A V_A\rangle$ in [22].

because we need to take into account one scalar/gauge couplings to the other fields. Indeed in $\langle V_C V_\phi V_{\bar\psi} V_\psi\rangle$ there is no external gauge field so these one scalar/gauge–two fermion couplings have to be searched just by comparison the field theory vertices with string theory S-matrix. Eventually all the coefficients of the field theory couplings must be fixed through comparisons field theory couplings with S-matrix elements.

We have already pointed out that the expansion of L_3 had infinite $(t+s+u)$-channel poles. Let us first carry out field theory computations to produce just the first simple $(t+s+u)$-channel gauge pole. First of all we need to have the coupling between one RR $(p-1)$-form field and one gauge field as follows

$$i\big(2\pi\alpha'\big)\mu_p\int d^{p+1}\sigma\frac{1}{(p)!}C_{p-1}\wedge F. \tag{56}$$

This coupling is derived in [12]. In the amplitude of one RR and three gauge fields [12], we saw that $V(C_{p-1},A)$ does not receive any correction. Hence, for producing all the infinite gauge poles for $p=n$ case, one expects that the only corrections are related to the corrections of one off-shell gauge, one on-shell scalar field and two fermion fields (to all orders in α' in IIB superstring theory). Thus the infinite higher derivative corrections should be explored by matching all the infinite gauge poles of the string amplitude of $C_{p-1}\bar\Psi\Psi\phi$ with field theory amplitude. The Feynman rule is

$$\mathcal{A}=V_\alpha^a(C_{p-1},A)G_{\alpha\beta}^{ab}(A)V_\beta^b(A,\bar\Psi,\Psi,\phi_1), \tag{57}$$

where the gauge propagator is obtained from the kinetic term of gauge fields and $V_\alpha^a(C_{p-1},A)$ is derived from (56) such that

$$\begin{aligned} G_{\alpha\beta}^{ab}(A)&=\frac{-i\delta_{\alpha\beta}\delta^{ab}}{T_p(2\pi\alpha')^2k^2}=\frac{-i\delta_{\alpha\beta}\delta^{ab}}{T_p(2\pi\alpha')^2(t+s+u)},\\ V_\alpha^a(C_{p-1},A)&=i\big(2\pi\alpha'\big)\mu_p\frac{1}{(p)!}\big(\varepsilon^v\big)^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}}\operatorname{Tr}(\lambda_\alpha). \end{aligned} \tag{58}$$

The gauge field in $V_\alpha^a(C_{p-1},A)$ must be Abelian so $\operatorname{Tr}(\lambda_\alpha)$ makes sense just for the Abelian matrix λ_α. By looking at (55) one understands that the first simple $(t+s+u)$-scalar pole has to be discovered with the couplings between an on-shell scalar, one off-shell gauge and two on-shell fermions such that they have to carry three momenta, (for more explanations see the previous section). Consider the following couplings:

$$\frac{T_p(2\pi\alpha')^3}{4}\big[\bar\Psi\gamma^iD_b\Psi D^a\phi_iF_{ab}+\bar\Psi\gamma^iD_b\Psi F_{ab}D^a\phi_i\big]. \tag{59}$$

In order to have general covariance in (59), one has to consider the multiplications of the kinetic term of the fermions, the field strength of gauge field and the covariant derivative of the scalar field. Notice that the connections (commutator terms) in the definitions of the covariant derivative of fermion field and scalar field must be overlooked. Let us work out (59). Unlike the previous section, the only possible orderings for the first and second term of (59) accordingly are $\operatorname{Tr}(\lambda_2\lambda_3\lambda_1\lambda_\beta)$, $\operatorname{Tr}(\lambda_2\lambda_3\lambda_\beta\lambda_1)$ where λ_β is related to Abelian gauge field in the propagator. To obtain the vertex of two on-shell fermions–one off-shell gauge and one on-shell scalar field, one has to extract couplings (59), apply momentum conservation along the world volume of brane and make use of the equations of motion for fermion fields, such that

$$V_\beta^b(A,\bar\Psi,\Psi,\phi_1)=i\frac{T_p(2\pi\alpha')^3}{2}\bar u^A\big(\gamma^j\big)_{AB}u^B\xi_{1j}\left(k_{1b}\frac{t}{2}\right)\operatorname{Tr}(\lambda_1\lambda_2\lambda_3\lambda_\beta). \tag{60}$$

Now if we replace (60) into (57) and consider the first term of the expansion of L_3 (which appeared in (45)) inside (55) then we are able to exactly produce the first $(t+s+u)$-channel gauge pole of (55).

It is clear from (45) that the string amplitude (55) has infinite massless $(t+s+u)$-channel gauge poles. Keeping in mind that $V^a_\alpha(C_{p-1}, A)$ and the simple gauge propagator do not require any corrections, one expects that all infinite gauge poles are related to higher derivative corrections of two fermions, one scalar and one gauge field of IIB superstring theory. In the previous section we introduced how to look for higher derivative corrections. Given the leading couplings in (59), one needs to apply the higher derivative operators $\mathcal{D}_{nm}, \mathcal{D}'_{nm}$ to (59) to be able to discover their all order corrections as follows

$$\begin{aligned}\mathcal{L}^{n,m} &= \pi^3 \alpha'^{n+m+3} \\ &\quad \times T_p\big(a_{n,m}\operatorname{Tr}\big[\mathcal{D}_{nm}\big(\bar{\Psi}\gamma^i D_b \Psi D^a\phi^i F_{ab}\big) + \mathcal{D}_{nm}\big(\bar{\Psi}\gamma^i D_b\Psi F_{ab} D^a\phi^i\big) + \text{h.c.}\big] \\ &\quad + i b_{n,m}\operatorname{Tr}\big[\mathcal{D}'_{nm}\big(\bar{\Psi}\gamma^i D_b \Psi D^a\phi^i F_{ab}\big) + \mathcal{D}'_{nm}\big(\bar{\Psi}\gamma^i D_b\Psi F_{ab} D^a\phi^i\big) + \text{h.c.}\big]\big). \qquad (61)\end{aligned}$$

Now we need to extract the terms carrying the coefficients $a_{n,m}$ in (61) to be able to derive the following vertex

$$\begin{aligned}&V^b_\beta(A, \bar{\Psi}, \Psi, \phi) \\ &\quad = i\frac{T_p(2\pi\alpha')^3}{4}\bar{u}^A\big(\gamma^j\big)_{AB} u^B \xi_{1j}\left(k_{1b}\frac{t}{2}\right)\big(t^n s^m + s^n t^m\big) a_{n,m}\operatorname{Tr}(\lambda_1\lambda_2\lambda_3\lambda_\beta). \qquad (62)\end{aligned}$$

Replacing (62) (instead of (60)) inside (57) and also substituting the second term of the expansion of L_3 which appeared in (17) inside (55) we might produce all infinite gauge poles of the amplitude. It is of high importance to mention the following remark as well. In order to consider $b_{n,m}$ coefficients, one needs to apply on-shell condition $(t+s+u=0)$ at each order of α' to the field theory vertices to obtain the desired terms in field theory amplitude.

6. Conclusions

In this paper we applied conformal field theory techniques and we found the complete form of the $\langle V_C V_{\bar{\psi}} V_\psi V_\phi\rangle$ amplitude in IIB superstring theory. All infinite scalar/gauge (for $p+2=n$, $p=n$ cases) and fermion poles have been explored. We observed that the vertices of $V^\alpha_i(C_{p+1}, \phi)$, $V^\alpha_a(C_{p-1}, A)$ do not require any corrections, hence, all infinite $(t+s+u)$-channel scalar (gauge) poles have provided worth information to determine infinite higher derivative corrections (to all orders in α') to two fermion–two scalar (two fermions–one scalar–one gauge) couplings which we have discovered them and particularly their coefficients are exactly fixed.

We also clarified that the same universal conjecture for all higher derivative corrections that appeared in [23] holds for two fermion–two scalar couplings of IIB superstring theory.

It is worth pointing out that in RR vertex operator there are no winding modes so applying T-duality to the known results is not effective. In particular in order not to miss any terms in superstring amplitudes and to be able to obtain all higher derivative corrections with their exact coefficients, one has to apply direct computations. Basically we proposed some patterns in this paper. Let us talk about a subtle issue regarding the relation of open/closed string vertices in type II superstring theory. For our amplitude which involves mixture open/closed strings, our calculations make sense of using path integral formalism such that propagators (Green functions) are found by conformal field theory methods while the closed string has $(\alpha_n, \tilde{\alpha}_n)$ oscillators. It is not obvious how to do calculations with oscillators, namely in the first quantization of strings it is subtle how to deal with $\tilde{\alpha}_n$. Some comments have been suggested in (3.4) of [39] such that both oscillators for closed string would be determined with open string's ones. In the other words some analytic continuation is needed and this kind of realization implies that the state for closed string should be considered as a composite state of the open strings. The interpretation in field theory might be useful mentioning. It reveals that all background fields in DBI action should be some functions of super Yang–Mills fields. In the other words background fields must become composite and these functions would be examined once we employ the complete open string formalism. Note also that supergravity background fields must include Taylor expansion as was argued in [8].

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