Note

The intricacy of avoiding arrays is 2

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Abstract

Let $A$ be any $n \times n$ array on the symbols $[n]$, with at most one symbol in each cell. An $n \times n$ Latin square $L$ avoids $A$ if all entries in $L$ differ from the corresponding entries in $A$. If $A$ is split into two arrays $B$ and $C$ in a special way, there are Latin squares $L_B$ and $L_C$ avoiding $B$ and $C$, respectively. In other words, the intricacy of avoiding arrays is 2, the number of arrays into which $A$ has to be split.

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1. The theorem

The concept of intricacy (for completing partial Latin squares) was introduced by Daykin and Häggkvist in [2], and a sample of applications to other problems can be found in [3]. An array $A$ is avoidable iff there is a Latin square $L$ that differs from $A$ in every cell. For the problem at hand, the intricacy is the natural number that answers the following question: “If we want to split an array into avoidable arrays, what is the maximum number of arrays we need to use?” In [1] it is proven that this number is at most 3.

There are unavoidable arrays, for example any array containing a whole row or column of just one symbol, so the intricacy is not 1.

Theorem 1. The intricacy of avoiding arrays is 2.

Proof. Let $A$ be any $n \times n$ array on the symbols $[n]$. Split $A$ into arrays $B$ and $C$, so that $C$ is empty. Certainly, there is a Latin square $L_C$ avoiding $C$. For each cell in $B$, move the entry to array $C$ iff it differs from the corresponding entry in $L_C$. Then $L_C$ will still avoid $C$, and the entries left in $B$ form a partial Latin square, which is completable (to $L_C$, for instance). By Theorem 2.1 in [1] $B$ is avoidable, and is avoided by some Latin square $L_B$, which in fact is $L_C$ with symbols permuted without fixed points.

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References