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Testing the RRPP vertex of effective Regge action

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ABSTRACT

We discuss the possibility of checking the vertex with creation of two real gluons in collision of two reggeized ones (RRPP) which can reveal themselves in process of scalar meson production in high energy peripheral nucleon collisions. Numerical estimations of the cross section of a pair of charged pion production for the LHC facility give the value of an order of 10 mb. We also estimate the excess of production of positively charged muons (as a decay of pions) created by cosmic ray proton collisions with the atmosphere gas nuclei to be in reasonable agreement with modern data. The effects of higher orders which reveal themselves as a screening factor are considered in terms of impact parameter representation. We estimate the cross section of pion pair production in central region to fall faster than factorial $\sigma_n \sim 1/(n^2 n!)$.

1. Introduction

The problem of unitarization of the BFKL pomeron as a composite state of two interacting reggeized gluons [1] is an urgent problem of QCD [2]. The problem of construction of three pomeron vertices, and more complicated ones including ordinary gluons can be solved using the terms of the vertices of effective Regge action formulated in the form of Feynman rules for effective Regge action built in [3]. In this Letter, the effective vertices containing the ordinary and the reggeized gluons were built.

Considerable resent attention has been focused on the description of the creation in the peripheral kinematics of bound states of heavy and light quarks was paid recent time [4]. Creation of gluon bound states (gluonium) in the peripheral kinematics was poorly considered in the literature. This is a motivation of this Letter.

To satisfy the requirement for the scattered nucleons to be colorless states (nonexcited baryons), we must consider the Feynman amplitudes corresponding to two reggeized gluon exchange. Moreover, in each nucleon both the reggeized gluons must interact with the same quark.

The creation of two gluons with their subsequent conversion to the bound state with quantum number of σ meson in the pionization region of kinematics of nucleon collision:

$$N_A(P_A) + N_B(P_B) \to N_A\left(P'_A\right) + N_B\left(P'_B\right) + g_1(p_1) + g_2(p_2); \quad g_1(p_1) + g_2(p_2) \to \sigma(p), \tag{1}$$

can be realized by two different mechanisms. One of them consists in the creation of two gluon of each of them is created in “collision” of two reggeized gluons RRP type vertices. The other contains the effective vertex with two reggeized and two ordinary gluons of the RRPP vertex. The second reggeized gluon in the scattering channel does not create real gluons. The relevant RRP vertex has the form

$$V_\mu\big(r_1(q_1, a) + r_2(q_2, b) \to g(\mu, c)\big) = \Gamma^\mu_{cad}(q_1, k, q_2) = g f_{cad} C_\mu(q_1, k, q_2), \tag{2}$$

with the coupling constant g, $g^2 = 4\pi\alpha_s$ and $k = q_1 + q_2$,

$$C_\mu(q_1, k, q_2) = 2\left[\left(n^-\right)^\mu\left(q_1^+ + \frac{q_1^2}{q_2^-}\right) - \left(n^+\right)^\mu\left(q_2^- + \frac{q_2^2}{q_1^+}\right) + (q_2 - q_1)_\mu\right]. \tag{3}$$

The 4-vector C_μ obey the gauge condition $k^\mu C_\mu = 0$.

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The effective RRPP vertex with the conservation law

$$r_1(q_1,c)+r_2(q_2;d)\to g_1(p_1,\nu_1,a_1)+g_2(p_2,\nu_2,a_2)$$

has the form [3]

$$\begin{aligned}\frac{1}{ig^2}\Gamma^{\nu_1\nu_2}_{ca_1a_2d}(q_1,p_1,p_2;q_2)={}&\frac{T_1}{p_{12}^2}c^\eta(q_1,q_2)\gamma^{\nu_1\nu_2\eta}(-p_1,-p_2,k)+\frac{T_3}{(p_2-q_2)^2}\Gamma^{\eta\nu_1-}(q_1,p_1-q_1)\Gamma^{\eta\nu_2+}(p_2-q_2,q_2)\\&-\frac{T_2}{(p_1-q_2)^2}\Gamma^{\eta\nu_2-}(q_1,p_2-q_1)\Gamma^{\eta\nu_1+}(p_1-q_2,q_2)-T_1\big[\big(n^-\big)^{\nu_1}\big(n^+\big)\nu_2-\big(n^-\big)^{\nu_2}\big(n^+\big)\nu_1\big]\\&-T_2\big[2g^{\nu_1\nu_2}-\big(n^-\big)^{\nu_1}\big(n^+\big)\nu_2\big]-T_3\big[\big(n^-\big)^{\nu_2}\big(n^+\big)\nu_1-2g_{\nu_1\nu_2}\big]-2q_2^2\big(n^+\big)^{\nu_1}(n_+)^{\nu_2}\left[\frac{T_3}{p_2^+q_1^+}-\frac{T_2}{p_1^+q_1^+}\right]\\&-2q_1^2\big(n^-\big)^{\nu_1}(n_-)^{\nu_2}\left[\frac{T_3}{p_1^-q_2^-}-\frac{T_2}{p_2^-q_2^-}\right],\end{aligned}\tag{4}$$

with the light-like 4-vectors

$$n^+=p_B/E,\qquad n^-=p_A/E,\qquad n^+n^-=2,\qquad \big(n^\pm\big)^2=0,$$

$\sqrt{s}=2E$ is the total center-of-mass energy,

$$\gamma_{\mu\nu\lambda}(p_1,p_2,p_3)=(p_1-p_2)_\lambda g_{\mu\nu}+(p_2-p_3)_\mu g_{\nu\lambda}+(p_3-p_1)_\nu g_{\lambda\mu},\quad p_1+p_2+p_3=0,\tag{5}$$

is the ordinary three gluon Yang–Mills vertex, and the induced vertices:

$$\Gamma^{\nu\nu'+}(q_1,q_2)=2q_1^+g^{\nu\nu'}-\big(n^+\big)^\nu(q_1-q_2)^{\nu'}-\big(n^+\big)^{\nu'}(q_1+2q_2)^\nu-\frac{q_2^2}{q_1^+}\big(n^+\big)^\nu\big(n^+\big)^{\nu'};\tag{6}$$

$$\Gamma^{\nu\nu'-}(q_1,q_2)=2q_2^-g^{\nu\nu'}+\big(n^-\big)^\nu(q_1-q_2)^{\nu'}+\big(n^-\big)^{\nu'}(-q_2-2q_1)^\nu-\frac{q_1^2}{q_2^-}\big(n^-\big)^\nu\big(n^-\big)^{\nu'}.\tag{7}$$

We use here the notation $k^\pm=(n^\pm)_\mu k^\mu$ and the light cone decomposition implies:

$$q_1=q_{1\perp}+\frac{q_1^+}{2}n^-;\qquad q_2=q_{2\perp}+\frac{q_2^-}{2}n^+;\qquad q_1^-=q_2^+=0,\qquad p_i=\frac{p_i^+}{2}n^-+\frac{p_i^-}{2}n^++p_{i\perp},\qquad p_\perp n^\pm=0.\tag{8}$$

The color structures are

$$T_1=f_{a_1a_2r}f_{cdr};\qquad T_2=f_{a_2cr}f_{a_1dr};\qquad T_3=f_{ca_1r}f_{a_2dr},\tag{9}$$

with f_{abc} being the structure constant of the color group; Jacobi identity provides the relation $T_1+T_2+T_3=0$. The conditions of Bose-symmetry and gauge invariance

$$\Gamma^{\nu_1\nu_2}_{ca_1a_2d}(q_1,p_1,p_2,q_2)p_{1\nu_1}=0,\qquad \Gamma^{\nu_1\nu_2}_{ca_1a_2d}(q_1,p_1,p_2,q_2)=\Gamma^{\nu_2\nu_1}_{ca_2a_1d}(q_1,p_1,p_2,q_2)\tag{10}$$

are satisfied.

2. Pomeron mechanisms of σ meson production

We consider the case when the hadrons after collision remains to be colorless. In the case of nucleon collisions it turns out that both exchanged reggeized gluons must interact with the same quark. The color coefficient associated with the RRPP vertex results in

$$\mathrm{Tr}\,t^nt^c\times\mathrm{Tr}\,t^nt^d\times\Gamma^{\nu_1\nu_2}_{ca_1a_2d}=\frac{1}{4}N\delta_{a_1a_2}\Pi_{02}^{\nu_1\nu_2},\tag{11}$$

$N=3$ is the rank of the color group. For the mechanism of creation of two separate gluons we have

$$\mathrm{Tr}\,t^nt^k\times\mathrm{Tr}\,t^mt^l\times f_{nma_1}\times f_{kla_2}\times\Pi_{11}^{\nu_1\nu_2}=\frac{1}{4}N\delta_{a_1a_2}\Pi_{11}^{\nu_1\nu_2}.\tag{12}$$

In projecting the two-gluon state to the colorless and spinless state we use the operator

$$\mathcal{P}=\frac{\delta_{a_1a_2}}{\sqrt{N^2-1}}\frac{g^{\nu_1\nu_2}}{4}.\tag{13}$$

The resulting expressions are

$$\frac{1}{16}N\sqrt{N^2-1}[\Pi_{02},\Pi_{11}]$$

with

$$\Pi_{02}=-12-\left[\frac{1}{(p_2-q_2)^2}\Gamma^{\eta\nu-}(q_1,p_1-q_1)\Gamma^{\eta\nu+}(p_2-q_2,q_2)+(p_1\leftrightarrow p_2)\right];\tag{14}$$

$q_1 = l_1$, $q_2 = p_1 + p_2 - l_1$ and

$$\Pi_{11} = C_\mu(l - l_1, l_1 - l + p_1) C_\mu(l_1, p_1 - l_1). \tag{15}$$

Here l_1 is the 4-momentum of the gluonic loop, and $l = P_A - P_{A'}$ is the transferred momentum.

In a realistic model describing interaction of two reggeized gluons with the transversal momenta $\vec{l}_1, \vec{l}_2$, which form a pomeron with quark [5] we have for the corresponding vertex:

$$\Phi_P(\vec{l}_1, \vec{l}_2) = -\frac{12\pi^2}{N} F_P(\vec{l}_1, \vec{l}_2), \tag{16}$$

with

$$F_P(\vec{l}_1, \vec{l}_2) = \frac{-3\vec{l}_1\vec{l}_2 C^2}{(C^2 + (\vec{l}_1 + \vec{l}_2)^2)(C^2 + \vec{l}_1^2 + \vec{l}_2^2 - \vec{l}_1\vec{l}_2)}, \tag{17}$$

and $C = m_\rho/2 \approx 400$ MeV. We note that this form of pomeron–quark coupling obeys the gauge condition: it turns to zero at zero transverse momenta of gluons. The factor 3 corresponds to three possible choices of quark into proton.

Matrix elements of peripheral processes are proportional to s. To see this, we can use Gribov's substitution into the gluon Green functions nominators $g_{\mu\nu} = (2/s) P_{A\mu} P_{B\nu}$ with the Lorentz index $\mu(\nu)$—associated with the $B(A)$ parts of the Feynman amplitude. In performing the loop momenta l_1 integration it is convenient to use the following form of phase volume:

$$d^4 l_1 = \frac{1}{2s} ds_1\, ds_2\, d^2\vec{l}_1, \quad s_1 = 2P_A l_1,\ s_2 = 2P_B l_1. \tag{18}$$

Simplifying the nucleon nominators as

$$\bar{u}(P'_A)\hat{P}_B\hat{P}_A\hat{P}_B u(P_A) = s^2 N_A, \quad N_A = \frac{1}{s}\bar{u}(P'_A)\hat{P}_B u(P_A),\ \sum |N_A|^2 = 2 \tag{19}$$

(with a similar expression for part B), we find all them to be equal. The integration over variables $s_{1,2}$ can be performed for the sum of all four Feynman amplitudes as

$$\int_{-\infty}^{\infty} \frac{ds_{1,2}}{2\pi i}\left[\frac{1}{s_{1,2} + a_{1,2} + i0} + \frac{1}{-s_{1,2} + b_{1,2} + i0}\right] = 1. \tag{20}$$

By combining all the factors the matrix element corresponding to the vertex RRPP can be written as

$$-iM = 2^5 3^2 s(\pi\alpha_s)^3 N_A N\sqrt{N^2 - 1}\frac{1}{N} F(\vec{\Delta}) f(\vec{l}, \vec{p}), \tag{21}$$

with $\vec{p} = \vec{p}_1 + \vec{p}_2$ and the relative momenta of real gluons $\vec{\Delta} = (\vec{p}_2 - \vec{p}_1)/2$,

$$f(\vec{l}, \vec{p}) = \int \frac{d^2 l_1\, C^4}{2\pi\vec{l}_1^2(\vec{l} - \vec{l}_1)^2(\vec{p} - \vec{l}_1)^2} F_P(\vec{l}_1, \vec{l} - \vec{l}_1) F_P(\vec{l}_1 - \vec{l}, \vec{p} - \vec{l}_1) \Pi_{02}(\vec{l}_1, \vec{p} - \vec{l}_1), \tag{22}$$

and a similar expression for another mechanism. Here we have introduced the factor $F(\vec{\Delta}) = [a^2\vec{\Delta}^2 + 1]^{-2}$ which describes the conversion of two gluon states to a bound state of the size $a \sim 1$ fm which is a gluonium component of a scalar meson.

Let us perform the phase volume of the final state as

$$\begin{aligned}
d\Gamma &= \frac{d^3 P'_B}{2E_{B'}}\frac{d^3 P'_A}{2E_{A'}}\frac{d^3 p_1}{2E_1}\frac{d^3 p_2}{2E_2}(2\pi)^{-8}\delta^4(P_A + P_B - P_{A'} - P_{B'} - p_1 - p_2) = d\Gamma_{AB}\, d\Gamma_{12}\,(2\pi)^{-8}, \\
d\Gamma_{AB} &= d^4 l\, d^4 P_{B'}\, d^4 P_{A'}\, \delta^4(P_A - P_{A'} - l)\delta^4(P_B + l - P_{B'} - p)\delta(P_{B'}^2 - M_B^2)\delta(P_{A'}^2 - M_A^2), \\
d\Gamma_{12} &= \frac{d^3 p}{2E_1}\frac{d^3\Delta}{2E_2}.
\end{aligned} \tag{23}$$

Using the relation

$$d^4 l = \frac{1}{2s} d(2lP_B)\, d(2lP_A)\, d^2\vec{l}, \tag{24}$$

we perform integration over the momenta of the scattered nucleons with the result:

$$d\Gamma_{AB} = \frac{d^2\vec{l}}{2s}. \tag{25}$$

Keeping in mind the almost collinearity of 3-momenta of a real gluon we can transform the gluon part of the phase volume as:

$$d\Gamma_{12} = d^4 p\, \delta(p^2 - M_\sigma^2)\frac{2}{M_\sigma} d^3\Delta. \tag{26}$$

Integration on over $\vec{\Delta}$ can be performed in the explicit form

$$\int d^3\Delta\, F^2(\Delta) = \frac{\pi^2}{4a^3} = \frac{2\pi^2 M_p^3}{10^3 a(\mathrm{fm})^3}, \tag{27}$$

where we used the conversion constant $M_p \times \mathrm{fm} = 5$, M_p is the nucleon mass. The last part of the phase volume of gluonic the system can be arranged using light cone form of 4-momentum p (see (8)):

$$d^4 p\, \delta\big(p^2 - M_\sigma^2\big) = \frac{dp^+}{2p^+} d^2\vec{p} = \frac{1}{2} L\, d^2\vec{p}, \tag{28}$$

with the so-called "boost logarithm" $L \approx \ln(\frac{2E}{M})$, E, M—energy and mass of proton in laboratory reference frame. For the LHC facility as well as for cosmic protons (in the knee region of spectra) we use below $L = 15$.

For the contribution to the total cross section we obtain

$$\sigma_1^P = A \frac{\alpha_s^6 L M_p^3}{M_\sigma M_\rho^4} J, \quad A = \frac{6^4}{5^3} \frac{N^2 - 1}{N^2} \frac{\pi^2}{a(\mathrm{fm})^3}, \tag{29}$$

with

$$J = \int \frac{d^2 l\, d^2 p}{(2\pi)^2 C^4} f^2(\vec{l}, \vec{p}). \tag{30}$$

Numerical integration gives $J = 7.4 \times 10^3$.

The corresponding contribution to the total cross section of the single σ meson production is of an order of 10 mb.

The contribution arising from the other mechanism of production (including the interference of amplitudes) turns out to be at least an order of magnitude less. It determines the accuracy of the result obtained on the level of 10%.

3. Screening effects. Several σ-production

Let us now generalize the result to include the screening effects as well as the possibility to produce several σ mesons.

In the large impact-parameters limit proton interacts with the whole gluon field of the nucleon (or nuclei) moving in the opposite direction coherently. So the main contribution arises from the many pomeron exchange mechanism (compare with the "chain" mechanism essential in BFKL equation) [1]. So we must consider pomeron s-channel iterations. Let consider three kinds of the iteration blocks. One is the pure pomeron exchange, the second is pomeron with the sigma meson emission from the central region. The third one is the "screening block": two blocks of the second type with the common virtual σ meson. The contribution to the amplitude of production at $n\ \sigma$ mesons of the blocks of the third kind is associated with the "large logarithm" which arises from boost freedom of these blocks in complete analogy with QED [6].

In a similar way the closed expression (omitting the terms of an order of $1/N^2$ compared with the ones of an order of 1) for the summed over the number of s-channel iteration ladders of the first and third type can be obtained using the relation

$$\int \Pi_1^n \frac{d^2 k_i}{(2\pi)^2} = \int \Pi_1^{n+1} \frac{d^2 k_i}{(2\pi)^2} \int d^2\rho \exp\left(i\vec{\rho} \sum_1^{n+1} (\vec{k}_i - \vec{q}) \right) = \int d^2\rho \exp(-i\vec{q}\vec{\rho}) \Pi_1^{n+1} \frac{d^2 k_i}{(2\pi)^2} e^{i\vec{k}_i \vec{\rho}}. \tag{31}$$

Accepting the assumption about colorless structure of the pomeron as a bound state of two reggeized interacting gluons and applying the same sequence of transformations, as was done in [6], we obtain for the cross section of $n\ \sigma$ meson production in the peripheral high-energy proton collisions

$$\sigma_n = \int \frac{d^2\rho}{(2\pi)^2} \frac{(L\sigma_0 Z(\rho))^n}{n!} \exp\big(-L\sigma_0 Z(\rho)\big), \tag{32}$$

with $\sigma_0 = 2.25 \times 10^{-3} \alpha_s^6 M_p^3 / (M_\sigma M_\rho^4 a(\mathrm{fm})^3)$ and

$$Z(\rho) = \int \frac{d^2 p}{(2\pi)^2} \big|B(\rho, p)\big|^2, \quad B(\rho, p) = \int \frac{d^2 l}{C^2} f(l, p) \exp(i\vec{l}\vec{\rho}). \tag{33}$$

Numerical estimations of the cross section of one sigma meson production give $\sigma_1 = 10$ mb.

Keeping in mind the exponential decrease of $Z(\rho)$ at large ρ: $Z(\rho) \sim C \cdot e^{-\frac{m_\rho \rho}{2}}$, we can estimate the behavior of σ_n at large n; $\sigma_n \sim \sigma_1 C^n (n^2 n!)^{-1}$, $n \gg 1$, $C \sim 1$.

4. Conclusion

The results given above can be applied to explain excess of the positive muons compared with the negative ones produced by cosmic ray interaction with the Earth surface. Really one can neglect QED mechanisms of production of the charged pions in favor at strong interactions. It turns out that the main mechanism is the peripheral production of the pion pairs (with the subsequent decay to muons) in the high-energy cosmic ray proton collisions with the nuclei of nitrogen or oxygen in the Earth atmosphere.

Keeping in mind the atmosphere to be density about $10^{19}\ \mathrm{cm}^{-3}$ with thickness ~ 10 km, effective proton size ~ 1.4 fm, we obtain number of effective collision $N_{\mathrm{eff}} = (1.4\ \mathrm{fm})^2 \times 10^5\ \mathrm{cm} \times 10^{19}\ \mathrm{cm}^{-3} \sim 1$.

The QED mechanism contribution is suppressed by at least a factor of $(\frac{\alpha^2 Z}{\alpha_s^3 N^2})^2 \sim 10^{-3}$ $N = 14$, $Z = 7$. The contribution of peripheral interaction of proton with gas nuclei with the impact parameter $\rho \gg A^{1/3}$ fm is small. For $\rho > 10$ fm suppression by a factor of $e^{m_\rho \cdot \rho/2} < 10^{-5}$ appears. So only the mechanism of penetration of proton through nuclei becomes dominant.

Travelling through the nuclei the cosmic proton has a direct collision with the protons and the neutrons of the nuclei $\rho \leqslant 1$ fm (this kind of collisions produces positive charged pions due to the decay of the excited resonances) or has the peripheral collisions when pairs of pions are produced ($1\ \mathrm{fm} < \rho < A^{1/3}$ fm).

The number of positive charged pions produced in direct collisions N_d, is proportional to $A^{1/3}$ with the characteristic atomic number $A = 14$. The number of pion pairs produced in the peripheral collisions can be estimated as

$$N_p = M_p^2 \sigma_1 \approx 7.8. \tag{34}$$

For the ratio of the positive charged muons to the negative charged ones $R = \frac{N_{\mu+}}{N_{\mu-}}$ we have

$$R_{\mathrm{th}} = 1 + \frac{N_d}{N_p} = 1.32. \tag{35}$$

This quantity can be compared with the recent experimental value [7] (here only hard muons are taken into account).

$$R_{\mathrm{exp}} = \frac{N_{\mu^+}}{N_{\mu_-}} = 1.4 \pm 0.003. \tag{36}$$

These values are in reasonable agreement.

s-channel iteration is enhanced by factor $(\alpha_s N^2)^n$ whereas t-iteration. Only $(\alpha_s L)^2$ with N—atomic number of nuclei $N^2 L \sim 10$; $L = \log \frac{s}{m_p^2}$. So we neglect t-channel iteration effects which leads to pomeron exchange contribution and enhanced pomeron contribution (triple pomeron mechanism).

By vertex counting we see that our effective expansion parameter is $(\alpha_s N^2)^n$ for screening effects and $\alpha_s(\alpha_s N^2)$ for each pair of gluon emission.

Mechanism RRPP vertex considered by [8] doubled (to produce 2 gluons) $\alpha_s^2(\alpha_s N^2)^2$ so it is suppressed by additional power of $\frac{\alpha_s}{\alpha} \sim 0.1$.

To take into account the screening effect one must consider insertion of (infinite) number of blocks each of which consist of two reggeized gluons exchange, block are connected by pair of ordinary gluons. Here we can use the analogy with QED [6] as well as here we accept the assumption of colorless final hadrons, so the pair of exchanged reggeized gluons interact with the same quark from nucleon.

We use the realistic model of pomeron–hadron interaction developed and intensively exploited in the literature (the same as use[5]).

The contribution of enhanced pomeron exchange (including three pomeron vertices) is estimated to be of an order 10% of the result obtained here. These reasons determine the level of accuracy of our results.

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