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Technical importance ratings in fuzzy QFD by integrating fuzzy normalization and fuzzy weighted average ☆

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ABSTRACT

Fuzzy quality function deployment (QFD) has been extensively used for translating customer requirements (CRs) into product design requirements (DRs) in fuzzy environments. Existing approaches, however, for rating technical importance of DRs in fuzzy environments are found problematic, either incorrect or inappropriate. This paper investigates how the technical importance of DRs can be correctly rated in fuzzy environments. A pair of nonlinear programming models and two equivalent pairs of linear programming models are developed, respectively, to rate the technical importance of DRs. The developed models are examined and illustrated with two numerical examples.

1. Introduction

Quality function deployment (QFD) [1] is a methodology for translating customer requirements (CRs), i.e. the voice of the customer (VoC), into product design requirements (DRs). In this translating process, a large number of subjective judgments have to be made by both customers and QFD team members. Due to uncertainty and vagueness involved in subjective judgments, fuzzy logic has been widely suggested for better capturing the relative importance of CRs and the relationships between CRs and DRs as well as the correlations among DRs. As a result, fuzzy QFD has been developed, researched and extensively applied [2–11].

For fuzzy QFD, one of the key issues is to derive the technical importance ratings of DRs in fuzzy environments and prioritize them so that limited resources such as budget can be reasonably or optimally allocated within DRs in terms of their priorities. Existing approaches for rating the technical importance of DRs in fuzzy environments are found problematic, either incorrect or inappropriate. Therefore, there is a need to develop a correct methodology for rating the technical importance of DRs. This paper investigates how the technical importance of DRs can be correctly rated in fuzzy environments. A pair of nonlinear programming (NLP) models is developed to correctly rate the technical importance of DRs in fuzzy environments, which is then broken down into two equivalent pairs of linear programming (LP) models for solution.

The paper is organized as follows. Section 2 gives a brief introduction to fuzzy sets and fuzzy weighted average that are or will be used in fuzzy QFD. Section 3 presents a literature review on the formulas and approaches for rating the technical importance of DRs in fuzzy environments and points out their incorrectness or inappropriateness. Section 4 develops correct NLP models for rating the technical importance of DRs. Section 5 shows how the NLP models can be simplified as two

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equivalent pairs of LP models for solution. The developed models, linear and nonlinear, are examined and illustrated with two numerical examples in Section 6. The paper concludes in Section 7.

2. Fuzzy sets and fuzzy weighted average

Fuzzy sets were introduced by Zadeh [12]. A fuzzy set is a collection of elements in a universe of discourse, with each element being assigned a value within [0,1] by a specified membership function. It can also be represented using α-level sets. The α-level sets, A_α, of a fuzzy set $\tilde{A}$ are defined as [13]

$$A_\alpha = \left\{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\right\} = \left[\min\{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}, \max\{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}\right], \tag{1}$$

where $\mu_{\tilde{A}}(x)$ is the membership function of $\tilde{A}$ and X is the universe of discourse. Accordingly, the fuzzy set $\tilde{A}$ can be equivalently expressed as

$$\tilde{A} = \cup_\alpha \alpha \cdot A_\alpha = \cup_\alpha \alpha \cdot \left[(A)_\alpha^L, (A)_\alpha^U\right], \quad 0 < \alpha \leq 1. \tag{2}$$

Fuzzy numbers are special cases of fuzzy sets, characterized by given intervals of real numbers. The most commonly used fuzzy numbers are triangular and trapezoidal fuzzy numbers, whose membership functions are, respectively, defined as

$$\mu_{\tilde{A}_1}(x) = \begin{cases} (x-a)/(b-a), & a \leq x \leq b, \\ (c-x)/(c-b), & b \leq x \leq c, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

$$\mu_{\tilde{A}_2}(x) = \begin{cases} (x-a)/(b-a), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ (d-x)/(d-c), & c \leq x \leq d, \\ 0, & \text{otherwise}. \end{cases} \tag{4}$$

For brevity, triangular and trapezoidal fuzzy numbers are often denoted as (a, b, c) and (a, b, c, d). Triangular fuzzy numbers can also be expressed as a special case of trapezoidal fuzzy numbers.

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two positive trapezoidal fuzzy numbers. Operations on the two fuzzy numbers, which are often called fuzzy arithmetics, are defined as [13]

Fuzzy addition: $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$;

Fuzzy subtraction: $\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$;

Fuzzy multiplication: $\tilde{A} \otimes \tilde{B} \approx (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$;

Fuzzy division: $\tilde{A} \div \tilde{B} \approx \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$.

Fuzzy sets are not easy to compare and often defuzzified for ranking purpose. Defuzzification is a transformation process which converts a fuzzy set into a crisp value. The most commonly used method for defuzzification is the centroid method, which is defined as [14]

$$\bar{x}_0(\tilde{A}) = \frac{\int_{-\infty}^{+\infty} x\mu_{\tilde{A}}(x)dx}{\int_{-\infty}^{+\infty} \mu_{\tilde{A}}(x)dx}, \tag{5}$$

where $\bar{x}_0(\tilde{A})$ is the centroid. In the case that fuzzy sets are characterized by α-level sets without knowing their explicit membership functions, their centroids can be computed by the following expressions [15]:

$$\begin{aligned} \int_{-\infty}^{+\infty} x\mu_{\tilde{A}}(x)dx = \; & \frac{1}{6n}\left[\left((x)_{\alpha_0}^{2U} - (x)_{\alpha_0}^{2L}\right) + \left((x)_{\alpha_n}^{2U} - (x)_{\alpha_n}^{2L}\right) + 2\sum_{i=1}^{n-1}\left((x)_{\alpha_i}^{2U} - (x)_{\alpha_i}^{2L}\right)\right] \\ & + \frac{1}{6n}\sum_{i=0}^{n-1}\left((x)_{\alpha_i}^{U} \cdot (x)_{\alpha_{i+1}}^{U} - (x)_{\alpha_i}^{L} \cdot (x)_{\alpha_{i+1}}^{L}\right), \end{aligned} \tag{6}$$

$$\int_{-\infty}^{+\infty} \mu_{\tilde{A}}(x)dx = \frac{1}{2n}\left[\left((x)_{\alpha_0}^{U} - (x)_{\alpha_0}^{L}\right) + \left((x)_{\alpha_n}^{U} - (x)_{\alpha_n}^{L}\right) + 2\sum_{i=1}^{n-1}\left((x)_{\alpha_i}^{U} - (x)_{\alpha_i}^{L}\right)\right], \tag{7}$$

where $\alpha_i = \frac{i}{n}$, $i = 0, \ldots, n$. In the case of $(x)_{\alpha_n}^L = (x)_{\alpha_n}^U$, the centroid can be computed by

$$\bar{x}_0(\tilde{A}) = \frac{1}{3} \cdot \frac{\left((x)_{\alpha_0}^{2U} - (x)_{\alpha_0}^{2L}\right) + 2\sum_{i=1}^{n-1}\left((x)_{\alpha_i}^{2U} - (x)_{\alpha_i}^{2L}\right) + \sum_{i=0}^{n-1}\left((x)_{\alpha_i}^{U} \cdot (x)_{\alpha_{i+1}}^{U} - (x)_{\alpha_i}^{L} \cdot (x)_{\alpha_{i+1}}^{L}\right)}{\left((x)_{\alpha_0}^{U} - (x)_{\alpha_0}^{L}\right) + 2\sum_{i=1}^{n-1}\left((x)_{\alpha_i}^{U} - (x)_{\alpha_i}^{L}\right)}. \tag{8}$$

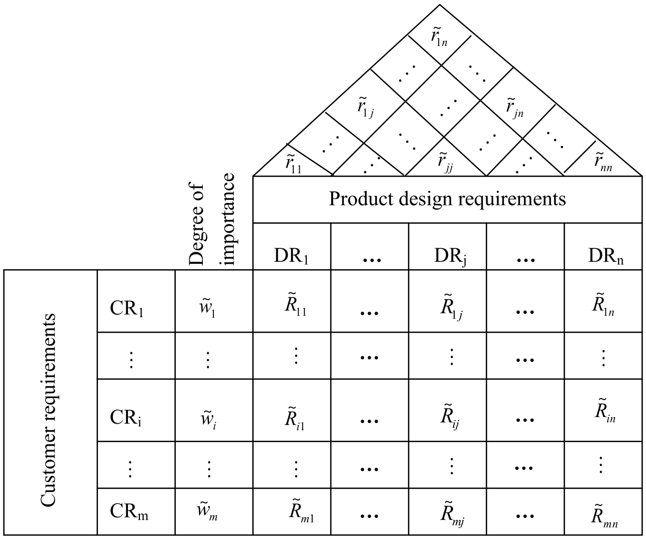

Fig. 1. The house of quality in fuzzy QFD.

Fuzzy weighted average (FWA) is a method for fuzzy multiple criteria decision analysis and is defined as [16]

$$\tilde{y} = \frac{\sum_{i=1}^{n} \tilde{w}_i \otimes \tilde{x}_i}{\sum_{i=1}^{n} \tilde{w}_i}, \tag{9}$$

where $\tilde{x}_1, \ldots, \tilde{x}_n$ are n positive fuzzy numbers and $\tilde{w}_1, \ldots, \tilde{w}_n$ are fuzzy weights. Fuzzy arithmetics are found not suitable for calculating $\tilde{y}$ because of occurrence of the weight variables in both denominator and numerator simultaneously. Quite a lot of research has been done on how to compute $\tilde{y}$. The most common approach is to calculate $\tilde{y}$ by the principle of extension. Let $x_{i\alpha} = [(x_i)_\alpha^L, (x_i)_\alpha^U]$, $w_{i\alpha} = [(w_i)_\alpha^L, (w_i)_\alpha^U]$ and $y_\alpha = [(y)_\alpha^L, (y)_\alpha^U]$ be the α-level sets of $\tilde{x}_i$, $\tilde{w}_i$ and $\tilde{y}$, respectively. Then, $y_\alpha = [(y)_\alpha^L, (y)_\alpha^U]$ can be derived by the following pair of fractional programming models [17]:

$$(y)_\alpha^L = \text{Min} \frac{w_1(x_1)_\alpha^L + w_2(x_2)_\alpha^L + \cdots + w_n(x_n)_\alpha^L}{w_1 + w_2 + \cdots + w_n}, \quad \text{s.t. } (w_i)_\alpha^L \le w_i \le (w_i)_\alpha^U, \quad i = 1, \ldots, n, \tag{10}$$

$$(y)_\alpha^U = \text{Max} \frac{w_1(x_1)_\alpha^U + w_2(x_2)_\alpha^U + \cdots + w_n(x_n)_\alpha^U}{w_1 + w_2 + \cdots + w_n}, \quad \text{s.t. } (w_i)_\alpha^L \le w_i \le (w_i)_\alpha^U, \quad i = 1, \ldots, n, \tag{11}$$

which can be converted into a pair of LP models for solution. After obtaining the α-level sets $y_\alpha = [(y)_\alpha^L, (y)_\alpha^U]$, the fuzzy weighted average, $\tilde{y}$, can then be expressed as

$$\tilde{y} = \cup_\alpha \alpha \cdot y_\alpha = \cup_\alpha \alpha \cdot [(y)_\alpha^L, (y)_\alpha^U], \quad 0 < \alpha \le 1. \tag{12}$$

3. Literature review and analysis

In fuzzy QFD as shown in Fig. 1, the relative importance weights of CRs, the relationships between CRs and DRs, and the correlations among DRs could all be fuzzy numbers. For example, Zhou [18] described the influences of DRs on CRs by fuzzy linguistic variables and calculated the technical importance ratings of DRs as the weighted aggregations of their influences on CRs, namely, $\tilde{v}_j = (1/m)\sum_{i=1}^{m}\left(w_i \otimes \tilde{R}_{ij}\right), j = 1, \ldots, n$, which were then defuzzified by using the maximizing set and minimizing set approach, proposed by Chen [19], where the weights for aggregations were the relative importance of CRs determined by using Saaty's analytical hierarchy process (AHP) [20] and were assumed to be crisp numbers.

Wang [21] used fuzzy arithmetics to compute the technical importance of DRs, i.e. $\tilde{t}_j = \sum_{i=1}^{m} \tilde{w}_i \otimes \tilde{R}_{ij}, j = 1, \ldots, n$, and the fuzzy ranking approach based on possibility and necessity measures to prioritize DRs. Shen et al. [22] also employed fuzzy arithmetics to calculate the technical importance ratings of DRs and defuzzified them using the Mean of Maxima (MOM) method and the centroid defuzzification method, respectively.

Chen and Weng [23] normalized the fuzzy relationships between CRs and DRs by the procedure proposed by Wasserman [24], namely,

$$\tilde{R}'_{ij} = \frac{\sum_{k=1}^{n} \tilde{R}_{ik} \otimes \tilde{r}_{kj}}{\sum_{l=1}^{n}\sum_{k=1}^{n} \tilde{R}_{ik} \otimes \tilde{r}_{kl}}, \quad i = 1, \ldots, m;\ j = 1, \ldots, n, \tag{13}$$

which were computed by using α-level sets rather than fuzzy arithmetics. As a result, they derived the following analytical formulas for the α-level sets of the normalized fuzzy relationships $\tilde{R}'_{ij}$, which also appeared in Chen and Weng [25] and Chen and Ko [26,27]:

$$(R'_{ij})^L_\alpha = \frac{\sum_{k=1}^{n} (R_{ik})^L_\alpha (r_{kj})^L_\alpha}{\sum_{\substack{l=1\\ l\neq j}}^{n}\sum_{k=1}^{n} (R_{ik})^U_\alpha (r_{kl})^U_\alpha + \sum_{k=1}^{n} (R_{ik})^L_\alpha (r_{kj})^L_\alpha}, \quad i = 1, \ldots, m;\ j = 1, \ldots, n, \tag{14}$$

$$(R'_{ij})^U_\alpha = \frac{\sum_{k=1}^{n} (R_{ik})^U_\alpha (r_{kj})^U_\alpha}{\sum_{\substack{l=1\\ l\neq j}}^{n}\sum_{k=1}^{n} (R_{ik})^L_\alpha (r_{kl})^L_\alpha + \sum_{k=1}^{n} (R_{ik})^U_\alpha (r_{kj})^U_\alpha}, \quad i = 1, \ldots, m;\ j = 1, \ldots, n, \tag{15}$$

where $[(R_{ik})^L_\alpha, (R_{ik})^U_\alpha](i = 1, \ldots, m; k = 1, \ldots, n)$ and $[(r_{kj})^L_\alpha, (r_{kj})^U_\alpha](k, j = 1, \ldots, n)$ are the α-level sets of $\tilde{R}_{ik}$ and $\tilde{r}_{kj}$, respectively. They then calculated fuzzy technical importance ratings of DRs as the weighted sum of the fuzzy normalized relationships with a set of crisp importance weights of CRs, i.e. $\tilde{Y}_j = \sum_{i=1}^{m} w_i \cdot \tilde{R}'_{ij}, j = 1, \ldots, n$. In the case where the relative importance weights of CRs were characterized by fuzzy numbers, Chen and Weng [25] calculated the fuzzy technical importance ratings of DRs as the fuzzy weighted average of the normalized fuzzy relationships, namely,

$$\tilde{Y}_j = \frac{\sum_{i=1}^{m} \tilde{w}_i \otimes \tilde{R}'_{ij}}{\sum_{i=1}^{m} \tilde{w}_i}, \quad j = 1, \ldots, n, \tag{16}$$

which were computed by solving a pair of LP models for different α-levels.

Karsak [28] utilized fuzzy Delphi method to determine the relative importance of CRs and the normalization procedure of Wasserman [24], i.e. Eq. (13), to normalize the fuzzy relationships between CRs and DRs, but the equation was solved by using fuzzy arithmetics rather than α-level sets. The technical importance ratings of DRs were then calculated as $\tilde{D}_j = \sum_{i=1}^{m} \tilde{w}_i \otimes \tilde{R}'_{ij}, j = 1, \ldots, m$.

Chen et al. [29] modified the fuzzy relationships between CRs and DRs by multiplying them by fuzzy correlations among DRs, i.e.

$$\tilde{R}'_{ij} = \sum_{k=1}^{n} \tilde{R}_{ik} \otimes \tilde{r}_{kj}, \quad i = 1, \ldots, m;\ j = 1, \ldots, n, \tag{17}$$

and then calculated fuzzy importance of DRs as $\tilde{v}_j = \sum_{i=1}^{m} \tilde{w}_i \otimes \tilde{R}'_{ij} = \sum_{i=1}^{m}\sum_{k=1}^{n} \tilde{w}_i \otimes \tilde{R}_{ik} \otimes \tilde{r}_{kj},\ j = 1, \ldots, n$. Chen, Fung and Tang [30] also calculated the fuzzy importance of DRs by using FWA without considering correlations among DRs, i.e. $\tilde{Y}_j = \sum_{i=1}^{m} \tilde{w}_i \otimes \tilde{R}_{ij} / \sum_{i=1}^{m} \tilde{w}_i,\ j = 1, \ldots, n$. Fuzzy expected value operator, proposed by Liu and Liu [31], was then utilized to rank or prioritize DRs.

Liu [32] considered the impacts of correlations among DRs by Eq. (17) and calculated the technical importance of DRs by Eq. (16), i.e. FWA. Since the derived membership functions of the technical importance of DRs were not explicitly known, a method that embeds the maximizing set and minimizing set approach of Chen [19] into FWA was devised to form a pair of nonlinear programs for prioritizing DRs.

From the above literature review, it has been found that existing approaches and formulas for calculating technical importance ratings of DRs exhibit some drawbacks. For example, formulas (14) and (15) derived by Chen and Weng [23,25] are both incorrect because none of R_{ik} $(k = 1, \ldots, n)$ can take two different values at the same time in numerator and denominator, respectively. If lower bound values $(R_{ik})^L_\alpha$ are taken in numerator, then R_{ik} should not take upper bound values $(R_{ik})^U_\alpha$ in denominator. As such, if upper bound values $(R_{ik})^U_\alpha$ appear in numerator, then lower bound values $(R_{ik})^L_\alpha$ should not been taken in denominator. In a word, R_{ik} $(k = 1, \ldots, n)$ cannot take lower and upper bound values simultaneously. As variables, each of them can only take one value at a time, either upper bound value or lower bound value or a value between lower and upper bounds.

The use of fuzzy arithmetics to perform fuzzy normalization or FWA is also inappropriate since fuzzy arithmetic operations increase the fuzziness of normalized fuzzy relationships $\tilde{R}'_{ij}$ and FWA, and make their support intervals much wider than actual ones.

Existing approaches that do not normalize fuzzy relationships between CRs and DRs cannot be correct either. According to Wasserman [24], the relationships between CRs and DRs need to be normalized; otherwise, the technical importance of DRs cannot be correctly rated. This is also true for fuzzy relationships. Besides, from the point of view of AHP, CRs could be viewed as evaluation criteria, DRs as decision alternatives, and their relationships or fuzzy relationships as local weights. Before the aggregation of local weights into global weights, AHP requires the local weights to be normalized. This also verifies that the relationships or fuzzy relationships between CRs and DRs need to be normalized before calculating technical importance ratings of DRs.

In the next two sections, we investigate how the technical importance of DRs can be correctly rated in fuzzy environments. A pair of NLP models based on α-level sets will be developed to rate the fuzzy technical importance of DRs. We then show how the pair of NLP models can be simplified as two equivalent pairs of LP models for solution.

4. NLP models for rating technical importance in fuzzy environments

In conventional QFD, the relative importance weights of CRs, the relationships between CRs and DRs, and the correlations among DRs are all deterministic. The technical importance of DRs can be rated by the weighted average of the normalized relationships $R'_{ij} = \frac{\sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{l=1}^{n}\sum_{k=1}^{n} R_{ik} r_{kl}}$, $i = 1, \dots, m; j = 1, \dots, n$, namely,

$$Y_j = \frac{\sum_{i=1}^{m} w_i R'_{ij}}{\sum_{k=1}^{m} w_k} = \sum_{i=1}^{m} \left(\frac{w_i \sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{l=1}^{n}\sum_{k=1}^{n} R_{ik} r_{kl}} \right) \Bigg/ \sum_{k=1}^{m} w_k, \quad j = 1, \dots, n, \tag{18}$$

which satisfies $\sum_{j=1}^{n} Y_j \equiv 1$. That is, the technical importance ratings $Y_j (j = 1, \dots, n)$ are normalized.

In fuzzy QFD, since the weights $\tilde{w}_i$ $(i = 1, \dots, m)$, the relationships $\tilde{R}_{ij}$ $(i = 1, \dots, m; j = 1, \dots, n)$ and the correlations $\tilde{r}_{kj}$ $(k, j = 1, \dots, n)$ are all fuzzy numbers, the technical importance ratings $\tilde{Y}_j$ $(j = 1, \dots, n)$ should be fuzzy numbers either. Let $[(w_i)^L_\alpha, (w_i)^U_\alpha]$, $[(R_{ij})^L_\alpha, (R_{ij})^U_\alpha]$ and $[(r_{kj})^L_\alpha, (r_{kj})^U_\alpha]$ be the α-level sets of the fuzzy weights, fuzzy relationships and fuzzy correlations, respectively. Then, the α-level sets of the fuzzy technical importance ratings $\tilde{Y}_j$ $(j = 1, \dots, n)$ can be correctly determined by the following pair of NLP models:

$$(Y_j)^L_\alpha = \text{Min} \sum_{i=1}^{m} \left(\frac{w_i \sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{l=1}^{n}\sum_{k=1}^{n} R_{ik} r_{kl}} \right) \Bigg/ \sum_{k=1}^{m} w_k, \tag{19}$$

$$\begin{aligned} \text{s.t. } & (w_i)^L_\alpha \le w_i \le (w_i)^U_\alpha, \quad i = 1, \dots, m, \\ & (R_{ik})^L_\alpha \le R_{ik} \le (R_{ik})^U_\alpha, \quad i = 1, \dots, m;\ k = 1, \dots, n, \\ & (r_{kl})^L_\alpha \le r_{kl} \le (r_{kl})^U_\alpha, \quad k, l = 1, \dots, n, \end{aligned}$$

$$(Y_j)^U_\alpha = \text{Max} \sum_{i=1}^{m} \left(\frac{w_i \sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{l=1}^{n}\sum_{k=1}^{n} R_{ik} r_{kl}} \right) \Bigg/ \sum_{k=1}^{m} w_k, \tag{20}$$

$$\begin{aligned} \text{s.t. } & (w_i)^L_\alpha \le w_i \le (w_i)^U_\alpha, i = 1, \dots, m, \\ & (R_{ik})^L_\alpha \le R_{ik} \le (R_{ik})^U_\alpha, \quad i = 1, \dots, m;\ k = 1, \dots, n, \\ & (r_{kl})^L_\alpha \le r_{kl} \le (r_{kl})^U_\alpha, \quad k, l = 1, \dots, n. \end{aligned}$$

By solving the above pair of NLP models for each given α-level and each $j = 1, \dots, n$, the fuzzy technical importance ratings of DRs can be correctly captured. According to Zadeh's fuzzy extension principle, $\tilde{Y}_j$ can be expressed as $\tilde{Y}_j = \cup_\alpha \alpha \cdot \left[(Y_j)^L_\alpha, (Y_j)^U_\alpha\right]$ for $0 < \alpha \le 1$ and $j = 1, \dots, n$, based on which the centroid defuzzification method based on α-level sets, introduced in Section 2, can be applied to prioritize DRs.

5. LP models for rating technical importance in fuzzy environments

It is seen from Eq. (16) that if the normalized fuzzy relationships $\tilde{R}'_{ij}$ ($i = 1, \ldots, m; j = 1, \ldots, n$) are known, then the fuzzy technical importance of DRs can be formulated as a FWA and can be calculated by solving a pair of LP models. In this section, we look into how the normalized fuzzy relationships can be correctly calculated using α-level sets.

We first rewrite fuzzy normalization equation (13) as

$$\tilde{R}'_{ij} = \frac{\sum_{k=1}^{n} \tilde{R}_{ik} \otimes \tilde{r}_{kj}}{\sum_{\substack{l=1 \\ l \neq j}}^{n} \sum_{k=1}^{n} \tilde{R}_{ik} \otimes \tilde{r}_{kl} + \sum_{k=1}^{n} \tilde{R}_{ik} \otimes \tilde{r}_{kj}}, \quad i = 1, \ldots, m;\ j = 1, \ldots, n. \tag{21}$$

Due to the reason that fuzzy relationships and fuzzy correlations appear in both numerator and denominator simultaneously, the normalization above cannot be performed using fuzzy arithmetics. The equation, however, can be equivalently characterized using α-level sets by the following pair of NLP models for each $i = 1, \ldots, m$ and $j = 1, \ldots, n$:

$$(R'_{ij})^L_\alpha = \text{Min} \frac{\sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{\substack{l=1 \\ l \neq j}}^{n} \sum_{k=1}^{n} R_{ik} r_{kl} + \sum_{k=1}^{n} R_{ik} r_{kj}}, \tag{22}$$

$$\text{s.t. } (R_{ik})^L_\alpha \le R_{ik} \le (R_{ik})^U_\alpha, \quad k = 1, \ldots, n,$$
$$(r_{kl})^L_\alpha \le r_{kl} \le (r_{kl})^U_\alpha, \quad k, l = 1, \ldots, n,$$

$$(R'_{ij})^U_\alpha = \text{Max} \frac{\sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{\substack{l=1 \\ l \neq j}}^{n} \sum_{k=1}^{n} R_{ik} r_{kl} + \sum_{k=1}^{n} R_{ik} r_{kj}}, \tag{23}$$

$$\text{s.t. } (R_{ik})^L_\alpha \le R_{ik} \le (R_{ik})^U_\alpha, \quad k = 1, \ldots, n,$$
$$(r_{kl})^L_\alpha \le r_{kl} \le (r_{kl})^U_\alpha, \quad k, l = 1, \ldots, n.$$

Let R'_{ij} be a deterministic function defined by

$$R'_{ij} = \frac{\sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{l=1}^{n} \sum_{k=1}^{n} R_{ik} r_{kl}} = \frac{\sum_{k=1}^{n} R_{ik} r_{kj}}{\sum_{\substack{l=1 \\ l \neq j}}^{n} \sum_{k=1}^{n} R_{ik} r_{kl} + \sum_{k=1}^{n} R_{ik} r_{kj}}.$$

Due to the fact that

$$\frac{\partial R'_{ij}}{\partial r_{kj}} = \frac{R_{ik} \left(\sum_{\substack{l=1 \\ l \neq j}}^{n} \sum_{k=1}^{n} R_{ik} r_{kl} + \sum_{k=1}^{n} R_{ik} r_{kj} \right) - \left(\sum_{k=1}^{n} R_{ik} r_{kj} \right) R_{ik}}{\left(\sum_{l=1, l \neq j}^{n} \sum_{k=1}^{n} R_{ik} r_{kl} + \sum_{k=1}^{n} R_{ik} r_{kj} \right)^2}$$
$$= \frac{R_{ik} \left(\sum_{l=1, l \neq j}^{n} \sum_{k=1}^{n} R_{ik} r_{kl} \right)}{\left(\sum_{l=1, l \neq j}^{n} \sum_{k=1}^{n} R_{ik} r_{kl} + \sum_{k=1}^{n} R_{ik} r_{kj} \right)^2} > 0, \quad k = 1, \ldots, n,$$

and that

$$\frac{\partial R'_{ij}}{\partial r_{kl}} = -\frac{\left(\sum_{k=1}^{n} R_{ik} r_{kj} \right) R_{ik}}{\left(\sum_{l=1, l \neq j}^{n} \sum_{k=1}^{n} R_{ik} r_{kl} + \sum_{k=1}^{n} R_{ik} r_{kj} \right)^2} < 0 \quad \text{for } k, l = 1, \ldots, n;\ l \neq j,$$

R'_{ij} is therefore an increasing function of r_{kj} ($k = 1, \ldots, n$), but decreases with r_{kl} ($k, l = 1, \ldots, n$; $l \neq j$). Based upon this conclusion, NLP models (22) and (23) can be simplified as

$$(R'_{ij})^L_\alpha = \text{Min} \frac{\sum_{k=1}^{n} R_{ik}(r_{kj})^L_\alpha}{\sum_{\substack{l=1\\l\neq j}}^{n}\sum_{k=1}^{n} R_{ik}(r_{kl})^U_\alpha + \sum_{k=1}^{n} R_{ik}(r_{kj})^L_\alpha}, \tag{24}$$

$$\text{s.t. } (R_{ik})^L_\alpha \le R_{ik} \le (R_{ik})^U_\alpha, \quad k = 1, \ldots, n,$$

$$(R'_{ij})^U_\alpha = \text{Max} \frac{\sum_{k=1}^{n} R_{ik}(r_{kj})^U_\alpha}{\sum_{\substack{l=1\\l\neq j}}^{n}\sum_{k=1}^{n} R_{ik}(r_{kl})^L_\alpha + \sum_{k=1}^{n} R_{ik}(r_{kj})^U_\alpha}, \tag{25}$$

$$\text{s.t. } (R_{ik})^L_\alpha \le R_{ik} \le (R_{ik})^U_\alpha, \quad k = 1, \ldots, n.$$

Theorem 1. *The pair of models (24) and (25) produces narrower intervals than the formulas (14) and (15) of Chen and Weng [23], reducing the fuzziness and uncertainties of normalized fuzzy relationships.*

The proof is trivial due to the inequalities:

$$\frac{\sum_{k=1}^{n} R_{ik}(r_{kj})^L_\alpha}{\sum_{\substack{l=1\\l\neq j}}^{n}\sum_{k=1}^{n} R_{ik}(r_{kl})^U_\alpha + \sum_{k=1}^{n} R_{ik}(r_{kj})^L_\alpha} \ge \frac{\sum_{k=1}^{n} (R_{ik})^L_\alpha(r_{kj})^L_\alpha}{\sum_{\substack{l=1\\l\neq j}}^{n}\sum_{k=1}^{n} R_{ik}(r_{kl})^U_\alpha + \sum_{k=1}^{n} (R_{ik})^L_\alpha(r_{kj})^L_\alpha}$$

$$\ge \frac{\sum_{k=1}^{n} (R_{ik})^L_\alpha(r_{kj})^L_\alpha}{\sum_{\substack{l=1\\l\neq j}}^{n}\sum_{k=1}^{n} (R_{ik})^U_\alpha(r_{kl})^U_\alpha + \sum_{k=1}^{n} (R_{ik})^L_\alpha(r_{kj})^L_\alpha}$$

and

$$\frac{\sum_{k=1}^{n} R_{ik}(r_{kj})^U_\alpha}{\sum_{\substack{l=1\\l\neq j}}^{n}\sum_{k=1}^{n} R_{ik}(r_{kl})^L_\alpha + \sum_{k=1}^{n} R_{ik}(r_{kj})^U_\alpha} \le \frac{\sum_{k=1}^{n} (R_{ik})^U_\alpha(r_{kj})^U_\alpha}{\sum_{\substack{l=1\\l\neq j}}^{n}\sum_{k=1}^{n} R_{ik}(r_{kl})^L_\alpha + \sum_{k=1}^{n} (R_{ik})^U_\alpha(r_{kj})^U_\alpha}$$

$$\le \frac{\sum_{k=1}^{n} (R_{ik})^U_\alpha(r_{kj})^U_\alpha}{\sum_{\substack{l=1\\l\neq j}}^{n}\sum_{k=1}^{n} (R_{ik})^L_\alpha(r_{kl})^L_\alpha + \sum_{k=1}^{n} (R_{ik})^U_\alpha(r_{kj})^U_\alpha}.$$

Since $R_{ik}(k = 1, \ldots, n)$ are all treated as decision variables, none of them can take two different values in both numerator and denominator simultaneously, correcting the mistakes in the formulas (14) and (15) of Chen and Weng [23].

Theorem 2. *If there is some* DR_j *which is not correlated to any other DRs, namely,* $r_{jk} \equiv 0$ *for* $k = 1, \ldots, n$ *but* $k \neq j$*, then*

$$(R'_{ij})^L_\alpha = \frac{(R_{ij})^L_\alpha(r_{jj})^L_\alpha}{\sum_{\substack{l=1\\l\neq j}}^{n} (R_{il})^U_\alpha(r_{ll})^U_\alpha + (R_{ij})^L_\alpha(r_{jj})^L_\alpha}, \quad i = 1, \ldots, m, \tag{26}$$

$$(R'_{ij})^U_\alpha = \frac{(R_{ij})^U_\alpha(r_{jj})^U_\alpha}{\sum_{\substack{l=1\\l\neq j}}^{n} (R_{il})^L_\alpha(r_{ll})^L_\alpha + (R_{ij})^U_\alpha(r_{jj})^U_\alpha}, \quad i = 1, \ldots, m. \tag{27}$$

Proof. In the case that DR_j is not correlated to any other DRs, there exist $\sum_{k=1}^{n} R_{ik}(r_{kj})^L_\alpha = R_{ij}(r_{jj})^L_\alpha$ and $\sum_{l=1,l\neq j}^{n}\sum_{k=1}^{n} R_{ik}(r_{kl})^U_\alpha = \sum_{l=1,l\neq j}^{n} R_{il}(r_{ll})^U_\alpha$. Accordingly, model (24) becomes minimizing $\frac{R_{ij}(r_{jj})^L_\alpha}{\sum_{l=1,l\neq j}^{n} R_{il}(r_{ll})^U_\alpha + R_{ij}(r_{jj})^L_\alpha}$, which achieves the minimum value at $R_{ij} = (R_{ij})_\alpha^L$ and $R_{il} = (R_{il})_\alpha^U$ ($l = 1, \dots, n; l \neq j$). The resultant minimum value is $(R'_{ij})_\alpha^L = \frac{(R_{ij})_\alpha^L (r_{jj})_\alpha^L}{\sum_{l=1,l\neq j}^n (R_{il})_\alpha^U (r_{ll})_\alpha^U + (R_{ij})_\alpha^L (r_{jj})_\alpha^L}$, which holds for all $i = 1, \dots, m$.

As such, in the case that DR_j is not correlated to any other DRs, there still exist $\sum_{k=1}^n R_{ik}(r_{kj})_\alpha^U = R_{ij}(r_{jj})_\alpha^U$ and $\sum_{l=1,l\neq j}^n \sum_{k=1}^n R_{ik}(r_{kl})_\alpha^L = \sum_{l=1,l\neq j}^n R_{il}(r_{ll})_\alpha^L$. As a result, model (25) becomes maximizing $\frac{R_{ij}(r_{jj})_\alpha^U}{\sum_{l=1,l\neq j}^n R_{il}(r_{ll})_\alpha^L + R_{ij}(r_{jj})_\alpha^U}$, which achieves the maximum value at $R_{ij} = (R_{ij})_\alpha^U$ and $R_{il} = (R_{il})_\alpha^L$ ($l = 1, \dots, n; l \neq j$). The corresponding maximum value is $(R'_{ij})_\alpha^U = \frac{(R_{ij})_\alpha^U (r_{jj})_\alpha^U}{\sum_{l=1,l\neq j}^n (R_{il})_\alpha^L (r_{ll})_\alpha^L + (R_{ij})_\alpha^U (r_{jj})_\alpha^U}$, which holds for all $i = 1, \dots, m$. □

Formulas (26) and (27) are only applicable to the DRs that are independent and not correlated to the other DRs, but cannot be used to normalize the fuzzy relationships among the DRs that are correlated to each other. So, there is a need to develop a universal solution procedure for models (24) and (25). Let

$$t = \frac{1}{\sum_{\substack{l=1\\l\neq j}}^n \sum_{k=1}^n R_{ik}(r_{kl})_\alpha^U + \sum_{k=1}^n R_{ik}(r_{kj})_\alpha^L} \quad \text{and} \quad z_{ik} = tR_{ik} \quad \text{for } k = 1, \dots, n, \tag{28}$$

$$s = \frac{1}{\sum_{\substack{l=1\\l\neq j}}^n \sum_{k=1}^n R_{ik}(r_{kl})_\alpha^L + \sum_{k=1}^n R_{ik}(r_{kj})_\alpha^U} \quad \text{and} \quad u_{ik} = sR_{ik} \quad \text{for } k = 1, \dots, n. \tag{29}$$

Through the above transformations, (24) and (25) can finally be converted into the following pair of LP models for solution:

$$\begin{aligned} (R'_{ij})_\alpha^L &= \text{Min} \sum_{k=1}^n z_{ik}(r_{kj})_\alpha^L, \\ &\quad \text{s.t.} \sum_{k=1}^n z_{ik}\left((r_{kj})_\alpha^L + \sum_{\substack{l=1\\l\neq j}}^n (r_{kl})_\alpha^U\right) = 1, \\ &\quad (R_{ik})_\alpha^L \cdot t \le z_{ik} \le (R_{ik})_\alpha^U \cdot t, \quad k = 1, \dots, n,\ t > 0, \end{aligned} \tag{30}$$

$$\begin{aligned} (R'_{ij})_\alpha^U &= \text{Max} \sum_{k=1}^n u_{ik}(r_{kj})_\alpha^U, \\ &\quad \text{s.t.} \sum_{k=1}^n u_{ik}((r_{kj})_\alpha^U + \sum_{\substack{l=1\\l\neq j}}^n (r_{kl})_\alpha^L) = 1, \\ &\quad (R_{ik})_\alpha^L \cdot s \le u_{ik} \le (R_{ik})_\alpha^U \cdot s, \quad k = 1, \dots, n,\ s > 0, \end{aligned} \tag{31}$$

where t, s, z_{ik} and u_{ik} for $k = 1, \dots, n$ are decision variables. By solving this pair of LP models for each given α-level, each $i = 1, \dots, m$ and each $j = 1, \dots, n$, the normalized fuzzy relationship matrix $\tilde{R}' = (\tilde{R}'_{ij})_{m\times n}$ can be obtained. Once the normalized fuzzy relationships are generated, the technical importance ratings of DRs can then be formulated as a FWA, as shown in Eq. (16). By using α-level sets, the fuzzy technical importance ratings of DRs can be determined by

$$\begin{aligned} (Y_j)_\alpha^L &= \text{Min} \frac{w_1(R'_{1j})_\alpha^L + w_2(R'_{2j})_\alpha^L + \dots + w_m(R'_{mj})_\alpha^L}{w_1 + w_2 + \dots + w_m}, \\ &\quad \text{s.t.}\ (w_i)_\alpha^L \le w_i \le (w_i)_\alpha^U, \quad i = 1, \dots, m, \end{aligned} \tag{32}$$

$$\begin{aligned} (Y_j)_\alpha^U &= \text{Max} \frac{w_1(R'_{1j})_\alpha^U + w_2(R'_{2j})_\alpha^U + \dots + w_m(R'_{mj})_\alpha^U}{w_1 + w_2 + \dots + w_m}, \\ &\quad \text{s.t.}\ (w_i)_\alpha^L \le w_i \le (w_i)_\alpha^U, \quad i = 1, \dots, m, \end{aligned} \tag{33}$$

which can be converted through variable transformations into the following pair of LP models for solution:

$$\begin{aligned} (Y_j)_\alpha^L &= \text{Min}\ v_1(R'_{1j})_\alpha^L + v_2(R'_{2j})_\alpha^L + \dots + v_m(R'_{mj})_\alpha^L, \\ &\quad \text{s.t.}\ v_1 + v_2 + \dots + v_m = 1, \\ &\quad\ z \cdot (w_i)_\alpha^L \le v_i \le z \cdot (w_i)_\alpha^U, \quad i = 1, \dots, m,\ z \ge 0, \end{aligned} \tag{34}$$

$$(Y_j)_\alpha^U = \text{Max } v_1(R'_{1j})_\alpha^U + v_2(R'_{2j})_\alpha^U + \cdots + v_m(R'_{mj})_\alpha^U,$$
$$\text{s.t. } v_1 + v_2 + \cdots + v_m = 1,$$
$$z \cdot (w_i)_\alpha^L \le v_i \le z \cdot (w_i)_\alpha^U, \quad i = 1, \ldots, m, \; z \ge 0. \tag{35}$$

By solving the above pair of LP models (34) and (35) for each given α-level and each $j = 1, \ldots, n$, the fuzzy technical importance ratings of DRs can finally be generated as $\tilde{Y}_j = \cup_\alpha \alpha \cdot \left[(Y_j)_\alpha^L, (Y_j)_\alpha^U\right]$, $0 < \alpha \le 1$, and $j = 1, \ldots, n$.

In summary, in order to correctly rate the technical importance of DRs in fuzzy environments, we need to solve two pairs of LP models, (30)–(31) and (34)–(35), for each given α-level. The following theorem shows the equivalence of the two pairs LP models and the pair of NLP models in Section 4. One can choose either of them for use.

Theorem 3. *The pair of LP models (34) and (35) and the pair of NLP models (19) and (20) are equivalent to each other.*

Proof. Let w_i^* $(i = 1, \ldots, m)$, R_{ik}^* $(i = 1, \ldots, m; k = 1, \ldots, n)$, r_{kl}^* $(k, l = 1, \ldots, n)$ and $(Y_j^*)_\alpha^L$ be, respectively, the optimal solution and optimum objective value of model (19). Then, w_i^* $(i = 1, \ldots, m)$ are a feasible solution to model (32) and R_{ik}^* $(i = 1, \ldots, m; k = 1, \ldots, n)$ and r_{kl}^* $(k, l = 1, \ldots, n)$ are a feasible solution to model (22). Denote by $(R_{ij}^{'*})_\alpha^L$ and $(Y_j^{**})_\alpha^L$ the optimum objective values of models (22) and (32), respectively. Then, there exist

$$(R_{ij}^{'*})_\alpha^L \le \frac{\sum_{k=1}^n R_{ik}^* r_{kj}^*}{\sum_{\substack{l=1\\ l\neq j}}^n \sum_{k=1}^n R_{ij}^* r_{kl}^* + \sum_{k=1}^n R_{ik}^* r_{kj}^*} = \frac{\sum_{k=1}^n R_{ik}^* r_{kj}^*}{\sum_{l=1}^n \sum_{k=1}^n R_{ik}^* r_{kl}^*}$$

and

$$(Y_j^{**})_\alpha^L \le \frac{w_1^*(R_{1j}^{'*})_\alpha^L + w_2^*(R_{2j}^{'*})_\alpha^L + \cdots + w_m^*(R_{mj}^{'*})_\alpha^L}{w_1^* + w_2^* + \cdots + w_m^*} \le \sum_{i=1}^m \left(\frac{w_i^* \sum_{k=1}^n R_{ik}^* r_{kj}^*}{\sum_{l=1}^n \sum_{k=1}^n R_{ik}^* r_{kl}^*} \right) \Bigg/ \sum_{k=1}^m w_k^* = (Y_j^*)_\alpha^L.$$

Similarly, let w_i^{**} $(i = 1, \ldots, m)$ and $(Y_j^{**})_\alpha^L$ be the optimal solution and optimum objective value of model (32), R_{ik}^{**} $(i = 1, \ldots, m; k = 1, \ldots, n)$, r_{kl}^{**} $(k, l = 1, \ldots, n)$ and $(R_{ij}^{'*})_\alpha^L$ be the optimal solution and optimum objective value of model (22), respectively. It is easy to see that w_i^{**} $(i = 1, \ldots, m)$, R_{ik}^{**} $(i = 1, \ldots, m; k = 1, \ldots, n)$ and r_{kl}^{**} $(k, l = 1, \ldots, n)$ are a feasible solution of model (19). Denote by $(Y_j^*)_\alpha^L$ the optimum objective value of model (19). Then, there exist

$$(R_{ij}^{'*})_\alpha^L = \frac{\sum_{k=1}^n R_{ik}^{**} r_{kj}^{**}}{\sum_{\substack{l=1\\ l\neq j}}^n \sum_{k=1}^n R_{ij}^{**} r_{kl}^{**} + \sum_{k=1}^n R_{ik}^{**} r_{kj}^{**}} = \frac{\sum_{k=1}^n R_{ik}^{**} r_{kj}^{**}}{\sum_{l=1}^n \sum_{k=1}^n R_{ik}^{**} r_{kl}^{**}}$$

and

$$(Y_j^*)_\alpha^L \le \sum_{i=1}^m \left(\frac{w_i^{**} \sum_{k=1}^n R_{ik}^{**} r_{kj}^{**}}{\sum_{l=1}^n \sum_{k=1}^n R_{ik}^{**} r_{kl}^{**}} \right) \Bigg/ \sum_{k=1}^m w_k^{**} = \frac{\sum_{i=1}^m w_i^{**} (R_{ij}^{'*})_\alpha^L}{\sum_{k=1}^m w_k^{**}} = (Y_j^{**})_\alpha^L.$$

From $(Y_j^{**})_\alpha^L \le (Y_j^*)_\alpha^L$ and $(Y_j^*)_\alpha^L \le (Y_j^{**})_\alpha^L$, it can be concluded that $(Y_j^*)_\alpha^L \equiv (Y_j^{**})_\alpha^L$. So, models (32) and (19) are equivalent to each other. As such, models (33) and (20) can also be proved equivalent to each other in the same way. Since the pair of (34) and (35) and the pair of (32) and (33) are equal, the pair of LP models (34) and (35) is thus equivalent to the pair of NLP models (19) and (20). This completes the proof. □

6. Numerical examples

In this section, we examine two fuzzy QFD examples using the proposed NLP or LP models to illustrate their applications and implementation processes.

Example 1. Consider the fuzzy QFD example investigated by Chen and Weng [25], which involves four CRs and five engineering DRs. The relative importance of the four CRs, the relationships between the four CRs and the five DRs, and the correlations among the five DRs are all described by linguistic terms, whose membership functions are defined in Fig. 2, where the linguistic terms for describing the technical importance of the CRs such as *Very important*, *Important*, *Fairly*

Table 1
Fuzzy relationships and fuzzy correlations for fuzzy QFD in Example 1.

⊚: Very strong/important	Correlation matrix	DR$_5$		□	⊚		⊚
⊙: Strong/Important		DR$_4$				⊚	
⊗: Fairly strong/important		DR$_3$		□	⊚		⊚
□: Medium		DR$_2$		⊚	□		□
△: Fairly weak/unimportant		DR$_1$	⊚				
◇: Weak/Unimportant	Degree of importance		Product or engineering design requirements				
▽: Very weak/unimportant			DR$_1$	DR$_2$	DR$_3$	DR$_4$	DR$_5$
Customer requirements	CR$_1$	◇	□			⊚	
	CR$_2$	□		□	⊚		⊚
	CR$_3$	⊗	◇	□	⊚		⊚
	CR$_4$	□	◇			⊚	

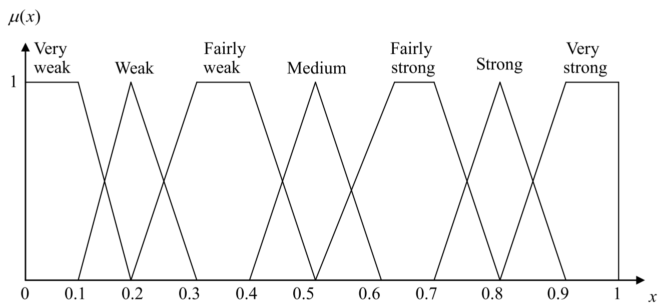

Fig. 2. Membership function of linguistic terms for relationships and correlations.

important, *Medium*, *Fairly unimportant*, *Unimportant* and *Very unimportant* share the same membership functions as those for describing relationships and correlations. Table 1 shows the relationship matrix and the correlation matrix of this fuzzy QFD example.

For this numerical example, we set α-level as 0, 0.1, ..., 1, respectively, and write the α-level sets of the fuzzy weights, fuzzy relationships, and fuzzy correlations for each given α-level as below:

$$w_{1\alpha} = [0.1 + 0.1\alpha, 0.3 - 0.1\alpha],$$
$$w_{2\alpha} = w_{4\alpha} = [0.4 + 0.1\alpha, 0.6 - 0.1\alpha],$$
$$w_{3\alpha} = [0.5 + 0.1\alpha, 0.8 - 0.1\alpha],$$
$$R_{11\alpha} = R_{22\alpha} = R_{32\alpha} = [0.4 + 0.1\alpha,\ 0.6 - 0.1\alpha],$$
$$R_{31\alpha} = R_{41\alpha} = [0.1 + 0.1\alpha, 0.3 - 0.1\alpha],$$
$$R_{14\alpha} = R_{23\alpha} = R_{25\alpha} = R_{33\alpha} = R_{35\alpha} = R_{44\alpha} = [0.8 + 0.1\alpha,\ 1],$$
$$R_{12\alpha} = R_{13\alpha} = R_{15\alpha} = R_{21\alpha} = R_{24\alpha} = R_{34\alpha} = R_{42\alpha} = R_{43\alpha} = R_{45\alpha} = 0,$$
$$r_{11\alpha} = r_{22\alpha} = r_{33\alpha} = r_{44\alpha} = r_{55\alpha} = r_{53\alpha} = r_{35\alpha} = [0.8 + 0.1\alpha,\ 1],$$

Table 2
Normalized fuzzy relationships for different α-levels in Example 1.

α		$(R'_{11})_\alpha$	$(R'_{14})_\alpha$	$(R'_{22})_\alpha$	$(R'_{23})_\alpha$	$(R'_{25})_\alpha$	$(R'_{31})_\alpha$	$(R'_{32})_\alpha$	$(R'_{33})_\alpha$	$(R'_{35})_\alpha$	$(R'_{41})_\alpha$	$(R'_{44})_\alpha$
0	Suf	0.4839	0.7576	0.3391	0.4375	0.4375	0.0725	0.3333	0.4308	0.4308	0.3191	0.9259
	Inf	0.2424	0.5161	0.2000	0.3016	0.3016	0.0121	0.1898	0.2846	0.2846	0.0741	0.6809
0.1	Suf	0.4735	0.7507	0.3321	0.4326	0.4326	0.0683	0.3259	0.4253	0.4253	0.3065	0.9182
	Inf	0.2493	0.5265	0.2044	0.3066	0.3066	0.0136	0.1944	0.2901	0.2901	0.0818	0.6935
0.2	Suf	0.4631	0.7438	0.3252	0.4278	0.4278	0.0643	0.3186	0.4199	0.4199	0.2940	0.9104
	Inf	0.2562	0.5369	0.2088	0.3117	0.3117	0.0151	0.199	0.2955	0.2955	0.0896	0.7060
0.3	Suf	0.4528	0.7370	0.3184	0.4231	0.4231	0.0605	0.3113	0.4146	0.4146	0.2816	0.9026
	Inf	0.2630	0.5472	0.2132	0.3167	0.3167	0.0167	0.2036	0.3009	0.3009	0.0974	0.7184
0.4	Suf	0.4425	0.7301	0.3116	0.4184	0.4184	0.0568	0.3042	0.4094	0.4094	0.2693	0.8948
	Inf	0.2699	0.5575	0.2176	0.3217	0.3217	0.0183	0.2082	0.3063	0.3063	0.1052	0.7307
0.5	Suf	0.4322	0.7233	0.3049	0.4137	0.4137	0.0533	0.2971	0.4042	0.4042	0.2571	0.8869
	Inf	0.2767	0.5678	0.2220	0.3267	0.3267	0.0200	0.2128	0.3117	0.3117	0.1131	0.7429
0.6	Suf	0.4220	0.7615	0.2983	0.4091	0.4091	0.0499	0.2902	0.3991	0.3991	0.2450	0.8790
	Inf	0.2835	0.5780	0.2263	0.3317	0.3317	0.0217	0.2174	0.3171	0.3171	0.1210	0.7550
0.7	Suf	0.4118	0.7098	0.2917	0.4046	0.4046	0.0467	0.2833	0.3941	0.3941	0.2331	0.8712
	Inf	0.2902	0.5882	0.2307	0.3367	0.3367	0.0235	0.222	0.3225	0.3225	0.1288	0.7669
0.8	Suf	0.4017	0.7030	0.2852	0.4001	0.4001	0.0436	0.2765	0.3892	0.3892	0.2212	0.8633
	Inf	0.2970	0.5983	0.2350	0.3416	0.3416	0.0253	0.2266	0.3279	0.3279	0.1367	0.7788
0.9	Suf	0.3917	0.6963	0.2788	0.3957	0.3957	0.0406	0.2698	0.3843	0.3843	0.2096	0.8554
	Inf	0.3037	0.6083	0.2394	0.3466	0.3466	0.0272	0.2312	0.3334	0.3334	0.1446	0.7904
1	Suf	0.3817	0.6897	0.2724	0.3913	0.3913	0.0378	0.2632	0.3794	0.3794	0.1980	0.8475
	Inf	0.3103	0.6183	0.2437	0.3515	0.3515	0.0291	0.2358	0.3388	0.3388	0.1525	0.8020

Table 3
Technical importance ratings of the five DRs by the proposed models.

α	DR_1	DR_2	DR_3	DR_4	DR_5
0	[0.0335, 0.2072]	[0.0972, 0.2474]	[0.1461, 0.3195]	[0.1705, 0.4349]	[0.1461, 0.3195]
0.1	[0.0377, 0.1961]	[0.1016, 0.2386]	[0.1520, 0.3112]	[0.1801, 0.4219]	[0.1520, 0.3112]
0.2	[0.0422, 0.1853]	[0.1062, 0.2301]	[0.1581, 0.3030]	[0.1900, 0.4091]	[0.1581, 0.3030]
0.3	[0.0468, 0.1749]	[0.1109, 0.2217]	[0.1643, 0.2950]	[0.2000, 0.3964]	[0.1643, 0.2950]
0.4	[0.0516, 0.1647]	[0.1157, 0.2135]	[0.1705, 0.2871]	[0.2103, 0.3838]	[0.1705, 0.2871]
0.5	[0.0565, 0.1549]	[0.1205, 0.2055]	[0.1769, 0.2793]	[0.2208, 0.3715]	[0.1769, 0.2793]
0.6	[0.0616, 0.1453]	[0.1255, 0.1978]	[0.1834, 0.2717]	[0.2315, 0.3592]	[0.1834, 0.2717]
0.7	[0.0669, 0.1360]	[0.1305, 0.1902]	[0.1900, 0.2643]	[0.2423, 0.3472]	[0.1900, 0.2643]
0.8	[0.0723, 0.1270]	[0.1357, 0.1828]	[0.1968, 0.2570]	[0.2534, 0.3353]	[0.1968, 0.2570]
0.9	[0.0778, 0.1184]	[0.1410, 0.1757]	[0.2036, 0.2498]	[0.2647, 0.3236]	[0.2036, 0.2498]
1	[0.0835, 0.1100]	[0.1463, 0.1687]	[0.2106, 0.2428]	[0.2761, 0.3120]	[0.2106, 0.2428]
Centroid	0.1102	0.1663	0.2306	0.2992	0.2306
Ranking	5	4	2	1	2

$$r_{23\alpha} = r_{32\alpha} = r_{25\alpha} = r_{52\alpha} = [0.4 + 0.1\alpha, 0.6 - 0.1\alpha],$$

$$r_{12\alpha} = r_{13\alpha} = r_{14\alpha} = r_{15\alpha} = r_{21\alpha} = r_{24\alpha} = r_{31\alpha} = r_{34\alpha} = r_{41\alpha} = r_{42\alpha} = r_{43\alpha} = r_{45\alpha} = r_{51\alpha} = r_{54\alpha} = 0.$$

We then solve the pair of NLP models (19) and (20) or the two equivalent pairs of LP models (30) and (31) as well as (34) and (35). Tables 2 and 3 show, respectively, the normalized fuzzy relationships obtained by solving the pair of LP models (30) and (31) and the normalized fuzzy technical importance ratings of the five DRs obtained by solving either the pair of NLP models (19) and (20) or the pair of LP models (34) and (35), where the centroids of the five fuzzy technical importance ratings are computed by formulas (5)–(8). For the sake of visualization, the fuzzy technical importance ratings of the five DRs are pictured in Fig. 3, from which it is seen that DR_4 has a top priority for resource allocation, followed by DR_3 and DR_5, while DR_1 is the least important engineering design requirement.

Table 4 shows the fuzzy technical importance ratings of the five DRs obtained by combining fuzzy normalization formulas (14) and (15) of Chen and Weng [23] with FWA. These ratings are essentially incorrect and make no sense at all in spite of the fact that there is no big difference between them and those in Table 3. It is observed that the technical importance ratings of DR_1 and DR_4 are not affected by the incorrect fuzzy normalization procedure of Chen and Weng [23]. This is because they are not correlated to any other DRs except for themselves. It is also observed that the ratings of DR_2, DR_3 and DR_5 in Table 4 have wider support intervals than those in Table 3, leading to more fuzziness and uncertainty. Quite evidently, Chen and Weng's approach for rating the technical importance of DRs in fuzzy QFD is inadvisable.

Example 2. Consider the fuzzy QFD example investigated by Chen et al. [30], which involves eight CRs and ten independent technical attributes (TAs), namely, product or engineering DRs. The relative importance weights of the eight CRs and the relationships between the eight CRs and the ten TAs are described by triangular fuzzy numbers, as shown in Table 5.

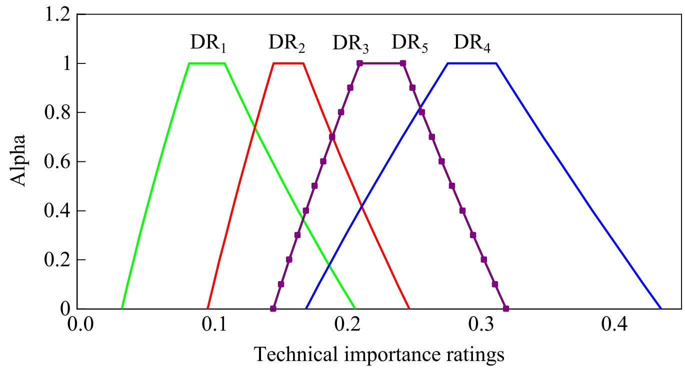

Fig. 3. Technical importance ratings of the five DRs by the proposed models.

Table 4
Incorrect technical importance ratings of the five DRs by combining Chen and Weng's fuzzy normalization procedure and FWA.

α	DR_1	DR_2	DR_3	DR_4	DR_5
0	[0.0335, 0.2072]	[0.0822, 0.2807]	[0.1249, 0.3619]	[0.1705, 0.4349]	[0.1249, 0.3619]
0.1	[0.0377, 0.1961]	[0.0871, 0.2689]	[0.1315, 0.3498]	[0.1801, 0.4219]	[0.1315, 0.3498]
0.2	[0.0422, 0.1853]	[0.0921, 0.2574]	[0.1382, 0.3381]	[0.1900, 0.4091]	[0.1382, 0.3381]
0.3	[0.0468, 0.1749]	[0.0973, 0.2463]	[0.1451, 0.3266]	[0.2000, 0.3964]	[0.1451, 0.3266]
0.4	[0.0516, 0.1647]	[0.1027, 0.2355]	[0.1523, 0.3155]	[0.2103, 0.3838]	[0.1523, 0.3155]
0.5	[0.0565, 0.1549]	[0.1083, 0.2250]	[0.1596, 0.3047]	[0.2208, 0.3715]	[0.1596, 0.3047]
0.6	[0.0616, 0.1453]	[0.1140, 0.2148]	[0.1672, 0.2942]	[0.2315, 0.3652]	[0.1672, 0.2942]
0.7	[0.0669, 0.1360]	[0.1199, 0.2050]	[0.1749, 0.2839]	[0.2423, 0.3472]	[0.1749, 0.2839]
0.8	[0.0723, 0.1270]	[0.1299, 0.1954]	[0.1829, 0.2740]	[0.2534, 0.3353]	[0.1829, 0.2740]
0.9	[0.0778, 0.1184]	[0.1321, 0.1862]	[0.1911, 0.2643]	[0.2647, 0.3236]	[0.1911, 0.2643]
1	[0.0835, 0.1100]	[0.1385, 0.1773]	[0.1995, 0.2550]	[0.2761, 0.3120]	[0.1995, 0.2550]
Centroid	0.1102	0.1716	0.2365	0.2992	0.2365
Ranking	5	4	2	1	2

Chen et al. [30] rated the technical importance of the ten TAs using FWA without normalizing the fuzzy relationships between the eight CRs and the ten TAs. The results are provided in Table 6. As we have pointed out in Section 3, the relationships or fuzzy relationships between CRs and DRs should be normalized before calculating technical importance ratings of DRs. The reason for this is similar to that for weight normalization in AHP. For this fuzzy QFD example, since correlations are not considered, we can normalize fuzzy relationships by using analytical formulas (26) and (27) without solving the pair of LP models (30) and (31). The technical importance ratings of the ten TAs can then be formulated as a FWA and be computed by solving the pair of LP models (34) and (35) for each given α-level. The results are shown in Table 7, where the centroids of the ten fuzzy technical importance ratings are calculated by formula (8). From Tables 6 and 7, it is observed that the main difference between the results in the two tables lies in that the fuzzy technical importance ratings in Table 7 are normalized, whereas those in Table 6 are not. There is also a minor difference between the priority rankings of TA_5 and TA_{10}, but the defuzzified ratings for the two TAs are nearly equal, indicating that they could be considered as equally important.

7. Conclusions

As a customer-oriented methodology, QFD has been widely applied to improve product quality to achieve higher or maximum customer satisfaction. To achieve maximum customer satisfaction, limited organizational resources such as budget have usually to be optimally allocated among DRs in terms of their technical importance ratings. Without correct technical importance ratings of DRs, it would be impossible to optimally allocate limited organizational resources among them to achieve maximum customer satisfaction. This requires the development of a method that can correctly rate the technical importance of DRs.

In this paper we have investigated how to correctly rate the technical importance of DRs in fuzzy QFD using α-level sets. We have presented a literature review on existing approaches for rating technical importance of DRs in fuzzy environments and pointed out their incorrectness or inappropriateness. We have then developed a pair of NLP models and two pairs of LP models, respectively, for rating the technical importance of DRs in fuzzy environments. We have proved the equivalence of the two pairs of LP models and the pair of NLP models. The developed models have finally been examined and illustrated with two numerical examples. The numerical examinations have clearly revealed that the developed models can correctly and accurately rate the technical importance of DRs in fuzzy environments.

Table 5
Fuzzy relationships and fuzzy weights for fuzzy QFD in Example 2.

$\tilde{R}_{ij}$	CR_1	CR_2	CR_3	CR_4	CR_5	CR_6	CR_7	CR_8
TA_1	(0.31, 0.54, 0.76)	(0.09, 0.25, 0.50)	(0.56, 0.82, 0.96)	(0.39, 0.64, 0.90)	(0.56, 0.82, 0.96)	(0.13, 0.36, 0.59)	(0.09, 0.32, 0.56)	(0.21, 0.43, 0.64)
TA_2	(0.16, 0.36, 0.60)	(0.47, 0.71, 0.91)	(0.27, 0.50, 0.73)	(0.27, 0.50, 0.73)	(0.59, 0.86, 1.00)	(0.13, 0.32, 0.56)	(0.33, 0.54, 0.74)	(0.09, 0.29, 0.53)
TA_3	(0.27, 0.50, 0.73)	(0.56, 0.82, 0.96)	(0.61, 0.89, 1.00)	(0.23, 0.46, 0.70)	(0.39, 0.61, 0.83)	(0.29, 0.50, 0.71)	(0.36, 0.57, 0.79)	(0.39, 0.61, 0.79)
TA_4	(0.37, 0.61, 0.80)	(0.13, 0.32, 0.56)	(0.13, 0.36, 0.63)	(0.24, 0.46, 0.69)	(0.59, 0.86, 0.96)	(0.39, 0.61, 0.83)	(0.04, 0.25, 0.50)	(0.31, 0.54, 0.76)
TA_5	(0.13, 0.29, 0.53)	(0.13, 0.29, 0.53)	(0.17, 0.36, 0.59)	(0.13, 0.32, 0.56)	(0.09, 0.25, 0.50)	(0.09, 0.32, 0.56)	(0.29, 0.50, 0.71)	(0.04, 0.25, 0.50)
TA_6	(0.17, 0.39, 0.61)	(0.24, 0.46, 0.69)	(0.13, 0.36, 0.59)	(0.17, 0.36, 0.59)	(0.13, 0.36, 0.59)	(0.17, 0.39, 0.61)	(0.31, 0.61, 0.83)	(0.17, 0.39, 0.61)
TA_7	(0.36, 0.57, 0.79)	(0.34, 0.57, 0.80)	(0.53, 0.79, 0.91)	(0.24, 0.46, 0.70)	(0.27, 0.50, 0.73)	(0.09, 0.29, 0.53)	(0.56, 0.82, 1.00)	(0.50, 0.75, 0.96)
TA_8	(0.17, 0.39, 0.61)	(0.27, 0.50, 0.73)	(0.17, 0.39, 0.61)	(0.20, 0.43, 0.66)	(0.31, 0.54, 0.76)	(0.09, 0.32, 0.56)	(0.36, 0.57, 0.79)	(0.36, 0.57, 0.79)
TA_9	(0.13, 0.29, 0.53)	(0.47, 0.71, 0.87)	(0.09, 0.25, 0.50)	(0.23, 0.46, 0.70)	(0.34, 0.57, 0.76)	(0.13, 0.36, 0.59)	(0.09, 0.32, 0.56)	(0.09, 0.32, 0.56)
TA_{10}	(0.17, 0.32, 0.56)	(0.04, 0.18, 0.44)	(0.09, 0.29, 0.53)	(0.09, 0.25, 0.50)	(0.41, 0.64, 0.83)	(0.13, 0.32, 0.56)	(0.04, 0.29, 0.53)	(0.16, 0.39, 0.63)
$\tilde{W}_i$	(0.71, 0.91, 0.98)	(0.40, 0.58, 0.75)	(0.61, 0.80, 0.94)	(0.57, 0.76, 0.88)	(0.30, 0.47, 0.64)	(0.47, 0.65, 0.83)	(0.52, 0.70, 0.86)	(0.63, 0.82, 0.93)

Table 6
Technical importance ratings of the 10 technical attributes by fuzzy weighted average.

Technical attribute		α											Centroid	Ranking
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1		
TA_1	Sup	0.7746	0.7496	0.7246	0.6996	0.6745	0.6493	0.6242	0.5989	0.5736	0.5483	0.5228	0.5163	3
	Inf	0.2518	0.2786	0.3054	0.3323	0.3593	0.3863	0.4135	0.4407	0.4680	0.4954	0.5228		
TA_2	Sup	0.7335	0.7088	0.6840	0.6593	0.6344	0.6095	0.5844	0.5592	0.5339	0.5084	0.4828	0.4839	5
	Inf	0.2342	0.2589	0.2837	0.3085	0.3333	0.3582	0.3831	0.4080	0.4392	0.4579	0.4828		
TA_3	Sup	0.8320	0.8109	0.7896	0.7683	0.7468	0.7252	0.7035	0.6817	0.6597	0.6377	0.6157	0.6028	1
	Inf	0.3599	0.3853	0.4107	0.4362	0.4617	0.4872	0.5128	0.5385	0.5641	0.5899	0.6157		
TA_4	Sup	0.7401	0.7158	0.6914	0.6669	0.6423	0.6175	0.5925	0.5675	0.5423	0.5170	0.4915	0.4881	4
	Inf	0.2312	0.2572	0.2832	0.3091	0.3350	0.3610	0.3870	0.4131	0.4392	0.4653	0.4915		
TA_5	Sup	0.5740	0.5490	0.5240	0.4991	0.4741	0.4491	0.4241	0.3991	0.3741	0.3491	0.3240	0.3405	10
	Inf	0.1233	0.1437	0.1638	0.1837	0.2037	0.2237	0.2437	0.2637	0.2838	0.3039	0.3240		
TA_6	Sup	0.6548	0.6304	0.6061	0.5819	0.5577	0.5335	0.5094	0.4854	0.4614	0.4374	0.4135	0.4143	7
	Inf	0.1761	0.1995	0.2230	0.2466	0.2702	0.2939	0.3176	0.3415	0.3654	0.3894	0.4135		
TA_7	Sup	0.8410	0.8178	0.7945	0.7712	0.7477	0.7242	0.7006	0.6768	0.6530	0.6291	0.6052	0.5958	2
	Inf	0.3409	0.3671	0.3934	0.4197	0.4460	0.4723	0.4986	0.5251	0.5517	0.5784	0.6052		
TA_8	Sup	0.7026	0.6783	0.6539	0.6296	0.6052	0.5808	0.5565	0.5321	0.5078	0.4834	0.4590	0.4599	6
	Inf	0.2175	0.2418	0.2661	0.2903	0.3145	0.3386	0.3628	0.3869	0.4110	0.4350	0.4590		
TA_9	Sup	0.6444	0.6191	0.5938	0.5684	0.5429	0.5174	0.4918	0.4662	0.4405	0.4148	0.3890	0.3962	8
	Inf	0.1553	0.1784	0.2016	0.2248	0.2481	0.2714	0.2948	0.3183	0.3418	0.3654	0.3890		
TA_{10}	Sup	0.5851	0.5590	0.5330	0.5070	0.4809	0.4549	0.4289	0.4029	0.3769	0.3510	0.3250	0.3414	9
	Inf	0.1144	0.1354	0.1564	0.1774	0.1985	0.2195	0.2406	0.2617	0.2827	0.3038	0.3250		

Table 7
Technical importance ratings of the 10 technical attributes by integrating fuzzy normalization and fuzzy weighted average.

Technical attribute		α											Centroid	Ranking
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1		
TA_1	Sup	0.2682	0.2440	0.2226	0.2037	0.1867	0.1715	0.1576	0.1449	0.1332	0.1223	0.1123	0.1319	3
	Inf	0.0392	0.0446	0.0503	0.0564	0.0628	0.0697	0.0771	0.0850	0.0934	0.1025	0.1123		
TA_2	Sup	0.2504	0.2275	0.2075	0.1896	0.1737	0.1592	0.1459	0.1337	0.1225	0.1121	0.1025	0.1224	5
	Inf	0.0362	0.0411	0.0464	0.0519	0.0578	0.0641	0.0708	0.0779	0.0855	0.0937	0.1025		
TA_3	Sup	0.2937	0.2688	0.2468	0.2272	0.2096	0.1937	0.1793	0.1661	0.1541	0.1430	0.1327	0.1525	1
	Inf	0.0561	0.0617	0.0677	0.0740	0.0808	0.0880	0.0957	0.1040	0.1129	0.1224	0.1327		
TA_4	Sup	0.2680	0.2427	0.2205	0.2008	0.1834	0.1678	0.1536	0.1406	0.1288	0.1178	0.1077	0.1290	4
	Inf	0.0364	0.0417	0.0472	0.0531	0.0594	0.0661	0.0732	0.0809	0.0892	0.0981	0.1077		
TA_5	Sup	0.2094	0.1876	0.1684	0.1514	0.1362	0.1225	0.1102	0.0990	0.0888	0.0794	0.0708	0.0930	9
	Inf	0.0192	0.0231	0.0271	0.0314	0.0359	0.0407	0.0458	0.0514	0.0574	0.0638	0.0708		
TA_6	Sup	0.2349	0.2122	0.1922	0.1745	0.1586	0.1444	0.1316	0.1199	0.1092	0.0994	0.0903	0.1101	7
	Inf	0.0277	0.0322	0.0371	0.0423	0.0478	0.0536	0.0599	0.0667	0.0740	0.0818	0.0903		
TA_7	Sup	0.2944	0.2692	0.2470	0.2271	0.2092	0.193	0.1782	0.1647	0.1523	0.1409	0.1303	0.1508	2
	Inf	0.0527	0.0585	0.0645	0.0709	0.0778	0.0851	0.0929	0.1012	0.1102	0.1199	0.1303		
TA_8	Sup	0.2474	0.2241	0.2037	0.1855	0.1693	0.1548	0.1416	0.1297	0.1187	0.1086	0.0992	0.1193	6
	Inf	0.0340	0.0389	0.0440	0.0495	0.0553	0.0615	0.0681	0.075	0.0825	0.0905	0.0992		
TA_9	Sup	0.2281	0.2057	0.1859	0.1683	0.1526	0.1384	0.1255	0.1138	0.1031	0.0933	0.0842	0.1050	8
	Inf	0.0242	0.0287	0.0334	0.0383	0.0436	0.0492	0.0552	0.0617	0.0686	0.0761	0.0842		
TA_{10}	Sup	0.2070	0.1855	0.1668	0.1501	0.1352	0.1216	0.1094	0.0982	0.0880	0.0786	0.0699	0.0915	10
	Inf	0.0177	0.0215	0.0256	0.0300	0.0346	0.0395	0.0447	0.0504	0.0564	0.0629	0.0699		

In comparison with existing approaches for rating the technical importance of DRs in fuzzy environments, the developed models have some significant merits such as being able to rate the technical importance of DRs in fuzzy environments accurately through α-level sets, producing normalized fuzzy technical importance ratings for DRs, offering two alternative options, linear and nonlinear programming, for technical importance rating. It is believed that the developed models make a good contribution to fuzzy QFD and lay a solid theoretical foundation for its development and applications. The future research work is to combine the developed models and limited resources to set targets for DRs. A belief-rule based (BRB) methodology will be developed.

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