Prediction of the diameter of jet grouting columns with artificial neural networks

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Abstract

The prediction of the diameter of columns is a fundamental step for the design of jet grouting applications, as harmful consequences may derive from an inadequate selection of the treatment setup. Starting from different perspectives, empirical or theoretical correlations between the mean diameter of columns, the treatment parameters and the mechanical properties of native soils have been provided in the literature. However, the margin of uncertainty with these relations is still relatively large, mostly because of arbitrary assumptions made in their formulation. In order to reduce as much as possible the role of preliminary choices, a method based on artificial neural networks (ANN) is proposed. It consists in training a computer code with a set of experimental observations and in using the established correlations between input and output variables to predict future occurrences. After a brief introduction of the principles and limitations of ANN’s, the paper describes the logical procedure followed for the selection of the variables which better describe the mechanism of columns formation. A database of more than 130 case studies, where jet grouting parameters, properties of soil and diameters are simultaneously reported, has been collected from the literature to train the network. Systematic analyses have been then performed, parametrically varying the structure of the network and the use of data, in order to improve the accuracy of prediction. The comparison with other methods recently published in the literature confirms the good predictive capability of the proposed method. For its practical application, a set of design charts has been produced where the mean diameters of columns are expressed, for all injection systems and soil types, as functions of the soil penetration index $N_{SPT}$ and the specific energy of treatment. Safety factors have been finally computed to take into account the inaccuracy of prediction.

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1. Introduction

The jet grouting is one of the most popular ground improvement methods, being adopted worldwide for the solution of various geotechnical problems (foundation, earth retaining, waterproofing etc.). The technique basically consists in cutting and mixing in place the soil with cement grout, ejected with high speed from small nozzles, in order to form sub-cylindrical columns of cemented material (Yahiro and Yoshida, 1973). Possible alternatives to the basic solution (named “single fluid” system), where only cement grout is injected, consist in protecting the action of the injected grout with a coaxial jet of air (“double fluid” system) or in using a coaxial jet of air and water to initially cut the soil and a second jet of grout to infill cementation (“triple fluid” system) (Croce et al., 2014). For a successful performance
of the jet grouted structures, either when they are formed by arrays of isolated columns (e.g. Modoni and Bzówka, 2012) or when they consist of continuous elements made of partially overlapped columns (Croce and Modoni, 2005; Eramo et al., 2012; Arroyo et al., 2012), it is fundamental to control the diameter of columns by tuning the power of injection in relation to the properties of native soils. In fact, if diameters are smaller than expected, as it may descend from an erroneous choice of the treatment parameters, the jet grouted elements (single columns or panels, circular shafts, tunnel canopies made by overlapped columns etc.) may be too weak or discontinuous, and hence the integrity and functionality of the overall structures may be threatened (e.g. Maertens and Maekelberg, 2001; Croce and Modoni, 2002; Lignola et al., 2008).

In general, the diameter of columns, depends on the ability of the jet to propagate its cutting/erosive action at larger distances from the nozzle, which is determined by the combination between the energy given with the injection and the resistance of soil (Bergschneider and Walz, 2003). The first is a function of the composition of the injected fluids, number and diameter of nozzles, injection flow rate, retraction speed of the monitor, whereas the latter depends on the mechanisms activated at the jet soil interface and is thus governed by the composition and initial state of the surrounding soil (Dabbagh et al., 2002).

In the past, the columns dimension has been controlled with different strategies. Former relations (e.g. Miki and Nakanishi, 1984; Botto, 1985; Tornaghi, 1989; Bell, 1993; Civil and Skinner, 1994; Croce and Flora, 2000) considered the dependency of diameters on soils properties and/or treatment parameters starting from site observation. The main limitations of such empirical studies stem from the reduced number of cases available to build relations but, moreover, from a non systematic analysis of the mechanisms taking place during injection. As a result, subjective and incomplete choices of the relevant factors turned into formulations lacking of generality. To fill this gap, an alternative strategy was undertaken by Chu (2005), Modoni et al. (2006) and Ho (2007), who analysed the mechanisms taking place during the diffusion of submerged jets and at the impact between jet and soil to formulate mathematical relations describing these phenomena. These functions are the basic components of theoretical models used to predict the diameters produced by single fluid jet grouting in cohesive soils (Chu, 2005) or in gravelly, sandy or clayey materials (Modoni et al., 2006).

Borrowing the concepts of the Modoni et al. model, Shen et al. (2013) formulated a theoretical solution applicable to cohesionless and cohesive soils treated with single, double and triple fluid systems. In this formulation the diameter of column \( D \) is obtained adding to the diameter of the monitor \( (D_r) \) the double of the erosion distance effectively covered by the jet. The latter is computed as product between its ultimate value \( x_L \), i.e. the distance which would be obtained for an unlimited action time of the jet, function of the soil properties and jet grouting parameters, and a reduction coefficient dependent on the injection time \( (\eta) \).

\[
D = 2 \cdot \eta \cdot x_L + D_r
\]

An alternative approach was proposed by Croce et al. (2011) who tried to reduce the mathematical complexity of the Modoni et al. model, which requires to iteratively integrate cascade equations, proving its equivalence with the following simpler power function:

\[
D = A \cdot s^\alpha \cdot E_n^\beta
\]

where the diameter of columns \( (D) \) is related to the resistance of soil \( (s) \) which is expressed by the undrained shear strength for cohesive materials, the product between vertical stress and tangent of friction angle for cohesionless soils) and to the specific energy at the nozzle \( (E'_n) \). The latter, which represents the energy given per unit length of columns, has the advantage of grouping all the relevant treatment variables into a unique parameters expressed by the following relations:

\[
E'_n = \frac{1}{2} \cdot \frac{m \cdot v_0^2}{L} = \frac{\pi}{8} \cdot \frac{M \cdot \rho \cdot d^2 \cdot v_o^3}{\nu_r}
\]

(3)

where \( M \) and \( d \) represent respectively the number and diameter of nozzles, \( \rho \) the density of grout, \( v_o \) the exit velocity of the grout at the nozzle, \( \nu_r \) the retraction rate of the monitor. Croce and Flora (2000), wrote the following relation between the specific energy at the nozzle and the specific energy at the pump, considering a 10% loss in the injection plant:

\[
E'_n \approx 0.9 \cdot E_p'
\]

(4)

\[
E_p = \frac{p \cdot Q}{\rho} \quad \text{where} \quad p' \quad \text{is the pressure at the pump and} \quad Q \quad \text{is the flow rate of the injected fluid.}
\]

The method was then extended to double and triple fluid systems by Flora et al. (2013) who proposed the following more complete equations:

\[
D = D_{ref} \cdot \left( \frac{d_E \cdot \Lambda^a \cdot E'_n}{7.5 \cdot 10} \right)^\beta \cdot \left( \frac{q_c}{1.5} \right)^\delta
\]

(for fine grained soils, \( E'_n \) in MJ/m and \( q_c \) in MPa) \hspace{1cm} (5.a)

\[
D = D_{ref} \cdot \left( \frac{d_E \cdot \Lambda^a \cdot E'_n}{7.5 \cdot 10} \right)^\beta \cdot \left( \frac{N_{SPT}}{10} \right)^\delta
\]

(for coarse grained soils, with \( E'_n \) in MJ/m) \hspace{1cm} (5.b)

where \( q_c \) represents the unit tip resistance measured with Cone Penetration Tests (expressed with MPa) and \( N_{SPT} \) the blow counts number measured with Standard Penetration Tests. \( \Lambda^a \) depends on the cement–water ratio \( (\Omega) \) by weight of the cutting fluid \( (\Lambda^a = 7.5 \quad \text{for} \quad \Omega = 1, \quad 16 \quad \text{for} \quad \text{water injected in the triple fluid system}) \). The parameter \( \alpha \) quantifies the effects of the shrouding air jet in double and triple fluid systems (it is equal to 1 for single fluid system, to 6 for double and triple fluid), the parameters \( \beta \) and \( \delta \) are found by calibration with the data obtained from literature and from the personal experience of the authors \( (\beta = 0.2 \quad \text{and} \quad \delta = -0.25) \). Finally, the parameter \( E_{ref} \) quantifies the role of grain size composition of the original soil, being equal to 0.5, 0.8 and 1.0 for respectively fine grained soil, coarse grained soil with and without a significant amount of fine material.

The influence of the finer fraction on the resistance of soil to erosion is also acknowledged by Shen et al., who quantify the resistance to erosion of fine and coarse grained soils with
different variables, and correct the resistance of coarse materials by a factor dependent of the fraction of particles smaller than 75 μm.

So far the methods proposed by Shen et al. (2013) and Flora et al. (2013) represent the most up to date and complete tools for the prediction of jet grouting columns diameters. However, in spite of a major conceptual robustness, the method proposed by Shen et al. is not very practical and immediate. Its application requires the complete list of injection parameters to be known, which is not so commonly found in literature reports, and thus it can be applied only in few limited cases.

On the other side, the relation proposed by Flora et al. has the advantage of a simpler mathematical formulation and of being validated with a wider set of data (the variables reported in Eq. (4) can be more easily known). However, the power functional law of Eq. (5.a,b) has been extended to double and triple fluid jet grouting without a physical analysis of the role of the air jet, and has been calibrated within a range of “normal” conditions (diameters, energies and soil resistances used by the authors are not particularly high). Therefore, extrapolation of Eq. (5.a,b) out of their range of calibration, i.e. to predict diameters larger than 2 m in difficult soils (large $N_{SPT}$ or $q_c$ values), which has recently become one of the most appealing frontiers of jet grouting, must be verified. At present, it can be said that unrealistic specific energies of thousand MJ/m come out from the application of these formulas to the above cases.

The present study explores the possibility of using a totally different approach, based on data mining strategy, to improve the confidence in prediction and limit the role of arbitrary assumptions. In their multiple expressions, data mining techniques are becoming more and more popular in many different fields thanks to their ability to solve high dimensional and complex problems where conventional approaches have failed (Hui, 2011). With reference to jet grouting, the support vector machine (SVM) technique has been successfully adopted (e.g. Tinoco et al., 2011, 2014) to predict the mechanical properties of the jet grouted material. In the present work, the artificial neural network (ANN) technique have been preferred considering its potential capabilities and its larger simplicity in comparison with SVM (Bengio and Le Cun, 2007). The basic principle of ANN’s is to build relations between causes and effects similarly to the human brain, i.e. learning from previous observation, and to adopt these relations for future predictions. The implicit correlation between input and output variables of a system are in fact found by training with a set of experimental observations a computer code reproducing in a simpler form the nervous system of living organisms. In spite of an initial suspet, basically deriving from inexactness in prediction and limit the role of arbitrary assumptions, the artiificial neural networks have been explored to define a maximum level of complexity compatible with the scope of the present work. Taking advantage of the above mentioned studies, a set of fundamental variables has been identified to quantify the effects of jet grouting in a simple and sufficiently complete form. Afterward, a large number of experimental observations, including single, double and triple fluid applications of jet grouting in different soil types, has been collected from the literature to design the network. Different possible structures of the network have been then attempted, testing each time the reliability of prediction, to find the most effective use and to compare the results with those obtained with alternative tools.

2. Artificial neural networks

2.1. Basic principles

The artificial neural networks are simplified mathematical descriptions of the extremely wide and complex nervous systems of living organisms (McCulloch and Pitts, 1943). Analogously to these systems, the basic purpose of ANN’s is to learn the behaviour of a system starting directly from observation, i.e. without arbitrary assumptions, and to use the connection between input and output variables to predict future occurrences. The goal is achieved by schematizing the problem to be studied with a number of input and output variables, by choosing the most appropriate computational algorithm and by training it with a number of cases where the response of the system is known.

In general, the nodes of a network are formed by logical operators, which are called neurons, set to perform simple computations. Each neuron processes the incoming information (a scalar quantity or a multidimensional vectors) and transfers it to the next neuron as described in Fig. 1: the generic input signal $x_i$ is firstly multiplied by a specific synaptic weight $w_{ki}$, then an external bias $w_{ki0}$ is added to amplify, when necessary, its importance; all weighted signals are summed up and processed with a predefined activation function, which specifies the node’s behaviour. The nodes of a network are arranged in a sequence of layers, a first one aimed at governing the input of the system, one or more intermediate hidden groups conceived to process data and a final layer for the managing of output. The hidden layers can be arranged with different alternative architectures depending on the problem to be studied, from the first and arguably the simplest.
one (feed-forward), where information moves in only one direction without cycles or loops, to a variety of more advanced algorithms (e.g. radial basis function, recurrent neural network etc.) conceived for more complex interactions among variables.

The number of hidden layers, of the neurons forming each layer and the activation functions of each neuron must be chosen by the operator at the beginning of the analysis. On the contrary, the synaptic weights and bias of each neuron, which form the so-called “memory” of the network, are automatically established during training of the network. This step is accomplished with a number of experimental observations where input and correspondent output variables are simultaneously known. Among the different possible ways to conduct a training process, the Levenberg–Marquardt learning technique (Marquardt, 1963) has proven to be one of the most effective. It involves an automatic sequence of calibration, validation and testing operations aimed at iteratively fitting the synaptic weights and bias in order to minimize the squared error between predicted and observed outputs. This technique consists of an initial calibration, where a first trial set of synaptic weights is computed from a smaller fraction of the available data, and a subsequent validation, where synaptic weights are progressively adjusted to best fit the remaining part of data. Therefore, the whole set of observations must be divided into two groups, one adopted for calibration, the other for validation. This choice must be done with particular care in order to avoid ‘data overfitting’, i.e. a very good fit of training data but with very poor interpolation ability.

Therefore, for a successful application of ANN it is of paramount importance that the complexity of the problem, dependent on the number of input and output variables and on the underlying functional relations, is adequately supported by a sufficiently rich and various sample of training observations (Haykin, 1999). If, on one side, the accuracy of prediction

<table>
<thead>
<tr>
<th>Soil type</th>
<th>ASTM D2487 classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse grained</td>
<td>Without fine: Gravels and sands with less than 5% fines</td>
</tr>
<tr>
<td></td>
<td>GW–GP–SW–SP</td>
</tr>
<tr>
<td></td>
<td>With fine: Gravels and sands with more than 5% fines</td>
</tr>
<tr>
<td></td>
<td>GM-GC-SM-SC</td>
</tr>
<tr>
<td>Fine grained</td>
<td>Silts, clay and organic soils</td>
</tr>
</tbody>
</table>

![Diagram](Fig. 2. Structure of the adopted artificial neural network.)
would be improved by creating more complex and refined networks based on a richer variety of descriptive variables, larger and more complete information would be on the other side needed to establish precise correlations. When the effort required to conduct experiments is particularly high, a compromise must be sought, sacrificing completeness and accuracy of the prediction to have a more stable solution. This goal can be achieved by simplifying as much as possible the structure of the network and by reducing the number of involved variables, neglecting those having minor importance, and assembling the others into functional groups able to take into account their respective role.

2.2. Structure of the adopted ANN

As previously discussed, the diameter of a jet grouting column depends on the set of injection parameters and on the properties of the original soil. From the complete list of these variables, reported in Fig. 2, it is argued that a large number of combinations is possible. The experiments necessary to establish correlations should thus reproduce numerous conditions to quantify the dependency of the diameter on the different variables and the mutual interaction among these latter. On the other side, the observation on jet grouting basically comes from a casuistry of field trials reported in the literature that, in spite of an increasing number of papers recently published by various authors, is not very big, assorted and enough documented.

Therefore, in order to simplify the logical structure of the network, the number of input variables has been reduced as shown in Fig. 2. Starting with the injection parameters, the rotational speed of the monitor has been neglected, as it is widely acknowledged to affect the homogeneity of jet grouted materials but to play a minor role on the dimension of columns. The other variables regarding the cutting fluid, (cement grout for single or double fluid system and water for triple fluid system), which are namely the injection velocity ($v_i$) and density ($\rho$), the number ($M$) and diameter ($d$) of nozzles and the monitor lifting speed ($v_m$), have been grouped into the previously defined specific energy at the nozzle ($E_s^i$ defined by Eqs. (3) and (4)) as suggested by Croce et al. (2011) and Flora et al. (2013).

Table 2a
List of observation adopted for training the neural network (single fluid system).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Case study</th>
<th>Soil properties</th>
<th>Soil type</th>
<th>USCS</th>
<th>$N_{spt}$</th>
<th>$E_s^i$ (MJ/m)</th>
<th>Diameter, $D_a$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Bianco and Santoro (1995)</td>
<td>Rio Matzeu, Italy</td>
<td>Coarse</td>
<td>Without fine</td>
<td>Sandy gravel</td>
<td>GW or GP</td>
<td>10</td>
<td>8.2</td>
</tr>
<tr>
<td>2–3 Croce et al. (1994)</td>
<td>Polcevera, Italy</td>
<td>Dense sandy gravel</td>
<td>SW or SP</td>
<td>20</td>
<td>13.2–14.6</td>
<td>1.10–1.20</td>
<td></td>
</tr>
<tr>
<td>4–8 Croce et al. (2011)</td>
<td>Barcelona, Spain</td>
<td>Gravelly sand</td>
<td>Pyroclastic silt and gravelly sand</td>
<td>23–34</td>
<td>15.4–29.4</td>
<td>0.76–1.08</td>
<td></td>
</tr>
<tr>
<td>9 Flora et al. (2013)</td>
<td>Caivano, Italy</td>
<td>Gravelly sand and gravelly silt</td>
<td>SM or SP or SW</td>
<td>14</td>
<td>16.9</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>10 Trento, Italy</td>
<td>Dense silty and gravelly sand</td>
<td>SW-SM or SP-SM</td>
<td>28</td>
<td>15.2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11–16 Croce and Flora (1998)</td>
<td>Vesuvio, Italy</td>
<td>Medium loose silty sand</td>
<td>SM or GC-GM</td>
<td>15</td>
<td>9–23.5</td>
<td>0.66–0.97</td>
<td></td>
</tr>
<tr>
<td>17 Tornaghi and Pettinaroli (2004)</td>
<td>S. Benedetto, Italy</td>
<td>Gravel in silty sand matrix</td>
<td>8</td>
<td>7.2</td>
<td>0.69</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18 Varallo P. (A), Italy</td>
<td>Silty sand</td>
<td>18</td>
<td>14.4</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19 Mazze, Italy</td>
<td>10</td>
<td>21.6</td>
<td>0.78</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 Casalmaio, Italy</td>
<td>Pyroclastic silty sand</td>
<td>13</td>
<td>11.5</td>
<td>0.83</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21 Flora et al. (2013)</td>
<td>Castellamare, Italy</td>
<td>Medium stiff clayey sand silt</td>
<td>CL-ML or ML</td>
<td>3</td>
<td>14.4</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>22 Tornaghi and Pettinaroli (2004)</td>
<td>Arezzo, Italy</td>
<td>Stiff sandy silt</td>
<td>7</td>
<td>5.9</td>
<td>0.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23 Varallo P. (B), Italy</td>
<td>Soft silty clay</td>
<td>3</td>
<td>9</td>
<td>0.63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24 Singapore</td>
<td>Very soft clayey silt</td>
<td>2</td>
<td>10.8</td>
<td>0.64</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 Qued Nil, Algeria</td>
<td>Clay</td>
<td>CL or CL-ML or CH</td>
<td>10–15</td>
<td>6.7–28.9</td>
<td>0.38–0.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26–48 Croce et al. (2011)</td>
<td>Barcelona, Spain</td>
<td>Clay</td>
<td>CL-ML or CH or MH</td>
<td>4</td>
<td>13.4</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>49 Davie et al. (2003)</td>
<td>Turkey</td>
<td>Stiff clay</td>
<td>10</td>
<td>9.2</td>
<td>0.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50 Bianco and Santoro (1995)</td>
<td>Rio Marzeu, Italy</td>
<td>Sandy silt</td>
<td>10</td>
<td>9.2</td>
<td>0.52</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The last aspect of concern for the injection is the role of the air jet shrouding the cutting fluid in double and triple fluid systems. In the present work, this role has been taken into account subdividing treatments into Single, Double and Triple fluid jet grouting, i.e. considering the injection system as an additional input for the network. In this way the presence of air

Table 2c
List of observation adopted for training the neural network (double fluid system).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Case study</th>
<th>Soil properties</th>
<th>USCS</th>
<th>Specific energy at nozzle, $E_n$ (MJ/m)</th>
<th>Diameter, $D_a$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>51–54</td>
<td>Flora et al. (2013)</td>
<td>Bojszowy Nowe, Poland</td>
<td>Coarse grained Without fine</td>
<td>Sand</td>
<td>SW or SP</td>
</tr>
<tr>
<td>57–60</td>
<td>Tornaghi and Pettinaroli (2004)</td>
<td>Kansas, USA</td>
<td>With fine</td>
<td>Silty sand</td>
<td>10</td>
</tr>
<tr>
<td>61–75</td>
<td>Tornaghi and Pettinaroli (2004)</td>
<td>Castellmare, Italy</td>
<td>Without fine</td>
<td>Silty sand</td>
<td>3</td>
</tr>
<tr>
<td>76–80</td>
<td>Tornaghi and Pettinaroli (2004)</td>
<td>Venezia, Italy</td>
<td>Fine grained</td>
<td>Soft clayey silt</td>
<td>CL or CL-ML or CH or MH</td>
</tr>
<tr>
<td>81–85</td>
<td>Flora et al. (2013)</td>
<td>Bologna, Italy</td>
<td>With fine</td>
<td>Soft silty clay</td>
<td>9</td>
</tr>
<tr>
<td>86–90</td>
<td>Sarno, Italy</td>
<td>Soft clay</td>
<td>7</td>
<td>31.6</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 2c
List of observation adopted for training the neural network (triple fluid system).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Case study</th>
<th>Soil properties</th>
<th>USCS</th>
<th>Specific energy at nozzle, $E_n$ (MJ/m)</th>
<th>Diameter, $D_a$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>91–95</td>
<td>Mauro and Santillan (2005)</td>
<td>Kansas, USA</td>
<td>Coarse grained Without fine</td>
<td>Medium to gravelly sand</td>
<td>GW or GP or SW or SP</td>
</tr>
<tr>
<td>96–99</td>
<td>Nikbakhtan and Osanloo (2009)</td>
<td>Shahriar Dam, Iran</td>
<td>With fine</td>
<td>Medium loose silty sand</td>
<td>SM or SW-SM or SP-SM</td>
</tr>
<tr>
<td>100–104</td>
<td>Tornaghi and Pettinaroli (2004)</td>
<td>S. Benedetto, Italy</td>
<td>With fine</td>
<td>From silty sand to medium sand</td>
<td>Silty sand</td>
</tr>
<tr>
<td>105–109</td>
<td>Stark et al. (2009)</td>
<td>Manhattan, Kansas, USA</td>
<td>With fine</td>
<td>Silty sand</td>
<td>10</td>
</tr>
<tr>
<td>110–114</td>
<td>Tornaghi and Pettinaroli (2004)</td>
<td>Casalmaicco, Italy</td>
<td>With fine</td>
<td>Silty sand</td>
<td>9</td>
</tr>
<tr>
<td>115–119</td>
<td>Shen et al. (2012)</td>
<td>Shanghai, China</td>
<td>Fine grained</td>
<td>Sandy silt and silty sand</td>
<td>CL or ML</td>
</tr>
<tr>
<td>120–124</td>
<td>Nikbakhtan and Osanloo (2009)</td>
<td>Shahriar Dam, Iran</td>
<td>Fine grained</td>
<td>Lean clay or plastic silt</td>
<td>CL or CL-ML or CH</td>
</tr>
<tr>
<td>125–129</td>
<td>Shen et al. (2012)</td>
<td>Shanghai, China</td>
<td>Stiff silty clay</td>
<td>Soft clay</td>
<td>15</td>
</tr>
</tbody>
</table>

The last aspect of concern for the injection is the role of the air jet shrouding the cutting fluid in double and triple fluid systems. In the present work, this role has been taken into account subdividing treatments into Single, Double and Triple fluid jet grouting, i.e. considering the injection system as an additional input for the network. In this way the presence of air
has been considered as an on–off variable, without introducing an explicit dependency on the flow rate (or pressure) of the air jet, which on the contrary is known to be important (Shen et al., 2013). While at present this choice has been necessary because of the lack of a sufficient number of data, it is hoped that new and more complete information will lead in the future to modify the structure of the neural network in order to more precisely quantify this issue.

The role of original soil has been taken into account making a distinction between fine and coarse grained materials, and separating the latter category into two subclasses (coarse with fine and coarse without fine) depending on the possible presence of a finer fraction in the coarse grained matrix (Fig. 2). The assimilation of the soils classified by the ASTM D2487 (2011) standard with the three above defined categories has been made consistently with Flora et al. (2013) (see Table 1).

This assumption has been made in accordance with the observations of different authors (Modoni et al., 2006; Flora et al., 2013; Shen et al., 2013), who observed different cutting or erosion mechanisms for respectively cohesive and cohesionless soils and noticed a reduction of the erosion capacity in coarse soils due to the presence of a finer matrix. In all cases, the resistance to the action of jet has been quantified with the number of blows in Standard Penetration Tests (N_{SPT}). Being aware that this quantity does not represent the best option for cohesive materials, the choice of adopting a unique resistance index for all materials has been made considering the noticeable formal simplification arising for the structure of the network. Correlations can be found in the literature to transform into \( N_{SPT} \) other resistance indexes like the \( q_{c} \) of Cone Penetration Tests, or the strength obtained with laboratory tests.

Therefore, considering the above simplifications, the input layer is then characterized by four variables, two (injection system and \( E_{n} \)) characterizing the treatment parameters, the other two (soil type and \( N_{SPT} \)) quantifying the role of soil (Fig. 2).

The architecture adopted for the neural network is a feed-forward type with a unique hidden layer. Such a choice, which allows a noticeable simplification of the computational algorithm (Masters, 1996), has been made considering that the governing factors (soil properties and treatment parameters) are independent on each other. Sigmoid activation functions defined in the library of the adopted computer code (Beale et al., 2013) have been used to characterize the neurons of the hidden layer because of their continuously positive derivative and of their capability in avoiding unstable convergence problems (which for instance occur with

![Table 3](image)

Table 3
MSE (m²) values computed for different number of neurons in the hidden layer and different percentages of calibrating (or validating) data.

<table>
<thead>
<tr>
<th>Number of neurons in the hidden layer</th>
<th>90–10</th>
<th>80–20</th>
<th>70–30</th>
<th>60–40</th>
<th>50–50</th>
<th>40–60</th>
<th>30–70</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.126</td>
<td>0.118</td>
<td>0.117</td>
<td>0.117</td>
<td>0.118</td>
<td>0.120</td>
<td>0.120</td>
</tr>
<tr>
<td>2</td>
<td>0.093</td>
<td>0.092</td>
<td>0.094</td>
<td>0.086</td>
<td>0.091</td>
<td>0.088</td>
<td>0.102</td>
</tr>
<tr>
<td>3</td>
<td>0.074</td>
<td>0.074</td>
<td>0.074</td>
<td>0.075</td>
<td>0.076</td>
<td>0.083</td>
<td>0.091</td>
</tr>
<tr>
<td>4</td>
<td>0.064</td>
<td>0.059</td>
<td>0.069</td>
<td>0.079</td>
<td>0.075</td>
<td>0.080</td>
<td>0.084</td>
</tr>
</tbody>
</table>
linear activation functions). On the contrary, a linear activation function have been used for the output layer because of its continuous variable. The numbers of neurons in the hidden layer and the subdivision of the training set of data for calibration and validation have been established with a parametric trial procedure, i.e. by performing the analysis with different combinations and checking the one giving the best prediction performance (see the next section).

3. Training

3.1. Experimental data

The above defined neural network has been trained on a total number of 131 observations taken from literature and summarily reported in Table 2. Each of the considered cases pertains to a field trial where the properties (composition and shear strength) of the original soil, the injection parameters (at the nozzle or at the pump) and the diameters of columns were simultaneously provided. According to previously described structure of the network, data have been subdivided into three main categories representing respectively Single (50 data in Table 2a), Double (43 data in Table 2b) and Triple (38 data in Table 2c) fluid systems. In each table, treatments performed in fine, coarse with fine and coarse without fine grained materials have been distinguished, giving for each case the specific energy at the nozzle, the measured diameter and the SPT blow count. In some cases this latter quantity has been directly provided by authors, while in some others has been obtained by converting other indexes quantifying soil resistance with the relations proposed in the literature. In particular, the correlation provided by Lunne et al. (1976) has been used to transform the undrained shear strength measured with laboratory tests into the unit CPT tip resistance, and the correlation given by Robertson et al. (1983) and Ismael and Jeragh (1989) has been used to convert CPT into SPT data.

3.2. Optimization of the neural network

As previously reported, some margin of freedom exists in the definition of the adopted neural network. In particular, the number of neurons forming the hidden layer and the subdivision of the training data into calibrating and validating portions have not been defined yet. To solve this indeterminacy without introducing subjective assumptions and to optimize the performance of the network, different possibilities have been

<table>
<thead>
<tr>
<th>Method</th>
<th>No. of data</th>
<th>MSE (m²)</th>
<th>MAPE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANN</td>
<td>Single fluid</td>
<td>50</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>Double fluid</td>
<td>43</td>
<td>0.050</td>
</tr>
<tr>
<td></td>
<td>Triple fluid</td>
<td>38</td>
<td>0.106</td>
</tr>
<tr>
<td></td>
<td>All</td>
<td>131</td>
<td>0.050</td>
</tr>
<tr>
<td>Flora et al. (2013)</td>
<td>Single fluid</td>
<td>50</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>Double fluid</td>
<td>43</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>Triple fluid</td>
<td>38</td>
<td>0.391</td>
</tr>
<tr>
<td></td>
<td>All</td>
<td>131</td>
<td>0.135</td>
</tr>
<tr>
<td>Shen et al. (2013)</td>
<td>Single fluid</td>
<td>7</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td>Double fluid</td>
<td>4</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>Triple fluid</td>
<td>6</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>All</td>
<td>17</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Fig. 4. Jet grouting column diameters predicted with ANN ((a) coarse grained soil without fine; (b) coarse grained soil with fine; (c) fine grained soil).
explored seeking the solution giving the lowest discrepancy between measurement and prediction.

The number of neurons in the hidden layer has been varied following the recommendation of Haykin (1999) who specifies that this number should be contained between number of input and output variables, while the percentage of validating data has been continuously varied between 10 and 70 (the fraction of calibrating data has been correspondingly varied between 90 and 30%). For each attempt, the level of correlation between measurement and prediction has been evaluated comparing each of the $n$ measured diameters of Table 2 with the one predicted for the corresponding input (treatment parameters and soil properties) by the network trained on the remaining $(n-1)$ data. Finally, the Mean Squared Error (MSE) has been computed as follows to provide an index describing the quality of prediction:

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^{n} \left( D_{i,\text{predicted}} - D_{i,\text{measured}} \right)^2 \quad (m^2)$$

where $D_{i,\text{measured}}$ is the $i$th measured diameter of the available sample of experimental observations, $D_{i,\text{predicted}}$ the value computed for the corresponding $i$th input by the network trained with the remaining $(n-1)$ observations. Since the data used for calibration and validation in the training process are randomly selected by the computer code, $D_{i,\text{predicted}}$ has been estimated as mean of twenty iteratively performed trials. It has to be noted that, the MSE has been adopted, instead of other indicators where errors are scaled by the current measured diameter, to emphasize the importance of the prediction made.

![Fig. 5. Safety factor accounting for the inaccuracy of prediction: (a) observed frequency distributions of the relative error; (b) definition of variables; (c) factors for single, double and triple fluid systems.](image-url)
for larger columns. As will be shown in next chapters, this choice does not affect the quality of prediction for smaller diameters.

From the list of MSE values, summarily reported in Table 3, it is seen that the lowest discrepancy between measurement and prediction (MSE = 0.059 m²) is obtained by a hidden layer with 4 neurons (i.e. the maximum possible number) and by subdividing the experimental dataset into 80% calibrating and 20% validating data.

4. Accuracy of prediction

The degree of correlation between measurement and prediction reached with the implemented neural network is here compared with that obtained by applying the two methods presented in Section 1, (Flora et al., 2013; Shen et al., 2013). Fig. 3 report the comparison between measured and predicted diameters (Fig. 3a for ANN, Fig. 3b for Flora et al. method, Fig. 3c for Shen et al. method). The comparison is also quantified in Table 4 where the values of MSE computed for the different prediction methods and for the different injection systems are summarily reported. This analysis has been performed separately for single, double and triple fluid systems and combining data all together. Additionally, in order to more completely depict the situation, the reliability of prediction has been quantified by the following quantity:

$$\text{MAPE} = \frac{100}{n} \sum_{i=1}^{n} \left[ \frac{D_{i,\text{predicted}} - D_{i,\text{measured}}}{D_{i,\text{measured}}} \right] \% \quad (7)$$

It is noted that, compared with MSE, this Mean Absolute Percentage Error gives an estimate of the average percentage error made with the prediction tools.

Although with some difference for the different injection systems, the agreement with measurement is generally good for all methods, with MAPE indexes ranging between 10 and 15%. However, while the prediction is very good for single fluid system, the deviation of dots from the 1:1 line obtained with ANN (Fig. 3a) and Flora et al. (Fig. 3b) methods tends to increase for the double and, moreover, for the triple fluid system. In particular, for this latter there is a tendency, particularly evident on the Flora et al. method (Fig. 3b), to underestimate the larger diameters (e.g. for triple fluid system MSE = 0.391 m² and MAPE = 22.8%). This result can be explained considering that the relations proposed by Flora et al. have a fixed (power function) structure, and that parameters are calibrated on a certain range of experimental conditions. The accuracy of prediction is good within this range (say less than 1.8 m, according to Fig. 3b), while extrapolation to higher energies leads to overly underestimate the jet grouting efficiency.

The prediction with the method proposed by Shen et al., plotted in Fig. 3c, seems to be better than for the previous two cases. However, it must be noted that the range of situations covered by Fig. 3c is much narrower than for Fig. 3a and b. In fact, while the ANN and Flora et al. methods have been tested on the 131 data (Table 2), the method of Shen et al. has been used only on 17 data, i.e. those for which the whole set of parameters at the nozzle is available.

5. Design charts

The neural networks created with the above described procedure and trained with the data reported in Table 2 are now used to predict the diameter of columns in a variety of conditions covering the most typical application of jet grouting. Following the previous subdivision, diameters have been predicted for single, double and triple fluid systems with variable specific energies $E_n^'$ and for different soil types (fine grained, coarse grained with and without fine) with increasing $N_{SPT}$ values.

Prediction has been performed by creating a unique network for the three injection system, i.e. by considering the injection system as an additional input parameter instead of creating separate networks for the three systems. Although the accuracy of prediction becomes slightly lower (MSE is 0.066 m², while the one obtained with separate networks is 0.050 m²), the use of a single network presents the advantage of learning the behaviour of the system simultaneously from all data, thus establishing a more robust correlation with the input variables common to all systems. This condition is particularly helpful to extend the range of validity of the prediction for each system.

In fact, with regard to the energy at nozzle $E_n^'$, the input conditions have been varied extending them slightly out of the ranges of experimental observations (100 MJ/m for single fluid, 200 MJ/m for double and triple fluid system). SPT blow counts equal to 5–10–15–20 have been considered for fine grained materials and for coarse grained materials with fine, equal to 10–20–30–40 for coarse grained materials without fine.

The diameters obtained with single, double and triple fluid systems are potted versus $E_n^'$ and $N_{SPT}$ in Fig. 4(a) for coarse grained soil without fine, (b) for coarse grained soil with fine, (c) for fine grained soil). The three figures show that the increase of the content of finer material has the effects of reducing the diameters and that the adoption of larger energies tends to mask the differences between double and triple fluid systems and to reduce the role of soil resistance.

The variability of the relative errors between the measured diameters of Table 2 and the values predicted with ANN has been finally studied to estimate the safety of prediction. Fig. 5a shows the statistical distribution of the error computed as follows:

$$\text{RE}_i = \frac{(D_{i,\text{predicted}} - D_{i,\text{measured}})}{D_{i,\text{predicted}}} \quad i = 1 \text{ to } n \quad (8)$$

where $n$ represents the number of measured data. This analysis, carried out separately for single double and triple fluid systems, reveals that Normal functions with zero mean and Standard Deviations respectively equal to 0.13, 0.17 and 0.20 can be assumed to model variation. Safety can be then introduced by correcting the estimate of Fig. 4 with a multiplying factor computed as function of the desire d level of conservativeness (CL, see Fig. 5b). In particular, CL (positioned on the vertical axis of Fig. 5c) expresses the probability that diameter of a column is larger than its predicted value, while the safety factor (reported on the horizontal axis) is equal to $\text{SF}=1+\text{RE}$, where RE is the relative error correspondent to CL via the cumulative probability Normal.
Function $F(=1−\text{CL})$. It is worth to observe that the relative position of the curves for single, double and triple fluid systems, reflects the different accuracy of the prediction for these three systems (see Fig. 3a).

6. Conclusions

The performed study has explored potentials and limits of the use of neural networks to improve the confidence in the prediction of the mean diameter of jet grouting columns. In order to balance the complexity of the problem with the amount of available information, the variables having minor importance have been neglected, while the others have been synthesized as much as possible. Finally, the prediction can be made for an injection system with a specific energy at the nozzle, knowing the soil type and N_SPT value. Mean absolute percentage errors of about $\pm 12\%$, $\pm 13\%$ and $\pm 15\%$ have been estimated for respectively single, double and triple fluid systems.

The comparison with other recently published methods has shown an improvement in the accuracy of prediction, particularly for double and triple fluid systems applied with high energies. This result has a particular relevance for applications where larger columns are adopted to speed up execution of treatment, which is one of the most recent trend of the jet grouting technology.

Considering this issue, the artificial neural networks have proven to be more flexible in comparison with formulas having predetermined structures. While the latter must be somehow arbitrarily changed to match new data, ANN’s possess the capability of adapting their architecture to fit new information. Thanks to this virtue, prediction is susceptible of significant improvement as far as new observation will be available. More comprehensive and detailed reports on field trials or site measurements will reduce the need of preliminary assumptions, allowing to separately identify the role of each variable distinguishing it in the predictive correlations. For instance, a more explicit quantification of the role of air will improve the performance of prediction for double and triple fluid systems.

References


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