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ESSAY REVIEW

Edited by KAREN HUNGER PARSHALL

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Histoire des mathématiques chinoises. By Jean-Claude Martzloff. Paris (Masson). 1988. xx + 375 pp.

Reviewed by Joseph W. Dauben

Department of History, Herbert H. Lehman College, City University of New York, Bedford Park Blvd. West, Bronx, New York 10468, and Ph.D. Program in History, The Graduate Center, City University of New York, 33 West 42nd Street, New York, New York 10036

On the cover of Jean-Claude Martzloff's *History of Chinese Mathematics*, a simple woodcut gravure from the late Qing dynasty depicts a master demonstrating the ease with which the abacus could be used. This one picture captures nicely the dual role of traditional mathematics in ancient China, for the abacus was an instrument not only of practical significance but also of theoretical importance. Martzloff's *History* demonstrates clearly that while the Chinese were adept in applying their mathematics to a host of practical problems, including astronomy and engineering as well as commercial transactions, they also paid attention to algorithmic techniques, methods of calculation, geometric constructions, and even certain purely logical problems. But above all, what sets this book apart from the usual histories of mathematics (in any language, Chinese or Western, of any period or country) is its emphasis first on context, then on content, in describing the long history of Chinese mathematics.

Often the approach taken to writing the history of science—including the history of mathematics—is a straightforward linear chronology of discoverers and discov-

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eries as they happened. For mathematics this is no less true of Dirk Struik's *Concise History of Mathematics* than it is of Morris Kline's comprehensive study of *Mathematics from Antiquity to the Present* [Struik 1948 and Kline 1972]. The most recent attempt to provide such a history for Chinese mathematics follows this familiar pattern as well, namely the (at times infelicitous) translation of Li Yan and Du Shiran's *History of Chinese Mathematics* [1987]. Fortunately, Martzloff understands that to capture the essence of Chinese mathematics requires much more than a chronological retelling, and thus his *History* is not a conventional chronology which simply recounts the step-by-step development of mathematics in China. Instead, Martzloff seeks to get to the heart of the matter by approaching the history of Chinese mathematics as a body of knowledge, setting forth its aims, and analyzing how Chinese mathematicians contributed to the advance of their discipline.

As Jacques Gernet, Professor of Chinese Social and Intellectual History at the Collège de France, notes in his preface to the book, traditionally the Chinese have understood their mathematics in terms of "things," not of "essence." As commonly understood, this meant that Chinese mathematics was practical and concrete in its approach to applications; it did not expect mathematics to reflect eternal truths in the same way that the Greeks came to appreciate mathematics as a reflection of the primordial "essence" of all things. Instead, the Chinese used mathematics as a means of "classifying and rendering manifest the organization of the universe" [Marcel Granet, quoted by Gernet, in Martzloff 1988, p. viii]. Gernet in turn relates the Chinese interest in diagrams and position, the former in geometry, the latter in algebra, to the development of the abacus. This device was remarkably flexible in the hands of Chinese mathematicians who used it both to expedite their arithmetic calculations (which could sometimes be of great complexity), and as a means of solving simultaneous equations for which the abacus became a tangible matrix of coefficients which could easily be manipulated to eliminate terms and solve for unknowns. Practice and theory became inextricably linked in a single instrument, the counting board or its more elaborate successor, the abacus.

In his capacity as President of the French History of Science Society (and Professor of Mathematics at the University of Nantes), Jean Dhombres also provides a brief preface to Martzloff's book. Dhombres emphasizes that while one may venerate the axiomatic method of Greek mathematics on abstract, conceptual grounds, Western mathematics also knew the value of developing practical methods like the calculus, even when its foundations were not as sound or its methods as rigorously justified as classical Euclidean geometry. Similarly, Dhombres finds it refreshing (and helpful) to examine a culture in which mathematics developed to a high degree but without the constraints or artificiality of axiomatic Euclidean-type mathematics. As Dhombres puts it, no Chinese or French gourmet would deny that *doufu* (tofu) and *haishen* (sea slugs) taste better after one has experienced foie gras and oysters! Perhaps the real question is whether eating with chop sticks or cutlery makes a difference—depending upon whether one is eating filet mignon or shredded pork and rice!

The first part of Martzloff's book is devoted to the context of Chinese mathematics, beginning with an historiographic overview of works on the subject in Western languages, in Japanese, and in Chinese. Then follow chapters on the historic context, the notion of Chinese mathematics, the applications of mathematics, and a lengthy discussion of the structure of Chinese works on mathematics.

Martzloff pays particular attention to the structure of Chinese texts because it is in their structure, as he shows, that they differ most remarkably from standard Western works. After discussing titles and prefaces, Martzloff distinguishes four kinds of problems considered in Chinese mathematical texts—real problems, pseudo-real problems, recreational problems, and speculative problems. He also analyzes in detail the rules prescribed for solving problems.

In their turn, Martzloff also considers terminology, modes of reasoning, Chinese mathematicians, and the transmission of mathematics, including the question of reciprocal influences and dispersion of knowledge between cultures. He not only considers contacts with Babylonia, India, and Islam, but notes the spread of Chinese mathematics to Korea and Japan, and last, but not of least interest, takes up the question of contacts with Europe.

Martzloff also discusses the major works of Chinese mathematics (up to 1600). These include the so-called "ten classics" of Chinese mathematics, although as he notes, this is something of a misnomer since there were actually more than 10 (some have been lost), and not all of them are "classics." In any case, the oldest of these is the *Zhoubi suanjing,* followed by the most famous, the *Jiuzhang suanshu.* In discussing the early works of Chinese mathematics, Martzloff also introduces some of the best-known commentators and analyzes their contributions, including Liu Hui, Jia Xian, Liu Yi, Li Zhi, Qin Jiushao, Ju Shijie, Yang Hui, and Cheng Dawei.

The second half of Martzloff's book is devoted to the content of Chinese mathematics, which the reader is all the more prepared to appreciate given the contextual emphasis of the first part of the book. Beginning with the basic problem of numbers and numeration (including the early use of knotted ropes for numerical record-keeping, similar to the quipus used by the Incas), the earliest examples of actual numbers, number symbols, and the evolution of different characters representing numbers are all discussed. These range from marks incised on bones and turtle shells, to traditional numbers and their intimate relation to number rods, which proved so flexible in Chinese hands when combined with the counting board and the abacus. Martzloff covers the history of units and measure, fractions, decimal measures, and above all the remarkable appearance at a comparatively early date of negative numbers (which arise naturally in the counting-rod system of arithmetic), as well as the role of zero in Chinese mathematics.

Calculating instruments—which again bear an obvious relation to the counting board, counting rods, and the abacus—receive special attention. Martzloff explains the techniques of numerical calculation in detail, beginning with elementary operations, the extraction of roots, solution of systems of equations of the first degree with many unknowns (where the Chinese *fangcheng* method was used, roughly equivalent to Western matrix methods for solving simultaneous equations—for details, see [Martzloff 1988, 232–241]). Chinese algebra of the 13th century, especially the *Tianyuan shu* (*Tianyuan* method), is also explained at length, along with examples of algebraic problems.

Geometry is treated next, beginning with planimetry, stereometry, and the right triangle. Indeterminate problems include the famous problem of 100 fowls. The problem of remainders accompanies approximation formulas including geometric formulas and interpolation formulas. Martzloff brings his account of the technical content of Chinese mathematics to an end with a discussion of the remarkable summation formulas of Li Shanlan, infinite series, arithmetic squares, and other "magic figures."

One difficulty that any history of Chinese mathematics must face is the fact that until the late Qing, at the end of the nineteenth century, there was no professional status or particular recognition accorded mathematics in China, at least not in the sense that professional mathematicians existed in the West. Indeed, the first truly "professional" mathematician in China was Li Shanlan, who is appropriately pictured at the beginning of Martzloff's book, in a frontispiece to the preface. But Li Shanlan did not come onto the scene until the 1850's, and his attempts to promote an indigenous, fully modern Chinese mathematics were not completely realized until the twentieth century.

In fact, if there is any disappointment in reading Martzloff's book, it is the scant attention paid to the modern period. Li Shanlan receives a few paragraphs at the very end of the book, hardly enough to explain the extraordinary transformation, largely social, that he witnessed in the fortunes of Chinese mathematics. It became a discipline, finally rooted in universities and employing teachers across the country, many of whom were trained in the United States or Europe before bringing home the new mathematics which soon took root and grew remarkably well in China. (Martzloff does consider some of these factors in an article that appeared recently in the *Mathematical Intelligencer;* see Martzloff [1992].) Without going into detail on the subject of Western influences on Chinese mathematics, Martzloff closes with an appendix listing Chinese "adaptations" of European mathematical works from the 17th through the 19th century.

As Martzloff says in his own introduction to this book, since the 19th century a variety of journals has increasingly often published articles on Chinese mathematics, and many books have devoted at least a chapter to the subject as well. Thanks to such unsystematic efforts, it slowly became clear that there was a convergence of certain results between East and West, and that such "Western" discoveries as the "Pythagorean" theorem, the formulas of Heron, and the rule of false position, for example, all have their Chinese counterparts. For the sake of argument, this has led some, like Frank Swetz and T. I. Kao, to pose such provocative questions as "Was Pythagoras Chinese?" [Swetz and Kao 1977].

Unfortunately, when it comes to considering the steps used by the Chinese to produce these results, the original texts (and this is true for Eastern as well as Western texts) are largely silent. In particular, the lack of "rational justifications" (in the Western sense of proofs) offered by Chinese mathematicians in their texts and commentaries has prompted various reconstructions of probable "demonstrations." In hands familiar with the convenient tools of elementary algebra, this has led to the dubious conclusion that Chinese mathematics is essentially "algebraic" in character.

The last few decades, fortunately, have added considerably to the stockpile of information at our disposal concerning Chinese mathematics—not only new archaeological evidence, but especially increasingly critical, scholarly literature produced by Japanese, Chinese, and a few Korean scholars who recently have begun to study, systematically, the history of mathematics in the Far East. (For some idea of the growing bibliography, see [Dauben 1985, 423–428; Lam and Ang 1992, 187–192; Li and Du 1987, 274–277; and of course Martzloff 1988, 340–366].) Of special interest are those who have examined the rare texts where justifications were given—from which it is possible to analyze not just the results but the mode—the methods—of Chinese reasoning. By studying the assumptions and arguments explicitly given in these texts, Martzloff considers the extent to which it is possible to recapture the intentions of ancient authors without necessarily involving what Martzloff terms the "convenient but deforming socioconceptual framework" [Martzloff 1988, 23] of the mathematics of our own time.

This requires, of course, some evaluation of the relative interest of different Chinese sources. Some texts previously considered as only of limited, secondary value, turn out to be fundamental due to the richness of their "demonstrations." But above all, this approach to reevaluating the nature of Chinese mathematics highlights the key role of certain *operational* procedures presumed to constitute the basic *modus operandi,* literally, of Chinese mathematics. Here, Martzloff points to graphic heuristic manipulations used for calculation, the extensive use of geometric dissections and rearrangements in dealing with areas and volumes, and Chinese reliance on nonwritten tabulation techniques in which the position of material objects (like counting rods) representing numbers was essential.

As a result of these carefully explored examples, it becomes clear that within Chinese mathematics the opposition between algebra and geometry, or between arithmetic and algebra, does not operate in the same way as it does in Western mathematics influenced by the Greek tradition. Instead, Martzloff finds a more close-knit "solidarity" unifying techniques of calculation without apparent connections (the structural analogy, for example, between the *practice* of arithmetic division and the determination of roots of polynomial equations, or between the evaluation of certain volumes and the summation of related series, etc.).

In addition to purely technical aspects of the history of mathematics, Martzloff's concern for context prompts him to consider various other questions as well. One question rarely asked concerns the basic definition of "mathematics" from the Chinese point of view. Is it the art of logical reasoning, or the art of calculation? Should it be viewed as arithmetic/logistic or a theory of numbers? What about surveying versus geometry? And what is the place of mathematics *vis-à-vis* history of mathematics?

Martzloff also deals with the important question of the ultimate destination of texts—some as he points out may be considered as *comptes rendus* of research. Others were clearly intended as teaching manuals, whereas some were designed as aids to memory, especially for mathematical formulas. If one fails to draw such distinctions, Martzloff warns that what is really "Chinese didactic or mnemonic thought" may be mistaken for actual "Chinese mathematical thought." Similarly, the fact that texts do not contain demonstrations does not mean that Chinese scholars could not reason carefully. Rather than taking the summary proofs one finds in Chinese texts to mean that the idea of a well-constructed proof was absent in China, Martzloff argues instead for the comparative importance of oral and written traditions over the desire for "proof" in the Western sense.

Martzloff confronts the history of Chinese reactions to Euclid's *Elements* (introduced at the beginning of the 17th century) as an opportunity to question the way Chinese mathematicians succeeded (or failed) to integrate cultural elements exterior to it, which in turn throws light on the differences between (sometimes) widely different systems of thought. Rather than present an encyclopedic history, however, Martzloff is more interested in considering various hypotheses meant to challenge the original texts, which also reveal unavoidable lacunae due to the lack of original sources and limited understanding of both the ancient and medieval world in China.

As Martzloff notes, prior to the second half of the 19th century what little was known of Chinese mathematics in the West was severely judged as "superficial" by Jesuit missionaries who were more interested in impressing the Chinese with the power of Western learning from which they hoped the superiority of Christian doctrines would also follow. As European historians of science like Moritz Cantor, Hankel, Vacca, and Zeuthen became increasingly interested in Chinese mathematics, their well-intentioned attempts to include it in their works led to inaccuracies and distortions. Not having access to original documents, they unfortunately attributed errors and inconsistencies to individual authors and not to the translators and interpreters of Chinese mathematics in the West.

An important change in the historiographic fortunes of Chinese mathematics came in 1913 with the publication of Yoshio Mikami's *Mathematics in China and Japan*. Despite Mikami's ability to read both Japanese and Chinese, this was not an entirely successful work, due in part to the difficulties of the time, but as well to the deficiencies of Japanese libraries and the inaccessability of European materials (except for Moritz Cantor's history of mathematics, it seems, upon which Mikami relied almost entirely). Equally limiting, as Martzloff explains, is the fact that Mikami's work was based for the most part on the *Chouren zhuan* (Notices on the astronomer–calenderists) written by Ruan Yuan in 1799, although Mikami did rely to a limited extent on various Chinese dynastic annals.

European historians of the early 20th century, like van Hee and Sédillot (due, it must be said, to both prejudice and a lack of familiarity with Chinese sources), argued that the Chinese borrowed most of their mathematics from the West. Here the one important exception was D. E. Smith, who corresponded with Mikami and worked with him in producing a number of articles on the history of Chinese mathematics. The two even co-authored *A History of Japanese Mathematics* [Mikami and Smith 1914].

In 1959, however, knowledge of the history of Chinese mathematics made a quantum leap forward when Joseph Needham and Wang Ling (who had completed his dissertation at Cambridge in 1955 on the history of Chinese mathematics in the Han dynasty) published Vol. 3 (on mathematics and astronomy) of Needham's monumental *History of Science and Civilization in China* [Needham 1959]. In terms of length, the section on mathematics is roughly comparable to Mikami's history, but there, as Martzloff notes, the similarity ends. Not only did Wang and Needham have access to many more resources than did Mikami, but their conception of the history of mathematics was also quite different.

Mikami proceeded unimaginatively, organizing his materials around works and authors. Needham, however, was inspired by a specific philosophy of history that influenced his entire conception of the history of science, including mathematics. From a staunchly Marxist perspective, Needham believes that there has been only one "universal science," and that all cultures have made their contributions to modern science thanks to natural categories of thought which are comparable, he maintains, to those of "science," whatever the culture. One of his favorite, recurring metaphors is the river of science, enriched by streams flowing from all cultures into the "sea of modern science" [Needham 1970, 113]:

> By a thousand capillary channels, like venules joining together to form a *vena cava magna*, influences came from all parts of the world [Needham 1973, 3].

Needham believes the originality of Chinese mathematics lies in its development of algebra, as opposed to what he takes to have been a Western preoccupation with geometry, or as he puts it, the "geometric spirit." Among mathematical discoveries the West owes to the East, he includes decimal notation, algebra, Horner's method, indeterminant analysis, etc.

Nevertheless, Needham asks a difficult question that goes well beyond the history of mathematics: why did modern science not develop in China, when paradoxically, at the end of the medieval period, it was well in advance of other civilizations? This is comparable to a question posed not long ago by Nathan Sivin, "Why the scientific revolution did not take place in China—or didn't it?" [Sivin 1982]. Although not included in Martzloff's bibliography (it is mentioned, but miscited, once in a footnote on p. 6), Sivin's article is well worth reading for historians of mathematics interested in such questions of broad cultural contrast and significance.

A decade later, A. P. Yushkevich [1961] devoted 100 or so pages of his history of medieval mathematics to Chinese mathematics, drawing to a large extent upon Wang and Needham's work as well as research of the Chinese historian of mathematics, Li Yan. But all of these works (even the most detailed, like that of Li Yan and Du Shiran, recently translated into English) focus for the most part on particulars [Li and Du 1987]. Basically they proceed chronologically to describe who did what, when, with emphasis and priority going to whoever made a particular discovery first. But as Nathan Sivin has cautioned, one should be wary of the value of examining isolated discoveries rather than the *context* within which Chinese science developed.

It is primarily this question of *context* that Martzloff approaches directly. Perhaps the greatest contribution his book makes is the chance it offers to consider issues of cultural context as significant, determining factors in the history of mathematics. Thus, Martzloff tries to get inside the Chinese mind, to explain how and why mathematics developed as it did in China, and often in ways strikingly different from its Western counterparts. Although he does not always account for these differences, he succeeds admirably in describing them, which results in a refreshingly rich sense of its evolution as well. Without the sort of contextual background Martzloff provides, it is impossible to understand why Chinese practitioners at first succeeded so well in fashioning their own peculiar style and methods in mathematics, but then, when presented with Western models, proved so adept at assimilating them. What is more, they went on immediately to make extraordinary contributions of their own when Western science came to China in a professional and systematic way in the 20th century.

As a major virtue of Martzloff's book, Jacques Gernet stresses his care to avoid (in a paraphrase of Martzloff) "robing Chinese mathematics in clothes it never wore" [Martzloff 1988, viii]. This book describes the Emperor's "new" clothes, that is, clothes the emperor actually wore—as opposed to the invisible trappings of a Chinese mathematics that never existed. Here, thanks to Martzloff's carefully crafted contextual setting, readers may actually begin to appreciate the true value of what Chinese mathematics accomplished—and indeed now continues to accomplish today—thanks to a long and venerable tradition that this book charts in all its diversity and richness.

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