Stability of Runge–Kutta methods in the numerical solution of equation $u'(t) = au(t) + a_0 u([t])$

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Abstract

This paper is concerned with the stability analysis of the Runge–Kutta methods for the equation $u'(t) = au(t) + a_0 u([t])$. The stability regions for the Runge–Kutta methods are determined. The conditions that the analytic stability region is contained in the numerical stability region are obtained and some numerical experiments are given.

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Keywords: Delay differential equation; Piecewise continuous arguments; Asymptotic stability

1. Introduction

This paper deals with the numerical solution of the delay differential equations with piecewise continuous arguments (EPCA)

$$u'(t) = f(t, u(t), u(\alpha(t))),$$  \hspace{1cm} (1.1)

where the argument $\alpha(t)$ has intervals of constancy. This kind of equations has been initiated in [3,9,11,12]. The general theory and basic results for EPCA have by now been thoroughly investigated in the book of Wiener [13].

EPCA describe hybrid dynamical systems, combine properties of both differential and difference equations and have applications in certain biomedical models in the work of Busenberg and Cooke.

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[1]. Furthermore, the equation
\[
\frac{dx(t)}{dt} = x(t) \left\{ r - \sum_{j=0}^{m} d_j x([t - j]) \right\}, \quad t \geq 0
\] (1.2)
is considered in [8], which may be viewed as a semi-discretization [7] of the delay logistic equation with several delays
\[
\frac{dx(t)}{dt} = x(t) \left\{ r - \sum_{j=0}^{m} d_j x(t - \tau_j) \right\}, \quad t \geq 0.
\] (1.3)

We consider the following equation:
\[
u'(t) = au(t) + a_0 u([t]), \quad t \geq 0,
\]
\[u(0) = u_0,\] (1.4)
where \(a, a_0, u_0\) are real constants and \([\cdot]\) denotes the greatest integer function. In the book [13], some properties of the solution of (1.4) are presented.

**Definition 1.1** (Wiener [13]). A solution of (1.4) on \([0, \infty)\) is a function \(u(t)\) that satisfies the conditions:

(i) \(u(t)\) is continuous on \([0, \infty)\).

(ii) The derivative \(u'(t)\) exists at each point \(t \in [0, \infty)\), with the possible exception of the points \([\cdot]\) \(\in [0, \infty)\) where one-sided derivatives exist.

(iii) (1.4) is satisfied on each interval \([n, n+1) \subset [0, \infty)\) with integral end-points.

**Theorem 1.2** (Wiener [13]). (1.4) has on \([0, \infty)\) a unique solution \(u(t) = m_0(\{t\})b_0^{[n]}u_0\), where \(\{t\}\) is the fractional part of \(t\) and
\[m_0(t) = e^{at} + (e^{at} - 1)a^{-1}a_0, \quad b_0 = m_0(1)\].

And (1.4) is asymptotically stable (the solution of (1.4) tends to zero as \(t \to \infty\)), for all \(u_0\), if and only if the inequalities
\[-a \frac{e^{at} + 1}{e^{at} - 1} < a_0 < -a\] (1.5)
hold.

2. Runge–Kutta methods

In this section we consider the adaptation of the Runge–Kutta methods \((A, b, c)\). Let \(h = 1/m\) be a given stepsize with integer \(m \geq 1\) and the gridpoints \(t_n\) be defined by \(t_n = nh\) \((n = 0, 1, 2, \ldots)\). For the Runge–Kutta methods we always assume that \(b_1 + b_2 + \cdots + b_v = 1\) and \(0 \leq c_1 \leq c_2 \leq \cdots \leq c_v \leq 1\).
The adaptation of the Runge–Kutta methods to (1.1) leads to a numerical process of the following type, generating approximations \( u_1, u_2, u_3, \ldots \) to the exact solution \( u(t) \) of (1.1) at the gridpoints \( t_n \) \( (n = 1, 2, 3, \ldots) \)

\[
\begin{align*}
u_{n+1} &= u_n + h \sum_{i=1}^{v} b_i f(t_n + c_i h, y_i^{(n)}, z_i^{(n)}), \\
y_i^{(n)} &= u_n + h \sum_{j=1}^{v} a_{ij} f(t_n + c_j h, y_j^{(n)}, z_j^{(n)})
\end{align*}
\]

(2.1)

where \( y_1^{(n)}, y_2^{(n)}, \ldots, y_v^{(n)} \) satisfy

\[
y_i^{(n)} = u_n + h \sum_{j=1}^{v} a_{ij} f(t_n + c_j h, y_j^{(n)}, z_j^{(n)})
\]

(2.2)

and the argument \( z_i^{(n)} \) denotes a given approximation to \( u(z(t_n + c_j h)) \) \( (i = 1, 2, \ldots, v, n = 0, 1, 2, \ldots) \).

We are interested in the application of (2.1) and (2.2) to (1.4). The application of the process (2.1) and (2.2), in the case of (1.4), yields

\[
\begin{align*}
u_{n+1} &= u_n + h \sum_{i=1}^{v} b_i (a y_i^{(n)} + a_0 z_i^{(n)}), \\
y_i^{(n)} &= u_n + h \sum_{j=1}^{v} a_{ij} (a y_j^{(n)} + a_0 z_j^{(n)})
\end{align*}
\]

(2.3)

where \( z_i^{(n)} \) is the approximation to \( u([t_n + c_j h]) \). If we denote \( n = km + l \) \( (l = 0, 1, \ldots, m - 1) \), then \( z_i^{(km+1)} \) can be defined as \( u_{km} \) according to Definition 1.1 \( (i = 1, 2, \ldots, v, l = 0, 1, \ldots, m - 1) \). Let 

\[
Y^{(km+l)} = (y_1^{(n)}, y_2^{(n)}, \ldots, y_v^{(n)})^T, \text{ then (2.3) reduces to}
\]

\[
\begin{align*}
u_{km+l+1} &= u_{km+l} + hab Y^{(km+l)} + ha_0 u_{km} \\
y^{(km+l)} &= u_{km+l} e + ha Y^{(km+l)} + ha_0 A e u_{km}
\end{align*}
\]

(2.4)

where \( e = (1, 1, \ldots, 1)^T \). Therefore, we have

\[
u_{km+l+1} = R(x) u_{km+l} + \alpha(x, y) u_{km}, \quad l = 0, 1, \ldots, m - 1,
\]

(2.5)

where \( x = ha, \ y = ha_0, \ R(x) = 1 + xb^T(I - xA)^{-1} e \) is the stability function of the method and \( \alpha(x, y) = y(1 + xb^T(I - xA)^{-1} A e) = yb^T(I - xA)^{-1} e \).

Let the Runge–Kutta method be of order \( p \). Then there is a constant \( C \) such that for sufficiently small \( h \) \( [2,4,5] \)

\[
|e^x - R(x)| \leq C h^{p+1}.
\]

(2.6)

For any given integer \( k \) and \( l \) with \( 0 \leq l \leq m - 1 \), from (1.4) and Definition 1.1 we have

\[
u(t_{km+l+1}) = e^{ah} u(t_{km+l}) + (e^{ah} - 1) a_0 u(t_{km}).
\]

(2.7)

It is easy to see from (2.5) that if \( u(t_{km}) = u_{km} \) and \( u(t_{km+l}) = u_{km+l} \), then

\[
|u(t_{km+l+1}) - u_{km+l+1}| = \left| (e^x - R(x)) \left( u(t_{km+l}) + \frac{a_0}{a} u(t_{km}) \right) \right|
\]

\[
\leq C h^{p+1} \left( 1 + \left| \frac{a_0}{a} \right| \right) \max_{k \leq i \leq k+1} |u(t)|,
\]

(2.8)

which implies that for (1.4) the Runge–Kutta method is also convergent of order \( p \).
In fact, in each interval \([k, k + 1)\), (1.4) can be seen as an ordinary differential equation. Hence the Runge–Kutta methods for (1.4) conserve their order of convergence.

3. Numerical stability

In this section we will discuss the stability of the Runge–Kutta methods. We introduce the set \(H\) consisting of all pairs \((a, a_0) \in \mathbb{R}^2\) which satisfy the condition (1.5), i.e.,

\[
H = \left\{ (a, a_0) : -\frac{e^a + 1}{e^a - 1} < a_0 < -a \right\}
\]

and divide the region \(H\) into three parts:

\[
H_0 = \{(a, a_0) : (a, a_0) \in H \text{ and } a = 0\},
\]

\[
H_1 = \{(a, a_0) : (a, a_0) \in H \text{ and } a < 0\},
\]

\[
H_2 = \{(a, a_0) : (a, a_0) \in H \text{ and } a > 0\}.
\]

We have from (2.5)

\[
\begin{pmatrix}
u_{km} \\ u_{km+1} \\ \vdots \\ u_{km+m-1} \\ u_{(k+1)m}
\end{pmatrix}
= 
\begin{pmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & \alpha(x, y) & 0 & \cdots & R(x)
\end{pmatrix}
\begin{pmatrix}
u_{km-1} \\ u_{km} \\ \vdots \\ u_{km+m-1} \\ u_{(k+1)m}
\end{pmatrix}
= B_1
\begin{pmatrix}
u_{km-1} \\ u_{km} \\ \vdots \\ u_{km+m-1} \\ u_{(k+1)m}
\end{pmatrix}
\]

\[
\begin{pmatrix}
u_{km-1} \\ u_{km} \\ \vdots \\ u_{km+m-2} \\ u_{km+m-1}
\end{pmatrix}
= 
\begin{pmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & \alpha(x, y) & 0 & \cdots & R(x)
\end{pmatrix}
\begin{pmatrix}
u_{km-2} \\ u_{km-1} \\ \vdots \\ u_{km+m-3} \\ u_{km+m-2}
\end{pmatrix}
= B_2
\begin{pmatrix}
u_{km-2} \\ u_{km-1} \\ \vdots \\ u_{km+m-3} \\ u_{km+m-2}
\end{pmatrix}
\]

\[
\begin{pmatrix}
u_{(k-1)m+1} \\ u_{(k-1)m+2} \\ \vdots \\ u_{km} \\ u_{km+1}
\end{pmatrix}
= 
\begin{pmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & \alpha(x, y) & 0 & \cdots & R(x)
\end{pmatrix}
\begin{pmatrix}
u_{(k-1)m} \\ u_{(k-1)m+1} \\ \vdots \\ u_{(k-1)m+3} \\ u_{km}
\end{pmatrix}
= B_m
\begin{pmatrix}
u_{(k-1)m} \\ u_{(k-1)m+1} \\ \vdots \\ u_{(k-1)m+3} \\ u_{km}
\end{pmatrix}
\]
Let $U_k = (u_{km}, u_{km+1}, \ldots, u_{km+m})^T$ and $B = \prod_{i=1}^{m} B_i$. It is easy to see

$$U_k = BU_{k-1}, \quad k = 1, 2, \ldots, \quad (3.1)$$

where

$$B = \begin{pmatrix}
0 & \cdots & 0 & b_{1,m+1} \\
0 & \cdots & 0 & b_{2,m+1} \\
& & \ddots & \vdots \\
0 & \cdots & 0 & b_{m+1,m+1}
\end{pmatrix}$$

and

$$b_{i,m+1} = \begin{cases}
1 + \left(1 + \frac{a_0}{a}\right) [R(x)]^{i-1} - 1, & a \neq 0, \\
1 + (i - 1)ha_0, & a = 0,
\end{cases} \quad i = 1, 2, \ldots, m + 1.$$

Let $\varphi(x) = b^T(I - xA)^{-1}e$. Then there exists $\delta > 0$ such that

$$\varphi(x) > 0 \quad \text{for all } x \text{ with } |x| \leq \delta,$$  

(3.2) since $\varphi(0) = 1$ and $\varphi(x)$ is continuous in a neighborhood of zero. In the rest of the paper we define

$$M = \begin{cases}
1, & a \leq 0, \\
a, & a > 0, \\
\frac{1}{\delta}, & a > 0.
\end{cases}$$

**Definition 3.1.** Process (2.1) for Eq. (1.4) is called asymptotically stable at $(a, a_0)$ if and only if for all $m \geq M$ and $h = 1/m$

(i) $(I - xA)$ is invertible,
(ii) for any given $u_i$ $(1 \leq i \leq m)$ relation (3.1) defines $U_k$ $(k = 1, 2, \ldots)$ that satisfy $U_k \to 0$ for $k \to \infty$.

**Definition 3.2.** The set of all pairs $(a, a_0)$ at which the process (2.1) for Eq. (1.4) is asymptotically stable is called asymptotical stability region denoted by $S$.

In the following we will investigate which conditions lead to

$$H \subseteq S.$$ 

It is well known that the process (2.1) for Eq. (1.4) is asymptotically stable if and only if all eigenvalues of $B$ have a modulus less than one. Clearly all eigenvalues of $B$ are zero other than $\lambda = b_{m+1,m+1}$. We denote

$$f(a, a_0) = \begin{cases}
\frac{a + a_0}{a} [R(x)]^m - 1, & a \neq 0, \\
a_0, & a = 0,
\end{cases}$$
\begin{align*}
g(a, a_0) &= \begin{cases} 
-2 - \frac{a + a_0}{a} [R(x)^m - 1], & a \neq 0, \\
-2 - a_0, & a = 0,
\end{cases} \\
J &= \left( -a \frac{e^a + 1}{e^a - 1}, -a \right). \tag{3.3}
\end{align*}

It is easy to see that \(|b_{m+1, m+1}| < 1\) is equivalent to
\[ f(a, a_0) < 0 \quad \text{and} \quad g(a, a_0) < 0. \tag{3.4} \]

Therefore, \((a, a_0) \in S\) if and only if
\[ f(a, a_0) < 0 \quad \text{and} \quad g(a, a_0) < 0 \quad \text{for all} \quad h = \frac{1}{m} \quad \text{with} \quad m \geq M. \]

We know that for the implicit Runge–Kutta method \(R(x)\) is a rational function with numerator and denominator of degree \(\leq v\). The following lemmas will be useful to prove our theorems in the paper.

**Lemma 3.3** (Butcher [2], Dekker and Verwer [4], Hairer et al. [5], Wanner et al. [10]). The \((r, s)\)-Padé approximation to \(e^z\) is given by
\[ R(z) = \frac{P_r(z)}{Q_s(z)}, \tag{3.5} \]

where
\[ P_r(z) = 1 + \frac{r}{r + s} z + \frac{r(r - 1)}{(r + s)(r + s - 1)} \frac{z^2}{2!} + \cdots + \frac{r! s!}{(r + s)! r!} \frac{z^r}{r!}, \]
\[ Q_s(z) = 1 - \frac{s}{r + s} z + \frac{s(s - 1)}{(r + s)(r + s - 1)} \frac{z^2}{2!} + \cdots + (-1)^s \frac{s! r!}{(r + s)! s!} \frac{z^s}{s!}, \]

with error
\[ e^z - R(z) = (-1)^s \frac{r! s!}{(r + s)!(r + s + 1)!} z^{r+s+1} + O(z^{r+s+2}). \tag{3.6} \]

It is the unique rational approximation to \(e^z\) of order \(r + s\), such that the degrees of numerator and denominator are \(r\) and \(s\), respectively. \(\square\)

Following [2,5,6,10], we define the order star
\[ D = \{ z \in \mathbb{C} : |R(z)| > |e^z| \}. \]

**Lemma 3.4** (Butcher [2], Hairer et al. [5], Iserles and Nørsett [6], Warner et al. [10]). If the Runge–Kutta method is of order \(p\), then for \(z \to 0\), \(D\) behaves like a star with \(p + 1\) sectors of equal width \(\pi/(p + 1)\), separated by \(p + 1\) similar white sectors of the complement of \(D\), each of the same width.
Lemma 3.5 (Butcher [2], Hairer et al. [5], Iserles and Nørsett [6], Warner et al. [10]). If $R(z)$ is the $(r,s)$-Padé approximation to $e^z$, then

(i) there are $s$ star sectors in the right-half plane, each containing a pole of $R(z)$,
(ii) there are $r$ white sectors in the left-half plane, each containing a zero of $R(z)$,
(iii) all sectors are symmetric with respect to the real axis.

According to Lemma 3.5 we have the following corollary:

Corollary 3.6. Suppose $R(z)$ is the $(r,s)$-Padé approximation to $e^z$. If $r$ is even, then the negative real axis is contained in a star sector in the left-half plane, which implies $R(x) > e^x$ for all $x < 0$. If $s$ is even, then the positive real axis is contained in a white sector in the right-half plane, which implies $R(x) < e^x$ for all $x > 0$.

Theorem 3.7. Suppose that the Runge–Kutta method is $A_0$-stable and the stability function is given by the $(r,s)$-Padé approximation to the exponential $e^x$. Then $H_1 \subseteq S$ if and only if $r$ is even.

Proof. Let $(a,a_0) \in H_1$, then $a + a_0 < 0$ and $a < 0$. Since the method is $A_0$-stable, $(I - xA)$ is invertible and $R(x) < 1$ which is equivalent to $f(a,a_0) < 0$ from (3.3).

Since $g(a,a_0)$ is a linear function of $a_0$ and $g(a,-a) = -2 < 0$, we only need to prove

$$g \left( a, -a \frac{e^a + 1}{e^a - 1} \right) = \frac{2}{e^a - 1} (R(x)^m - e^a) \leq 0,$$

which is equivalent to $R(x) \geq e^x$. According to Corollary 3.6 the proof is completed. \qed

Theorem 3.8. Suppose that the stability function of the Runge–Kutta method is given by the $(r,s)$-Padé approximation to the exponential $e^x$. Then $H_2 \subseteq S$ if and only if $s$ is even.

Proof. Let $(a,a_0) \in H_2$, then $a + a_0 < 0$ and $a > 0$. Since $s$ is even, we can obtain that $(I - xA)$ is invertible and $R(x) \leq e^x$ according to Lemma 3.5 and Corollary 3.6, which is equivalent to (3.7). In a way analogous to the discussion in Theorem 3.7, it follows that $g(a,a_0) < 0$. We have from (3.2) $R(x) > 1$ for $h = 1/m$ and $m \geq M$, in view of (3.3), which is equivalent to $f(a,a_0) < 0$. \qed

Theorem 3.9. For all Runge–Kutta methods, we have $H_0 \subseteq S$.

Proof. In fact, if $a = 0$, then $J = (-2,0)$. Therefore from (3.3) it is easily seen that $f(0,a_0) < 0$ and $g(0,a_0) < 0$ for all $a_0 \in J$. \qed

Using the above theorems we can formulate the following result.

Theorem 3.10. Suppose that the Runge–Kutta method is $A_0$-stable and the stability function is given by the $(r,s)$-Padé approximation to the exponential $e^x$. Then $H_0 \subseteq S$ and $H \subseteq S$ if and only if both $r$ and $s$ are even.
Table 1
The higher order Runge–Kutta methods

<table>
<thead>
<tr>
<th>(r,s)</th>
<th>Gauss–Legendre</th>
<th>Radau IA, IIA</th>
<th>Lobatto IIIA, IIIB</th>
<th>Lobatto IIIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₁ ⊆ S</td>
<td>r is even</td>
<td>v is odd</td>
<td>v is odd</td>
<td>v is even</td>
</tr>
<tr>
<td>H₂ ⊆ S</td>
<td>r is even</td>
<td>v is odd</td>
<td>v is odd</td>
<td>v is even</td>
</tr>
</tbody>
</table>

H₁ ⊆ S if and only if r is even,
H₂ ⊆ S if and only if s is even.

Remark 3.11. For the A-stable higher order Runge–Kutta methods, it is easy to see from Theorem 3.10 (see Table 1).

(i) For the v-stage Gauss–Legendre and Lobatto IIIC methods, H ⊆ S if and only if v is even.
(ii) For the v-stage Lobatto IIIA and IIIB methods, H ⊆ S if and only if v is odd.
(iii) For the v-stage Radau IA and IIA methods, H₁ ⊆ S if and only if v is odd and H₂ ⊆ S if and only if v is even.

4. Numerical experiments

In order to give a numerical illustration to the conclusions in the paper, we consider the following two problems:

\begin{align}
\quad u₁′(t) &= -20u₁(t) - 10.3u₁([t]), \quad u₁(0) = 1, \quad (4.1) \\
\quad u₂′(t) &= 10u₂(t) - 10.0001u₂([t]), \quad u₂(0) = 1. \quad (4.2)
\end{align}

It can be seen that J ≈ (−20.00000008245, 20), J ≈ (−10.00090839820, −10) for the problem (4.1) and (4.2), respectively. Hence (−20, −10.3) ∈ H₁ and (10, −10.0001) ∈ H₂.

We shall use four methods with the stepsize h = 1/m, 2-stage Gauss–Legendre, 2-stage, 3-stage Radau IA and 2-stage Lobatto IIIC, to get the numerical solution at t = 10, where the true solutions are u₁(10) ≈ 1.312418259626160E − 3 and u₂(10) ≈ 8.308600759955528E − 2 from Theorem 1.2.

In Tables 2 and 3, we have listed the absolute errors (AE) and relative errors (RE) at t = 10 of the four methods. We can see from these tables that the methods conserve their order of convergence.

For these methods we may expect a stable behavior, i.e.,

( a, a₀ ) ∈ S. \quad (4.3)

It follows from Theorem 3.10 that (4.3) is fulfilled, in the situation of (4.1), for all M = 1, whereas (4.3) is fulfilled, in the situation of (4.2), when M > 2, M > 5, M > 5, for 2-stage Radau IA, 2-stage Lobatto IIIC and 2-stage Gauss–Legendre methods, respectively.

All above numerical experiments are in agreement with the conclusions in the paper.
Table 2
Problem (4.1)

<table>
<thead>
<tr>
<th>m</th>
<th>3-Radau IA</th>
<th>2-Lobatto IIIC</th>
<th>2-Gauss–Legendre</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AE</td>
<td>RE</td>
<td>AE</td>
</tr>
<tr>
<td>3</td>
<td>1.4451E-3</td>
<td>1.1011E-3</td>
<td>1.0390E-7</td>
</tr>
<tr>
<td>5</td>
<td>7.2970E-10</td>
<td>5.5600E-7</td>
<td>3.8739E-9</td>
</tr>
<tr>
<td>10</td>
<td>6.2577E-12</td>
<td>4.7681E-9</td>
<td>3.4492E-10</td>
</tr>
<tr>
<td>50</td>
<td>6.5088E-17</td>
<td>5.1880E-14</td>
<td>6.9070E-12</td>
</tr>
</tbody>
</table>

Table 3
Problem (4.2)

<table>
<thead>
<tr>
<th>m</th>
<th>2-Radau IA</th>
<th>2-Lobatto IIIC</th>
<th>2-Gauss–Legendre</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AE</td>
<td>RE</td>
<td>AE</td>
</tr>
<tr>
<td>2</td>
<td>9.1680E-1</td>
<td>1.1034E+1</td>
<td>9.1701E-1</td>
</tr>
<tr>
<td>3</td>
<td>9.1325E-1</td>
<td>1.0992E+1</td>
<td>9.1700E-1</td>
</tr>
<tr>
<td>5</td>
<td>6.4497E-1</td>
<td>7.7626E+0</td>
<td>9.1691E-1</td>
</tr>
<tr>
<td>20</td>
<td>4.8089E-3</td>
<td>5.7879E-2</td>
<td>1.9263E-1</td>
</tr>
<tr>
<td>50</td>
<td>2.7612E-4</td>
<td>3.3233E-3</td>
<td>1.9246E-2</td>
</tr>
<tr>
<td>100</td>
<td>3.3507E-5</td>
<td>4.0328E-4</td>
<td>4.2750E-3</td>
</tr>
</tbody>
</table>

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References