Available online at www.sciencedirect.com

ScienceDirect

Physics Letters B 660 (2008) 72–79

PHYSICS LETTERS B

www.elsevier.com/locate/physletb

# $SU(3)$ dibaryons in the Einstein–Skyrme model

Hisayuki Sato *, Nobuyuki Sawado

*Department of Physics, Faculty of Science and Technology, Tokyo University of Science, Noda, Chiba 278-8510, Japan*

Received 1 October 2007; received in revised form 12 October 2007; accepted 21 October 2007

Available online 3 December 2007

Editor: L. Alvarez-Gaumé

**Abstract**

$SU(3)$ collective coordinate quantization to the regular solution of the $B=2$ axially symmetric Einstein–Skyrme system is performed. For the symmetry breaking term, exact diagonalization method called Yabu–Ando approach are used. The effect of the gravity on the mass spectra of the $SU(3)$ dibaryons and the symmetry breaking term is studied in detail. In the strong gravity limit, the symmetry breaking term significantly reduces and exact $SU(3)$ flavor symmetry is recovered.


*PACS:* 12.39.Dc; 21.10.-k; 04.20.-q

## 1. Introduction

The Skyrme model [1] is considered as an unified theory of hadrons by incorporating baryons as topological solitons of pion fields, called skyrmions. The topological charge is identified as the baryon number $B$. Performing collective quantization for a $B=1$ skyrmion, one can obtain proton and neutron states within 30% error [2]. Correspondingly multi-skyrmion ($B>1$) solutions are expected to represent nuclei [3,4].

The Einstein–Skyrme (ES) model can be thought as a model of hadrons in which baryons interact with black holes. The early studies [5–7] have shown that the Schwarzschild black hole can support spherically symmetric (hedgehog) Skyrme hair which is a first counter example to the no-hair conjecture. Axially symmetric regular and black hole skyrmion solutions with $B=2$ were found in Ref. [8]. Also some multi-skyrmion solutions $B>2$ have been obtained [9].

If a skyrmion is regarded as nucleon or nuclei, it must be quantized to assign quantum numbers like spin, isospin, etc. Study of gravitational effects on the quantum spectra of skyrmions was initiated in Ref. [10] by performing collective quantization of a $B=1$ gravitating skyrmion. The study about a $B=2$ axially symmetric skyrmion immediately followed to it [11]. In Ref. [11], we investigated the gravity effects to the $SU(2)$ solutions of the ES model and found out that the effects are seen especially in the heavier dibaryon spectra. In this Letter we shall study the gravity effects to the $SU(3)$ dibaryons because they have much larger mass compared with that of $SU(2)$ dibaryons in terms of the large flavor symmetry breaking. The effect will be more apparent on such dihyperons.

The extensions of the Skyrme model into $SU(3)$ have extensively been studied. The solutions are classified into two categories: embedded and nonembedded ones. For the nonembedded solutions, in Ref. [12] the classical $SO(3)$ chiral field is simply extended into three flavor space. The authors constructed the solutions in the ES system and found particle-like and black hole solutions. More systematic analyses in this direction have been done in Refs. [9,13]. By using the harmonic map ansatz, the authors have investigated various large flavor and multi winding number solitons.

* Corresponding author.
*E-mail addresses:* sugar@dec.rikadai.jp (H. Sato), sawado@ph.noda.tus.ac.jp (N. Sawado).

0370-2693 
doi:10.1016/j.physletb.2007.10.084

For the embedded solutions, collective coordinate method, which is essentially a natural extension of the $SU(2)$ collective quantization scheme, has often been used. Yabu–Ando (YA) treatment [14] is a sort of the collective quantization but the collective Hamiltonian is exactly diagonalized by using Euler-angle parameterization of the $SU(3)$ rotations. As a result, baryon appears to be its lowest irreducible representation (irrep) but contain significant admixture of higher representations. The application to the $B = 2$ axially symmetric skyrmions [15] and to the multi-skyrmions ($3 \leqslant B \leqslant 8$) [16] in flat space have been studied. They observed large mixing of higher irreps especially in excited or higher winding number states.

In this Letter, we investigate $SU(3)$ axially ES system in terms of the collective quantization and YA approach. We estimate mass spectra of the dibaryons down to strangeness $S = -6$. Since our concern is about contributions of higher irreps to the lowest states and we shall show change of the mixing probabilities about varying coupling strength of the gravity.

## 2. Classical gravitating $B = 2$ skyrmions

The $SU(3)$ extended Skyrme Lagrangian coupled with gravity can be written as

$$L = L_G + L_S + L_{\mathrm{SB}} + L_{\mathrm{WZ}} \tag{1}$$

where $L_G$ is the standard Einstein–Hilbert Lagrangian

$$L_G = \int d^3r\,\sqrt{-g}\,\frac{1}{16\pi G}R \tag{2}$$

and the remaining parts are about the $SU(3)$ extension of the Skyrme Lagrangian, which are defined in terms of the chiral field $U(x) \in SU(3)$ and using the notation $l_\mu = U^\dagger\partial_\mu U$ as follows:

$$L_S = \int d^3r\,\sqrt{-g}\left[\frac{1}{16}F_\pi^2 g^{\mu\nu}\,\mathrm{Tr}(l_\mu l_\nu) + \frac{1}{32e^2}g^{\mu\rho}g^{\nu\sigma}\,\mathrm{Tr}\big([l_\mu, l_\nu][l_\rho, l_\sigma]\big)\right], \tag{3}$$

$$L_{\mathrm{SB}} = \frac{1}{16}F_\pi^2 m_\pi^2\int d^3r\,\sqrt{-g}\,\mathrm{Tr}\big(U + U^\dagger - 2\big) + \frac{1}{24}\big(F_\kappa^2 m_\kappa^2 - F_\pi^2 m_\pi^2\big)\int d^3r\,\sqrt{-g}\,\mathrm{Tr}(1 - \sqrt{3}\lambda_8)\big(U + U^\dagger - 2\big), \tag{4}$$

$$L_{\mathrm{WZ}} = -\frac{iN_c}{240\pi^2}\int_Q d\Sigma^{\mu\nu\lambda\rho\sigma}\,\mathrm{Tr}[l_\mu l_\nu l_\lambda l_\rho l_\sigma], \tag{5}$$

where $F_\pi$ and $e$ are basic model parameters which indicate the pion decay constant, a dimensionless parameter, respectively. The $L_{\mathrm{SB}}$ is comprised of all chiral and flavor symmetry breaking terms. The $L_{\mathrm{WZ}}$ is usual Wess–Zumino term which concerns with the topological charge of the soliton. $F_\kappa, m_\kappa$ are the kaon decay constant and the mass. $N_c$ means a number of color.

In this Letter, we employ the solution of the chiral field with axially symmetric, winding number two which is considered as a possible candidate for the $B = 2$ minimal energy configuration [17]

$$U_0(\boldsymbol{r}) = \exp\big[iF(r,\theta)\boldsymbol{\tau}\cdot\boldsymbol{n}_R\big], \tag{6}$$

$$\boldsymbol{n}_R = \big(\sin\Theta(r,\theta)\cos n\varphi, \sin\Theta(r,\theta)\sin n\varphi, \cos\Theta(r,\theta)\big), \tag{7}$$

and $n \in \mathbb{Z}$ is the winding number. We explore the solution with $n = 2$. $SU(3)$ chiral field is constructed by trivial embedding:

$$U(\boldsymbol{r}) = \begin{pmatrix} U_0(\boldsymbol{r}) & 0 \\ 0 & 1 \end{pmatrix}. \tag{8}$$

Correspondingly, the following axially symmetric ansatz is imposed on the metric [18]

$$ds^2 = -f\,dt^2 + \frac{m}{f}\big(dr^2 + r^2\,d\theta^2\big) + \frac{l}{f}r^2\sin^2\theta\,d\varphi^2 \tag{9}$$

where the metric functions $f$, $m$ and $l$ are the function of coordinates $r$ and $\theta$. This metric is symmetric with respect to the $z$-axis ($\theta = 0$). Substituting these ansatz to the Lagrangian (1), one obtains the following static (classical) energy for the chiral fields

$$\begin{aligned} M_{\mathrm{class}} = \frac{2\pi F_\pi}{e}\int dx\,d\theta\Bigg[&\frac{\sqrt{l}\sin\theta}{8}\left\{x^2\big((\partial_x F)^2 + (\partial_x\Theta\sin F)^2\big) + (\partial_\theta F)^2 + (\partial_\theta\Theta\sin F)^2 + \frac{n^2 m}{l\sin\theta}\sin^2 F\sin^2\Theta\right\} \\ &+ \frac{\sqrt{l}\sin\theta}{2}\Bigg[\frac{f}{m}(\partial_x F\partial_\theta\Theta - \partial_\theta F\partial_x\Theta)^2\sin^2 F + \frac{n^2 f}{l\sin^2\theta}\sin^2 F\sin^2\Theta\Bigg\{\left((\partial_x F)^2 + \frac{1}{x^2}(\partial_\theta F)^2\right) \\ &+ \left((\partial_x\Theta)^2 + \frac{1}{x^2}(\partial_\theta\Theta)^2\right)\sin^2 F\Bigg\}\Bigg] + \frac{1}{4}\frac{m\sqrt{l}}{f}x^2\sin\theta\beta_\pi^2(1 - \cos F)\Bigg], \end{aligned} \tag{10}$$

where dimensionless variable $x = eF_\pi r$ and $\beta_\pi = \frac{m_\pi}{eF_\pi}$ are introduced.

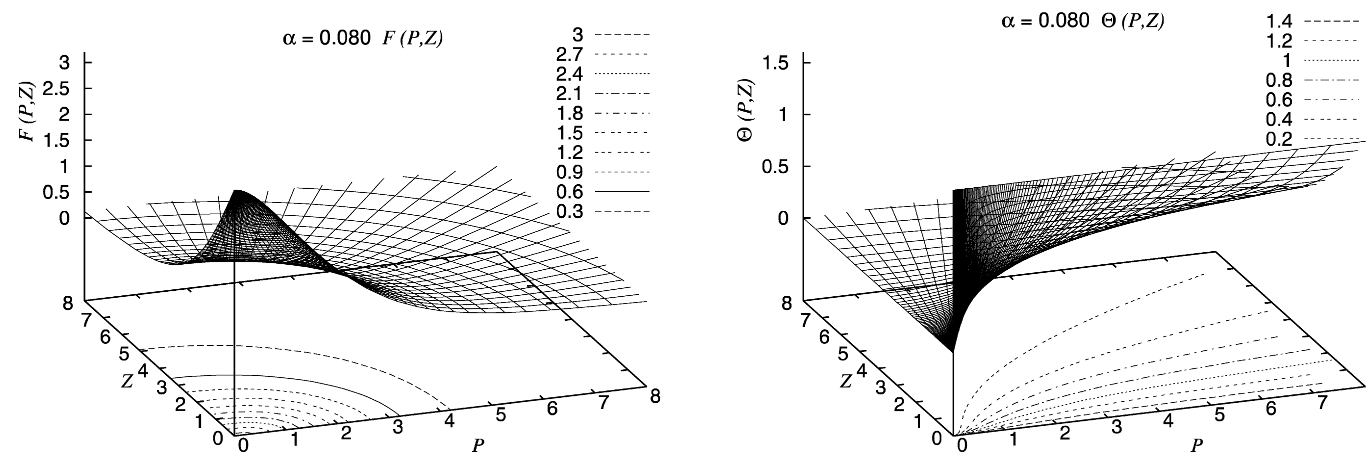


Fig. 1. The profile functions $F$, $\Theta$ for $\alpha = 0.080$ in the cylindrical coordinate with dimensionless variables $P := ef_\pi \rho$, $Z = ef_\pi z$.

For the profile functions, the boundary conditions at the $x = 0, \infty$ are imposed

$$F(0,\theta) = \pi, \qquad F(\infty,\theta) = 0, \tag{11}$$

$$\partial_x \Theta(0,\theta) = \partial_x \Theta(\infty,\theta) = 0. \tag{12}$$

At $\theta = 0$ and $\pi/2$,

$$\partial_\theta F(x,0) = \partial_\theta F\left(x, \frac{\pi}{2}\right) = 0, \tag{13}$$

$$\Theta(x,0) = 0, \qquad \Theta\left(x, \frac{\pi}{2}\right) = \frac{\pi}{2}. \tag{14}$$

For the solutions to be regular at the origin $x = 0$ and to be asymptotically flat at infinity, the following boundary conditions must be imposed

$$\partial_x f(0,\theta) = \partial_x m(0,\theta) = \partial_x l(0,\theta) = 0, \tag{15}$$

$$f(\infty,\theta) = m(\infty,\theta) = l(\infty,\theta) = 1. \tag{16}$$

For the configuration to be axially symmetric, the following boundary conditions are imposed at $\theta = 0, \pi/2$

$$\partial_\theta f(x,0) = \partial_\theta m(x,0) = \partial_\theta l(x,0) = 0, \tag{17}$$

$$\partial_\theta f\left(x, \frac{\pi}{2}\right) = \partial_\theta m\left(x, \frac{\pi}{2}\right) = \partial_\theta l\left(x, \frac{\pi}{2}\right) = 0. \tag{18}$$

By taking a variation of the static energy (10) with respect to $F$ and $\Theta$, one obtains the equations of motion for the profile functions. The field equations for the metric functions $f$, $m$ and $l$ are derived from the Einstein equations. The explicit form of the equations is essentially same (except for contribution of the mass term) as reported in Ref. [11].

The effective coupling constant of the ES system, the only free parameter of the model, is given by

$$\alpha = 4\pi G F_\pi^2. \tag{19}$$

To solve the equations of motion numerically, the relaxation method with the typical grid size $100 \times 30$ are performed. We observe that the solution survives at $0 \leqslant \alpha \leqslant 0.120$. Including the mass term, the range becomes a little narrow. We show examples of our numerical results for the profile functions $F$, $\Theta$ in Fig. 1 and also for the metric functions $f$, $l$, $m$ in Fig. 2.

## 3. The $SU(3)$ collective quantization

We study the $SU(3)$ extension of solutions in ES system by Yabu–Ando approach. We start by embedded chiral field into $SU(3)$ which is of the form

$$\tilde{U}(\boldsymbol{r},t) = A(t) \begin{pmatrix} U_0(R(t)\boldsymbol{r}) & 0 \\ 0 & 1 \end{pmatrix} A^\dagger(t) \tag{20}$$

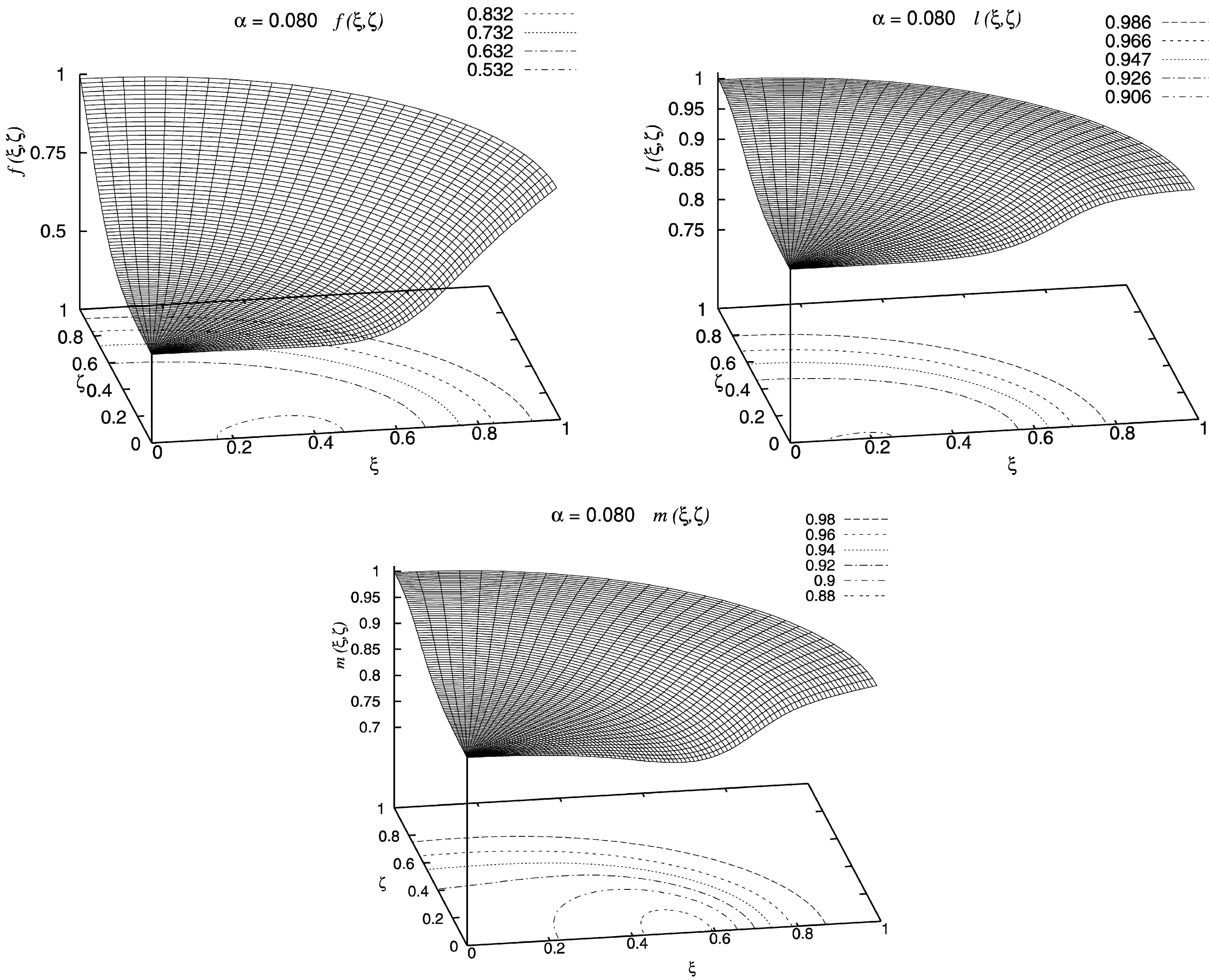


Fig. 2. The metric functions $f, l, m$ in the cylindrical coordinate with dimensionless, rescaled variables $\xi = P/(1+P)$, $\zeta = Z/(1+Z)$, $P = ef_\pi \rho$, $Z = ef_\pi z$.

where $U_0$ is introduced in Eq. (6). $A(t)$ is time dependent $SU(3)$ rotational matrix and $R(t)$ describes a spatial rotation of the soliton. We introduce the angular velocities $\Omega_a$, $\omega_l$ which are defined by

$$A^\dagger \dot{A} = \frac{i}{2}\sum_{a=1}^{8} \lambda_a \Omega_a, \qquad \big(\dot{R}R^\dagger\big)_{ik} = \sum_{l=1}^{3} \varepsilon_{ikl}\omega_l. \tag{21}$$

Substituting the chiral field (20) into the Lagrangians (3)–(5) and after a lengthy calculation, one finally obtain the effective Lagrangian of the form:

$$L = -M_{\text{class}} + \frac{1}{2} I_N \sum_{p=1}^{2} \Omega_p^2 + \frac{1}{2} I_J \sum_{p=1}^{2} \omega_p^2 + \frac{1}{2} I_3\big(\Omega_3^2 + n\omega_3\big)^2 + \frac{1}{2} I_S \sum_{k=4}^{7} \Omega_k^2 - \frac{N_c}{2\sqrt{3}}\Omega_8 + \frac{1}{2}\gamma\big(1 - D_{88}(A)\big), \tag{22}$$

where $I_N$, $I_J$, $I_3$, $I_S$ are called the moments of inertia and their explicit forms are

$$\begin{aligned} I_N = \frac{\pi}{F_\pi e^3} \int dx\, d\theta \bigg[ & \frac{m\sqrt{l}}{4f^2} x^2 \sin\theta \sin^2 F\big(1+\cos^2\Theta\big) + \frac{\sqrt{l}}{f} \sin\theta \sin^2 F \bigg\{ \big(1+\cos^2\Theta\big)\big(x^2(\partial_x F)^2 + (\partial_\theta F)^2\big) \\ & + \sin^2 F \cos^2\Theta\big(x^2(\partial_x \Theta)^2 + (\partial_\theta \Theta)^2\big) + \frac{n^2 m}{l \sin^2\theta} \sin^2 F \sin^2\Theta \bigg\} \bigg], \end{aligned} \tag{23}$$

$$I_3 = \frac{\pi}{F_\pi e^3}\int dx\, d\theta\bigg[\frac{m\sqrt{l}}{2f^2}x^2\sin\theta\sin^2 F\sin^2\Theta + \frac{2\sqrt{l}}{f}\sin\theta\sin^2 F\sin^2\Theta\big\{x^2(\partial_x F)^2 + (\partial_\theta F)^2 + \sin^2 F\big(x^2(\partial_x\Theta)^2 + (\partial_\theta\Theta)^2\big)\big\}\bigg], \tag{24}$$

$$I_J = \frac{\pi}{F_\pi e^3}\int dx\, d\theta\bigg[\frac{m\sqrt{l}}{4f^2}x^2\sin\theta\big((\partial_x F)^2 + (\partial_\theta\Theta\sin F)^2 + (n\cot\theta\sin F\sin\Theta)^2\big) + \frac{\sqrt{l}}{f}x^2\sin\theta\sin^2 F\big\{(\partial_x F\partial_\theta\Theta - \partial_\theta F\partial_x\Theta)^2 + n^2\big((\partial_x F)^2 + (\partial_x\Theta\sin F)^2\big)\cot^2\theta\sin^2\Theta\big\} + \frac{n^2\sqrt{l}}{f\sin\theta}\left(\cos^2\theta + \frac{m}{l}\right)\big((\partial_\theta F)^2 + (\partial_\theta\Theta\sin F)^2\big)\sin^2 F\sin^2\Theta\bigg], \tag{25}$$

$$I_S = \frac{\pi}{F_\pi e^3}\int dx\, d\theta\,(1-\cos F)\bigg[\frac{m\sqrt{l}}{4f^2}x^2\sin\theta + \frac{\sqrt{l}}{4f}x^2\sin\theta\big\{(\partial_x F)^2 + (\partial_x\Theta\sin F)^2\big\} + \frac{\sqrt{l}}{4f}\sin\theta\bigg\{(\partial_\theta F)^2 + (\partial_\theta\Theta\sin F)^2 + \frac{mn^2}{l\sin^2\theta}\sin^2 F\sin^2\Theta\bigg\}\bigg]. \tag{26}$$

$\frac{1}{2}\gamma(1-D_{88})$ exhibits strength of the symmetry breaking and the explicit form of $\gamma$ is

$$\gamma = \frac{2\pi F_\pi}{3e}\big(\beta_\kappa^2 - \beta_\pi^2\big)\int dx\, d\theta\, x^2\sin\theta(\cos F - 1), \tag{27}$$

where $\beta_\kappa = \frac{m_\kappa F_\kappa}{eF_\pi^2}$. $D_{88}$ is a component of Wigner function which is defined as

$$D_{ab}(A) = \frac{1}{2}\operatorname{Tr}\big(\lambda_a A^\dagger\lambda_b A\big). \tag{28}$$

From (22) the Hamiltonian reads

$$H = M_{\text{class}} + \frac{J(J+1)}{2I_J} + \frac{1}{2}\left(\frac{1}{I_N} - \frac{1}{I_S}\right)N(N+1) + \frac{1}{2}\left(\frac{1}{I_3} - \frac{1}{I_N} - \frac{n^2}{I_J}\right)L^2 - \frac{3}{8I_S}B^2 + \frac{1}{2I_S}\varepsilon_{\text{SB}} \tag{29}$$

where eigenvalues of diagonal operators are already inserted. Here, the eigenvalue of $J$ is spin, $I$ is isospin, $N$ is right isospin derived from $N = \frac{1}{2}p_0$ where $(p_0, q_0)$ is the minimal irrep and $L$ is the third component of the body fixed spin operator which determine parity $P$ of the state by the relation of $P = (-1)^L$. The eigenvalue of the $\varepsilon_{\text{SB}}$ is derived from following eigenequation

$$\big\{C_2\big[SU(3)\big] + I_S\gamma(1-D_{88})\big\}\Psi = \varepsilon_{\text{SB}}\Psi \tag{30}$$

where $C_2[SU(3)]$ is Casimir operator of $SU(3)$.

We shall diagonalize (30) via a basis of the $SU(3)$ Wigner functions. We introduce a wave function of the form [19]

$$\Phi^{(m)}_{II_3Y,NN_3Y_R,JJ_3}(A,R) = \sqrt{d^{(m)}}(-1)^{\frac{Y_R}{2}+N_3}D^{(m)*}_{II_3Y,NN_3Y_R}\big(A^{-1}\big)D^{J*}_{J_3,-nN_3}\big(R^{-1}\big) \tag{31}$$

where the dimension of the $(p,q)$ irrep is expressed by $d^{(m)} = (p+1)(q+1)(p+q+2)/2$, the $m$ is representation of $SU(3)$ group, and subscript of the $Y$ and $Y_R$ is hypercharge and right hypercharge, respectively. According to Yabu–Ando, the wave function $\Psi$ is expanded via (31)

$$\Psi := \sum_m \beta^{(m)}\Phi^{(m)}_{II_3Y,NN_3Y_R,JJ_3}(A,R). \tag{32}$$

In terms of the basis, the eigenvalue problem (30) can be reduces to a matrix diagonalization problem. In YA, state of baryon appears to be its lowest irrep but contain large admixture of higher irreps. We shall see that such mixing reduces in large gravity limit.

With the operation of the collective quantization, the angular velocity $\Omega_8$ appears linear in Eq. (22). Therefore we obtain a constraint

$$Y_R = \frac{1}{3}N_c B, \tag{33}$$

which means that the symmetry $U_R(1)$ is redundant. Thus we obtain $Y_R = 2$.

In terms of Eq. (31), the expectation value of the Casimir invariants $C_2(SU(3))$ in Eq. (30) is easily obtained

$$\big\langle C_2\big(SU(3)\big)\big\rangle = \frac{1}{3}\big(p^2 + q^2 + pq + 3(p+q)\big). \tag{34}$$

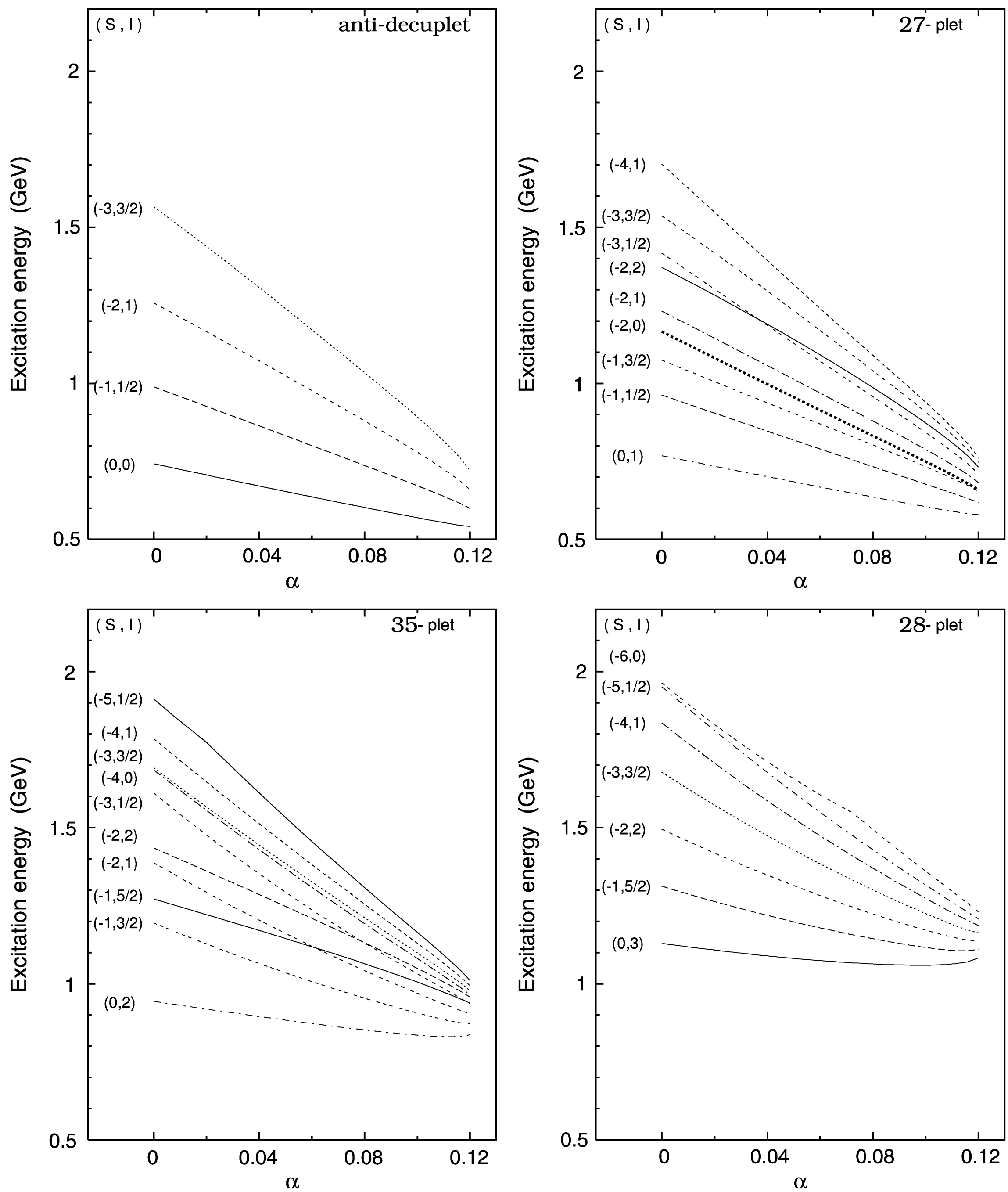


Fig. 3. The coupling constant dependence of the mass difference from the classical energy for the multiplets $\{\overline{10}\}$, $\{27\}$, $\{35\}$, $\{28\}$ ($(p,q) = (0,3), (2,2), (4,1), (6,0)$, respectively) are shown in the unit of GeV. The quantized energy is computed in terms of YA.

For the symmetry breaking term $I_S\gamma(1 - D_{88})$, the estimation of the expectation value can be done by performing the integral of the three Wigner rotation matrices [20,21] which is evaluated by the $SU(3)$ Clebsch–Gordan coefficient, or the isoscalar factor

$$\int dA\, D^{(m_3)*}_{\nu_3\nu_3'}(A) D^{(m_1)}_{\nu_1\nu_1'}(A) D^{(m_2)}_{\nu_2\nu_2'}(A) = \frac{1}{d^{(m)}} \sum_{\mu} \begin{pmatrix} m_1 & m_2 & m_{3\mu} \\ \nu_1 & \nu_2 & \nu_3 \end{pmatrix} \begin{pmatrix} m_1 & m_2 & m_{3\mu} \\ \nu_1' & \nu_2' & \nu_3' \end{pmatrix}. \tag{35}$$

Computations of the Clebsch–Gordan coefficients can be performed by the numerical algorithm of Ref. [22].

## 4. Numerical results

For the numerical analysis, we fix $F_\pi = 108$ MeV, $e = 4.84$, $\beta_\pi = 0.263$. The kaon decay constant, experimentally, is $F_\kappa \approx \sqrt{2}F_\pi$, but for the simplicity, we employ $F_\kappa = F_\pi$. For the kaon mass, we employ the experimental value, i.e., $\beta_\kappa = 0.952$.

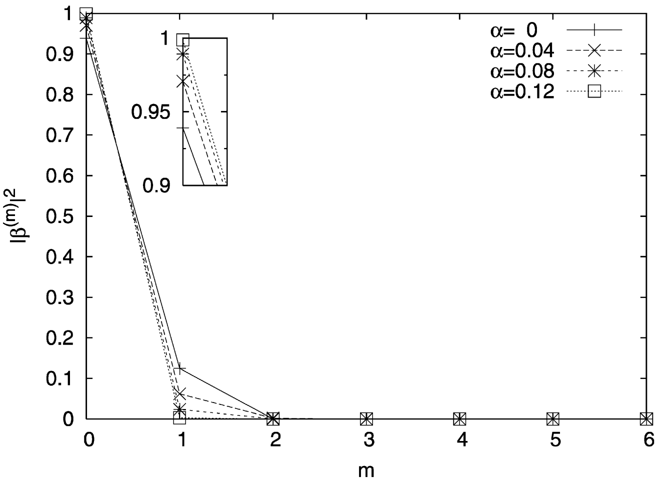


Fig. 4. Mixing probability of the higher representations to the lowest energy state of $NN$ channel in $\{\overline{10}\}$.

We estimate mass spectra belonging to $SU(3)$ multiplets $\{\overline{10}\}$, $\{27\}$, $\{35\}$, $\{28\}$ (or in the $(p,q)$ representation, $(0,3)$, $(2,2)$, $(4,1)$, $(6,0)$, respectively). The Finkelstein–Rubinstein constraints [23] tells us that for $\{\overline{10}\}$, $\{35\}$, $J=1$ is chosen for the ground state, otherwise one can set $J=0$ [24]. The eigenvalue of $L$ concerns with the third component of body fixed spin operator [3]. Substantially it is related to the orbital angular momentum but no experimental identification has been done. Therefore in our analysis we put $L=0$ for all multiplets states.

In YA treatment, one needs to truncate the base in finite size. We expand the collective wave function with $N \leqslant 3$, except for the states $(S,I)=(-5,1/2)$ and $(-6,0)$ in $\{28\}$, $(-4,0)$, $(-4,1)$ and $(-5,1/2)$ in $\{35\}$. In those states, we expand the base with $N \leqslant 4$ for obtaining sufficient convergence.

In Fig. 3 presents the $\alpha$ dependence of the mass spectra. Actually, we show mass difference between the quantized mass spectra and the classical energy. One easily see that the spectra as well as their differences within each multiplet decrease monotonically with increasing $\alpha$. On the other hand, mass differences between the different multiplet but with same quantum numbers $(S,I)$ increase, which have been already observed in the calculation of $SU(2)$ [11]. In Fig. 4 we illustrate the mixing probability of the multiplet for $NN$ channel in $\{\overline{10}\}$ for various $\alpha$. For increasing $\alpha$, the mixing of higher representations are significantly decreased; as a result, in a strong gravity region the baryons appear to be their lowest irreps and a naive perturbation treatment is sufficient. In our analysis, the pion and the kaon mass and the coupling constants are fixed by their experimental values (we simply set $F_\kappa = F_\pi$ for the coupling constant) and if we take into account variations of the mesonic data about change of gravity, exact $SU(3)$ flavor symmetry will recover at a strong gravity limit.

## 5. Conclusion

In this Letter, we have studied the gravitational effect to the dibaryons in the axially symmetric ES model. In particular, we have investigated gravity coupling constant dependence of the energy spectra for the $SU(3)$ dibaryons. We have used the collective quantization in three flavor space. For the symmetry breaking term, we employ the Yabu–Ando treatment. The result have shown that mass differences between spectra with different strangeness decrease monotonically and increase within different multiplet but with same quantum numbers $(S,I)$ with increasing $\alpha$. In the strong gravity limit, the $SU(3)$ flavor symmetry recovers; all the spectra degenerate in each multiplet and no mixing between the multiplets occur. Such symmetry restorations may observe in high energy experiment at LHC.

In this Letter, we treat $\alpha$ as a free parameter. In the ES theory, the Planck mass is related to the pion decay constant $f_\pi$ and coupling constant $\alpha$ by $M_{\mathrm{pl}} = f_\pi\sqrt{4\pi/\alpha}$. To realize the realistic value of the Planck mass, the coupling constant should be extremely small with $\alpha \sim O(10^{-39})$. However, we have shown that the effects of gravity can be observed only in large $\alpha$. Some theories such as scalar-tensor gravity theory [25] and "brane world scenario" [26] predict possibility of huge gravitational constant. There may have been an epoch in the early universe and may observe at an ultra high energy experiment where the gravitational effects on hadrons are crucial.

## Acknowledgements

We would like to thank Rajat K. Bhaduri for drawing our attention to this subject and useful comments. Also we deeply thank Noriko Shiiki for valuable discussions.

## References

[1] T.H.R. Skyrme, Proc. R. Soc. London, Ser. A 260 (1961) 127.
[2] G.S. Adkins, C.R. Nappi, E. Witten, Nucl. Phys. B 228 (1983) 552.
[3] E. Braaten, L. Carson, Phys. Rev. D 38 (1988) 3525.
[4] O.V. Manko, N.S. Manton, S.W. Wood, arXiv: 0707.0868.
[5] H. Luckock, I. Moss, Phys. Lett. B 176 (1986) 341.
[6] S. Droz, M. Heusler, N. Straumann, Phys. Lett. B 268 (1991) 371.
[7] P. Bizon, T. Chmaj, Phys. Lett. B 297 (1992) 55.
[8] N. Sawado, N. Shiiki, K. Maeda, T. Torii, Gen. Relativ. Gravit. 36 (2004) 1361.
[9] T. Ioannidou, B. Kleihaus, W. Zakrzewski, Phys. Lett. B 600 (2004) 116.
[10] N. Shiiki, N. Sawado, S. Oryu, Phys. Rev. D 70 (2004) 114023.
[11] H. Sato, N. Sawado, N. Shiiki, Phys. Rev. D 75 (2007) 014011.
[12] B. Kleihaus, J. Kunz, A. Sood, Phys. Lett. B 352 (1995) 247.
[13] Y. Brihaye, B. Hartmann, T. Ioannidou, W. Zakrzewski, Phys. Rev. D 69 (2004) 124035.
[14] H. Yabu, K. Ando, Nucl. Phys. B 301 (1988) 601.
[15] V.B. Kopeliovich, B. Schwesinger, B.E. Stern, Nucl. Phys. A 549 (1992) 485.
[16] C.L. Schat, N.N. Scoccola, Phys. Rev. D 62 (2000) 074010.
[17] V.B. Kopeliovich, B.E. Stern, JETP Lett. 45 (1987) 203, Pis'ma Zh. Eksp. Teor. Fiz. 45 (1987) 165.
[18] B. Kleihaus, J. Kunz, Phys. Rev. Lett. 78 (1997) 2527.
[19] H. Weigel, B. Schwesinger, G. Holzwarth, Phys. Lett. B 168 (1986) 321.
[20] A. Blotz, D. Diakonov, K. Goeke, N.W. Park, V. Petrov, P.V. Pobylitsa, Nucl. Phys. A 555 (1993) 765.
[21] N. Toyota, Prog. Theor. Phys. 77 (1987) 688.
[22] T.A. Kaeding, H.T. Williams, Comput. Phys. Commun. 98 (1996) 398, nucl-th/9511025.
[23] D. Finkelstein, J. Rubinstein, J. Math. Phys. 9 (1968) 1762.
[24] V.B. Kopeliovich, J. Exp. Theor. Phys. 93 (2001) 435, Zh. Eksp. Teor. Fiz. 120 (2001) 499.
[25] C. Brans, R.H. Dicke, Phys. Rev. 124 (1961) 925.
[26] N. Arkani-Hamed, S. Dimopoulos, D. Dvali, Phys. Lett. B 429 (1998) 263;
N. Arkani-Hamed, S. Dimopoulos, D. Dvali, Phys. Rev. D 59 (1999) 086004.