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The analysis of the behaviour of an innovative pedestrian steel bridge

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Abstract

The article dwells on a new structural solution for pedestrian steel suspension bridges. This new structural system of pedestrian stress-ribbon bridges includes suspension members with bending stiffness and a pre-stressed tie. The article looks into the behaviour of such suspension bridge under symmetrical and asymmetrical loads; it also presents analytical expressions for displacements, thrust forces and bending moments of such stiff suspension members. Then the article explains the effect of the cable’s bending stiffness and the tie’s axial stiffness on the bridge’s stresses as well as the effect of the tie’s pre-stressing on horizontal and vertical displacements of the bridge structure. The article then proceeds to comparative analysis, which considers the new structure and a traditional pedestrian suspension bridge. Numerical experiment determines the accuracy of the new engineering method developed for the analysis of stress-ribbon bridges. The efficiency of steel stress-ribbon bridge displacement stabilization through the bending stiffness is being discussed.

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1. Introduction

Stress-ribbon suspension structure of pedestrian bridges is among the most elegant and light bridge structures. Because of their static and dynamic properties, these stress-ribbon structures are predominantly used to carry pedestrian and bicycle traffic (Strasky J. 2005; Schlaich J. et. al. 2005).

The main load-carrying members of modern pedestrian suspension bridges are high-strength steel sheets or pre-stressed reinforced concrete structures (Juozapaitis, A., Norkus, A. 2007; Michailov, V. V. 2002; Schlaich J., Bergerman R. 1992; Strasky J. 2005; Kulbach, V. 2007). Heavy decks and pre-stressing mean that such bridge structures require considerable amounts of load-carrying structural materials. In recent years, along the
ordinary single-span stress-ribbon suspension bridges, multi-span bridges have been used as well (Atanasovski, S., Markovski, G.; Troyano L.F. 2003; Schlaich J., Bergherman R. 1992; Caetano E., Cunha A. 2004; Strasky J. 2005). Asymmetrical loads cause these structures to behave in more complex ways. The central support may be displaced horizontally and, in turn, cause additional horizontal and vertical kinematic displacements.

A suggestion to stabilize the initial shape of a stress-ribbon suspension structure is to use so-called stiff suspension members instead of usual flexible steel sheets or cables (Juozapaitis, A., Norkus, A. 2007; Juozapaitis A. et al. 2006). Plainly, the main load-carrying structures of such bridges absorb the loads by tension, but also by bending (Moskalev N. S. 1981). On top of the efficient stabilisation of the initial shape of a suspension bridge, the load-carrying suspension members make it possible to do without expensive pre-stressing and heavy decks. The materials used in making such load-carrying suspension members are typical rolled or welded steel cross sections. Importantly, flexible support is recommended for the said suspension structures (Juozapaitis A. et al. 2006). The behaviour of structures in multi-span stress-ribbon suspension bridges has not been discussed enough, particularly taking into account the bending stiffness of their cross-sections.

The article discusses an innovative two spans structure of pedestrian bridges: it includes suspension members with bending stiffness and a pre-stressed tie connecting the central support to the outer supports. The behaviour of the structure affected by symmetrical and asymmetrical loads is analysed. Then the article presents the analytical expressions to calculate the horizontal and vertical displacements, the thrust force and the bending moments of such load-carrying suspension structures of a bridge. It then proceeds to the results of our numerical experiment and finally assesses the efficiency of the displacement stabilisation in the suggested innovative structural solution.

2. Analysis of the innovative pedestrian-bridge structure affected by symmetrical loads

The innovative two spans suspension structure of a pedestrian bridge consists of two flexibly supported load-carrying suspension members, a flexible tie connects the central and outer supports. Each load-carrying suspension member is with low initial sag and finite bending stiffness. Their initial shape is a square parabola. The flexible tie is a pre-stressed flexible cable. The design diagram of the innovative stress-ribbon bridge, affected by symmetrical and asymmetrical loads, is shown in Fig. 1.

![Analytical model of the innovative stress-ribbon suspension bridge](image)

Fig.1. Analytical model of the innovative stress-ribbon suspension bridge: (a) symmetrical load; (b) asymmetrical load

In order to find out the vertical displacements of the load-carrying suspension member with finite bending stiffness, a known equation will be used (Sandović et al. 2011). The shape of the strained member in question is assumed to be a square parabola. In such case the length of the suspension member under elastic strain $s_1$ is:

$$s_1 = L + \frac{8 \cdot (f_0 + \Delta f)^2}{3 \cdot L},$$

where $f_0$ – initial sag of the suspension member, $\Delta f$ – vertical displacement, $L$ – span length.
After known algebraic operations (Sandovič G. et al. 2011) we obtain the expression for mid-span displacements of the suspension member:

$$\Delta f^2 + 2f_0 \cdot \Delta f - \frac{3 \cdot H \cdot L^2}{8 \cdot E \cdot A} = 0,$$

(2)

where:

$$H = \frac{(g + p) \cdot L^2}{5 \cdot L^2} \cdot \frac{48 \cdot E \cdot I \cdot \Delta f}{(f_0 + \Delta f)},$$

(3)


The second member in the numerator of the formula (3) takes into account the effect of the bending stiffness. We take formula (3), insert it into formula (2) and then have a 3rd degree equation for mid-span vertical displacements of the suspension member:

$$\Delta f^3 + 3 \cdot f_0 \cdot \Delta f^2 + 2f_0^2 \cdot \Delta f - \frac{15 \cdot (g + p) \cdot L^4 - 1152 \cdot E \cdot I \cdot \Delta f}{320 \cdot E \cdot A} = 0.$$

(4)

Importantly, under symmetrical load the tie has no influence, because the central support is still.

3. Structural analysis of the innovative pedestrian bridge affected by asymmetrical loads

The bridge structure subjected by asymmetrical loads is shown in Figure 1(b). Importantly, the central support affected by this load moves by the value $\Delta h$ towards the (left) span affected by asymmetrical load (see Fig. 1(b)). Hence, the tie of the first (left) span is affected by compression and therefore has no influence. Taking into account any potential asymmetrical loads and in order to stabilise (reduce) the horizontal displacement $\Delta h$ of the central support, both ties must be pre-stressed. The pre-stress force is considered to be 1.1 times higher than the internal compression force caused in the tie by the asymmetrical load.

The thrust forces acting on the first (left) and the second (right) span are calculated as follows:

$$H_l = \frac{g \cdot (L + \Delta h)^2 - 48 \cdot E \cdot I \cdot \Delta f_r}{8 \cdot (f_0 + \Delta f_r)} - \frac{5 \cdot L^2}{5 \cdot L^2} + \frac{2 \cdot \Delta h \cdot E_t \cdot A_t}{L},$$

(5)

$$H_r = \frac{(g + p) \cdot (L - \Delta h)^2 - 48 \cdot E \cdot I \cdot \Delta f_l}{8 \cdot (f_0 + \Delta f_l)} - \frac{5 \cdot L^2}{5 \cdot L^2} - \frac{2 \cdot \Delta h \cdot E_t \cdot A_t}{L},$$

(6)

where $H_l$ and $H_r$ – the thrust forces acting on the first (left) and the second (right) spans, $\Delta f_l$ and $\Delta f_r$ – the vertical mid-span displacements of the first and the second spans, $E_t$ – the tie’s elastic modulus, $A_t$ – the tie’s cross-section area.
The mid-span bending moments of the first (left) and the second (right) spans are calculated using the expression (27) – (28) (Sandovič G. et al. 2011).

We take the thrust force expression of the first span, insert it into the equation of strain balance (Sandovič et al. 2011) and then have a 2nd degree equation for mid-span vertical displacements of the first span:

\[
\Delta f_l^2 + 2 \cdot f_0 \cdot \Delta f_l - 0.375 \cdot L \cdot \Delta h - \frac{15 \cdot g \cdot (L + \Delta h)^2 \cdot L^2 + 240 \cdot \Delta h \cdot E_i \cdot A_i \cdot (f_0 + \Delta f_r) \cdot L - 1152 \cdot E \cdot I \cdot \Delta f_r}{320 \cdot (f_0 + \Delta f_r) \cdot E \cdot A}, (7)
\]

The balance of thrust forces \((H_l = H_r)\) gives the equation for mid-span general displacements of the second span:

\[
\Delta f_r = \frac{g \cdot L^2 \cdot (L + \Delta h)^2 \cdot (f_0 + \Delta f_l) + 76.8 \cdot E \cdot I \cdot \Delta f_l \cdot f_0}{(g + p) \cdot (L - \Delta h)^2 \cdot L^2 - 16 \cdot \Delta h \cdot E_i \cdot A_i \cdot (f_0 + \Delta f_l) \cdot L + 76.8 \cdot E \cdot I \cdot f_0} + \frac{8 \cdot (f_0 + \Delta f_l) \cdot \Delta h \cdot E_i \cdot A_i \cdot L - (g + p) \cdot (L - \Delta h)^2 \cdot L^2 \cdot f_0}{(g + p) \cdot (L - \Delta h)^2 \cdot L^2 - 16 \cdot \Delta h \cdot E_i \cdot A_i \cdot (f_0 + \Delta f_l) \cdot L + 76.8 \cdot E \cdot I \cdot f_0}. (8)
\]

The horizontal displacement of the central support is determined by calculating the difference between the cable lengths of the first and the second spans after the deformation (Sandovič et al. 2011):

\[
\Delta h = \frac{4}{3 \cdot L} \left( (f_0 + \Delta f_l)^2 - (f_0 + \Delta f_r)^2 \right) - \frac{H_l \cdot L}{2 \cdot E \cdot A} + \frac{H_r \cdot L}{2 \cdot E \cdot A}, (9)
\]

Vertical displacements are defined by approximation; for the first iteration, the values \(\Delta f_r\) and \(\Delta h\) are obtained by calculating the kinematic displacements of the innovative structure, they are \(\Delta f_r = \Delta f_{r, \text{kinematic}}\) and \(\Delta h = \Delta h_{\text{kinematic}}\).

4. Numerical experiment

In order to determine the accuracy of the engineering method of analysis, a numerical experiment was carried out. A two spans bridge structure of a total length of 80 m, 40 m for each span, was analysed. The values of the initial sag varied as follows: \(f_0 = 0.8\ m\), \(f_0 = 1\ m\) or \(f_0 = 1.25\ m\). The values of the asymmetrical load distributed evenly were determined taking into account the range of variation \(-\gamma = 1\) of the ratio \(\gamma = p / g\) between the live load and the dead load.

The mid-span vertical displacements of the first span were calculated analytically and by the BEM software; the difference (in percent) between the obtained values is shown in Fig. 2(a) for symmetrical load and in Fig. 2(b) for asymmetrical load. It has been already mentioned that a tie affected by symmetrical load has no influence on structural deformations.

Figure 2(a) and 2(b) shows that the difference between the vertical displacements calculated by the BEM software and those calculated with the help of the engineering methods, taking into account the slenderness parameter \(kL\) \((kL = L \cdot \sqrt{H/E \cdot I})\) of the suspension member, does not exceed 4% (the highest error).
The BEM software and the engineering methods were also used to calculate the horizontal displacement of the central support and the thrust force of the first span; at the highest error, the difference between the obtained values was about 2% in case of the horizontal displacement and about 1% in case of the thrust force.

The numerical experiment shows that, both in case of symmetrical and asymmetrical loads, the engineering methods have sufficient accuracy. The highest relative error does not exceed 4%.

5. Efficiency of displacement stabilisation

In order to determine the efficiency of the innovative suspension bridge, a numerical experiment was carried out by comparing the behaviour of structures in the innovative two spans suspension bridge and its traditional version. The length of spans was considered to be 40 m, while the initial span sag varied as follows: \(f_0 = 0.8 \, m\), \(f_0 = 1 \, m\) and \(f_0 = 1.25 \, m\). The cross-section areas of the ties of the innovative bridge also varied as follows: \(7.64 \, cm^2\), \(16.43 \, cm^2\) and \(53.8 \, cm^2\).

Under asymmetrical load, the tie restricts horizontal displacements of the central support and, hence, stabilises the initial shape of the bridge. Importantly, the thrust forces acting on the first and the second spans of the innovative bridge are not equal—the thrust force acting on the first span is always higher than that acting on the second span \(H_1 > H_2\). In order to prove that the displacement stabilisation in the innovative bridge is efficient, the ratio \(a\) between the axial stiffness of the tie and that of the suspension member was used, thus taking into account the axial stiffness of the bridge’s members. The ratio can be calculated as follows:

\[
a = \frac{A_t \cdot E_t}{A \cdot E}
\]

The diagrams illustrating the stabilisation of the vertical displacement of the first span and the horizontal displacement of the central support (the initial sag of the bridges is \(f_0 = 0.8 \, m\)) are shown in Fig. 3: the innovative bridge is compared to a traditional bridge. Fig. 3(a) shows that when the values of the axial stiffness ratio \(a\) (relative increase of the tie’s axial stiffness \(E_t \cdot A_t\)) of the innovative bridge increase, the stability of vertical displacements becomes higher: when the axial stiffness ratio equals 0.06, the displacements decrease by 6.47%, but when \(a = 0.40\), the displacements decrease by 22.44%. The higher the initial sag of both the innovative and traditional bridge structures, the better the stabilising effect, taking into account the ratio \(a\).

Even when the axial stiffness ratio is low (\(a = 0.06\)), the tie, depending on the ratio \(\gamma\) between the live load and the dead load, stabilises horizontal displacements of the central support by up to 18.46%. When the axial
stiffness ratio and the initial sag of the load-carrying member increase, the displacement stabilisation in the central support also increases respectively (see Fig. 3(b)).

Fig.3. (a) Stabilisation of the vertical displacement of the first span, taking into account the axial stiffness; (b) stabilisation of the horizontal displacement of the central support, taking into account the initial sag of the structure

6. Concluding remarks

The article presents an approximate (engineering) method to analyse the innovative structure of a two spans steel pedestrian bridge affected by symmetrical and asymmetrical loads. The method makes it possible to determine, by gradual approximation, vertical and horizontal displacements, thrust forces and bending moments affecting a suspension bridge. A numerical experiment was carried out to compare the results obtained by the engineering method and the results delivered by the BEM software; the experiment proved sufficient accuracy of this method: the error was below 4%. The comparative analysis of structures of the innovative bridge and a traditional bridge revealed that the efficiency of displacement stabilisation in the innovative structure depends on the initial sag of the stress-ribbon structure and the axial stiffness ratio $a$. When we take the innovative bridge with $f_0 = 0.8 \, m$ and $a = 0.06$, the vertical displacement of the part affected by load is more stable by only 6.47%, while the horizontal displacement of the central support by 18.46%. But when $a = 0.40$, the vertical displacement of the part affected by load is more stable by load is more stable by only 6.47%, while the horizontal displacement of the central support by 18.46%. But when $a = 0.40$, the vertical displacement of the part affected by load is more stable by 22.44%, while the horizontal displacement of the central support by 62.97% (compared to a traditional bridge).

References