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Scotogenic Z_2 or $U(1)_D$ model of neutrino mass with $\Delta(27)$ symmetry

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ABSTRACT

The scotogenic model of radiative neutrino mass with Z_2 or $U(1)_D$ dark matter is shown to accommodate $\Delta(27)$ symmetry naturally. The resulting neutrino mass matrix is identical to either of two forms, one proposed in 2006, the other in 2008. These two structures are studied in the context of present neutrino data, with predictions of *CP* violation and neutrinoless double beta decay.

To understand the pattern of neutrino mixing, non-Abelian discrete symmetries have been used frequently in the past several years, starting with A_4 [1–6]. Another symmetry $\Delta(27)$ was also studied [7–9] some years ago. Using the fact that it admits geometric *CP* violation [10], it has been proposed recently for understanding the *CP* phases in the mixing of quarks [11,12] and of leptons [13,14].

In a parallel development, there is a large body of literature on the radiative generation of neutrino mass through dark matter. The simplest original (scotogenic) one-loop model [15] adds one extra scalar doublet (η^+, η^0) and three neutral fermion singlets $N_{1,2,3}$ together with an exactly conserved Z_2 symmetry under which the new particles are odd and the standard-model (SM) particles are even. The resulting one-loop diagram for Majorana neutrino mass is shown in Fig. 1.

A variation of this mechanism was recently proposed [16], using two extra scalar doublets $(\eta^+_{1,2}, \eta^0_{1,2})$ transforming as ± 1 under a $U(1)_D$ gauge symmetry together with three Dirac fermion singlets $N_{1,2,3}$ transforming as $+1$. The resulting one-loop diagram for Majorana neutrino mass is shown in Fig. 2.

Combining these two ideas, it is shown in this paper that $\Delta(27)$ is naturally adapted to realize the two neutrino mass matrices proposed earlier [7,9] without additional particle content in the loop, in either the Z_2 or $U(1)_D$ case. We then study their implications in the context of present neutrino data.

The group $\Delta(27)$ has nine one-dimensional representations $\underline{1}_i$ $(i = 1, \ldots, 9)$ and two three-dimensional ones $\underline{3}, \underline{3}^*$. Their multiplication rules are

$$\underline{3} \times \underline{3}^* = \sum_{i=1}^{9} \underline{1}_i, \qquad \underline{3} \times \underline{3} = \underline{3}^* + \underline{3}^* + \underline{3}^*. \tag{1}$$

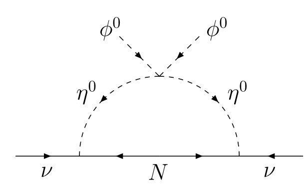

Fig. 1. One-loop generation of neutrino mass with Z_2 symmetry.

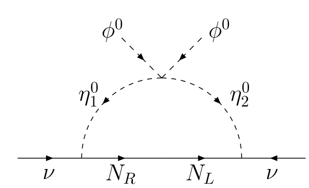

Fig. 2. One-loop generation of neutrino mass with $U(1)_D$ symmetry.

In the decomposition of $\underline{3} \times \underline{3} \times \underline{3}$, there are three invariants: $111 + 222 + 333$ and $123 + 231 + 312 \pm (132 + 213 + 321)$. In Fig. 1, let $\Phi, \eta \sim \underline{1}_1$, $\nu \sim \underline{3}$, $N \sim \underline{3}^*$, then the 3×3 Majorana neutrino mass matrix is proportional to the 3×3 Majorana N mass matrix, which is nonzero from the vacuum expectation values of a neutral scalar $\zeta \sim \underline{3}$ under $\Delta(27)$ with the Yukawa couplings $f_{ijk} N_i N_j \zeta^*_k$. In Fig. 2, let $\Phi, \eta_{1,2} \sim \underline{1}_1$, $\nu \sim \underline{3}$, $N_R \sim \underline{3}$, $N_L \sim \underline{3}^*$, then the same result is obtained with $f_{ijk} \bar{N}_{Li} N_{Rj} \zeta_k$. In both cases, the neutrino mass matrix is of the form

$$\mathcal{M}_\nu = \begin{pmatrix} fa & c & b \\ c & fb & a \\ b & a & fc \end{pmatrix}, \tag{2}$$

where a, b, c are proportional to the three arbitrary vacuum expectation values of ζ.

In Ref. [7], with $l^c \sim \underline{3}^*$ and $\phi_{1,2,3} \sim \underline{1}_{1,2,3}$, the charged-lepton mass matrix is diagonal, whereas in Ref. [9], with $l^c \sim \underline{3}$ and $\phi \sim \underline{3}$, it is given by

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$$\mathcal{M}_l = U_\omega \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} U_\omega^\dagger, \tag{3}$$

where U_ω is the familiar

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{4}$$

with $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. In this second model, the vacuum expectation values of the three components of ϕ under $\Delta(27)$ are set equal, in analogy to what was done in A_4 [17]. Both of these $\Delta(27)$ models are consistent with $\theta_{13} \neq 0$, but they were proposed before its determination in 2012.

Consider first the case where the charged-lepton mass matrix is diagonal. In Ref. [7], two solutions were found with $\theta_{13} = 0$; one with $f \simeq 1$, the other with $f \simeq -0.5$. The former turns out to be unacceptable because θ_{13} is always very small. The latter has a solution as shown below. Let $f = -0.5 + \epsilon$, $a = b(1+\eta)$ and $c = b(1-\kappa)$, then in the tribimaximal basis, defined as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{TB} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{5}$$

the neutrino mass matrix becomes

$$\mathcal{M}_\nu^{TB} = U_{TB}^T \mathcal{M}_\nu U_{TB}$$
$$= \begin{pmatrix} -\frac{3}{2} + \epsilon + \frac{3}{4}\kappa & -\frac{(2\eta+\kappa)}{2\sqrt{2}} & \frac{\sqrt{3}}{4}\kappa \\ -\frac{(2\eta+\kappa)}{2\sqrt{2}} & \frac{3}{2} + \epsilon + \frac{1}{2}\eta - \frac{1}{2}\kappa & \frac{\sqrt{3}}{2\sqrt{2}}\kappa \\ \frac{\sqrt{3}}{4}\kappa & \frac{\sqrt{3}}{2\sqrt{2}}\kappa & -\frac{3}{2} + \epsilon - \eta + \frac{1}{4}\kappa \end{pmatrix} b, \tag{6}$$

where ϵ, η, κ are all assumed to be small compared to one. We define $\kappa + 2\eta = \zeta$ and assume all parameters to be real, then

$$\Delta m_{21}^2 \simeq m_{22}^2 - m_{11}^2 \simeq \frac{3}{4}(8\epsilon + \zeta) b^2, \tag{7}$$

$$\Delta m_{31}^2 \simeq m_{33}^2 - m_{11}^2 \simeq \frac{3}{2}\zeta b^2, \tag{8}$$

$$\sin\theta_{13} \simeq \pm\left(\sqrt{\frac{2}{3}}\theta'_{13} + \sqrt{\frac{1}{3}}\theta'_{23}\right) \simeq \pm\frac{\kappa}{\sqrt{2}\zeta}, \tag{9}$$

$$\tan\theta_{12} \simeq \frac{\sqrt{1/3} - \sqrt{2/3}\theta'_{12}}{\sqrt{2/3} + \sqrt{1/3}\theta'_{12}} \simeq \frac{1}{\sqrt{2}}\left[\frac{1-\zeta/6}{1+\zeta/12}\right], \tag{10}$$

where

$$\theta'_{ij} = \frac{m_{ij}}{m_{ii} - m_{jj}}. \tag{11}$$

Note the important fact that the neutrino mass matrix under consideration is quasi-degenerate, with $m_1 \simeq -m_{22} \simeq m_{33}$. This means that θ'_{23} is much smaller than θ'_{13}. In the limit $\theta'_{23} = 0$, we obtain

$$\sin^2 2\theta_{23} \simeq 1 - 2\sin^2\theta_{13}, \tag{12}$$

which is consistent with data. If θ'_{23} dominates, then it would be $1 - 8\sin^2\theta_{13}$ and be ruled out by data. This was first pointed out in Ref. [18]. Using $\tan^2\theta_{12} = 0.45$, we find $\zeta = 0.209$. Hence $\Delta m_{31}^2 > 0$, i.e. normal ordering of neutrino masses. Using $\Delta m_{31}^2 = 2.32 \times 10^{-3}$ eV^2, we find $b = 0.086$ eV. Using $\sin\theta_{13} = \pm 0.16$, we find $\kappa = \pm 0.047$. Using $\Delta m_{21}^2 = 7.50 \times 10^{-5}$ eV^2, we find $8\epsilon + \zeta = 0.0135$. This predicts $\sin^2 2\theta_{23} = 0.966$, which takes into account both θ'_{13} and θ'_{23}, and $m_{ee} = |fa| = 0.05$ eV for the effective Majorana neutrino mass in neutrinoless double beta decay.

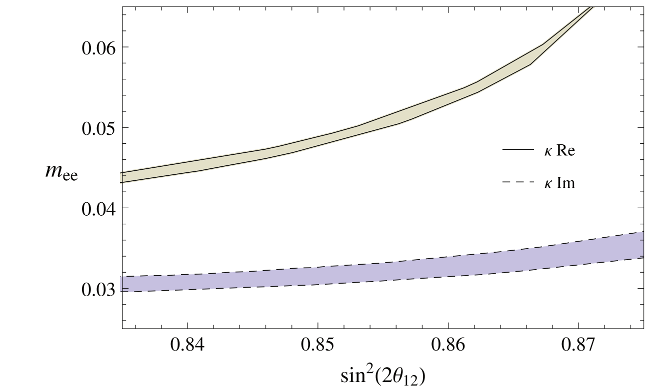

Fig. 3. Predictions of m_{ee} versus $\sin^2 2\theta_{12}$ for $\sin^2 2\theta_{13} = 0.095 \pm 0.010$.

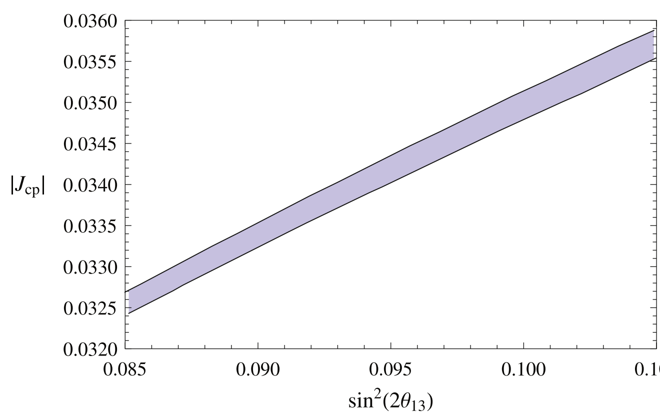

Fig. 4. Predictions of $|J_{CP}|$ versus $\sin^2 2\theta_{13}$ for κ purely imaginary and $\sin^2 2\theta_{12} = 0.857 \pm 0.024$.

Although ζ and κ/ζ are small, they are not so small compared to one, which means that our approximation may have significant corrections. However, the figures presented here are not based on the above approximation, but rather on exact numerical diagonalizations of $\mathcal{M}_\nu$.

We consider also the case with κ purely imaginary, in which case $\sin^2 2\theta_{23} = 1$ is guaranteed in the limit of a symmetry based on a generalized *CP* transformation [19]. Using a complete numerical analysis, we plot in Fig. 3 the predictions of this model for m_{ee} as a function of $\sin^2 2\theta_{12}$ for $\sin^2 2\theta_{13} = 0.095 \pm 0.010$. In this range, $\sin^2 2\theta_{23}$ varies only slightly from the estimated value of 0.966 for κ real and approaches one as the imaginary part of κ increases. The higher (lower) band corresponds to κ real (purely imaginary), with the allowed region in between for any arbitrary phase. We plot in Fig. 4 the invariant J_{CP} as a function of $\sin^2 2\theta_{13}$ for κ purely imaginary and $\sin^2\theta_{12} = 0.857 \pm 0.025$. We verify that $\sin^2 2\theta_{23}$ is indeed unity as required. We see also that m_{ee} is mostly sensitive to θ_{12}, whereas J_{CP} is mostly sensitive to θ_{13} as expected. In Figs. 3 and 4, we have used the latest Particle Data Group numbers [20], i.e.

$$\Delta m_{21}^2 = 7.50 \pm 0.20 \times 10^{-5}\ \text{eV}^2, \tag{13}$$

$$\Delta m_{32}^2 = 2.32 + 0.12 - 0.08 \times 10^{-3}\ \text{eV}^2, \tag{14}$$

$$\sin^2 2\theta_{12} = 0.857 \pm 0.025, \tag{15}$$

$$\sin^2 2\theta_{23} > 0.95, \tag{16}$$

$$\sin^2 2\theta_{13} = 0.095 \pm 0.010. \tag{17}$$

Consider next the case where $\mathcal{M}_l$ is given by Eq. (3). In the tribimaximal basis,

$$\mathcal{M}_\nu^{TB} = \begin{pmatrix} a+f(b+c)/2 & (b+c)/\sqrt{2} & f(-b+c)/2 \\ (b+c)/\sqrt{2} & fa & (b-c)/\sqrt{2} \\ f(-b+c)/2 & (b-c)/\sqrt{2} & a-f(b+c)/2 \end{pmatrix}. \quad (18)$$

Let $f=-1+\epsilon'$, $\eta'=(b+c)/2a$, $\kappa'=(b-c)/2a$, then

$$\mathcal{M}_\nu^{TB} \simeq \begin{pmatrix} 1-\eta' & \sqrt{2}\eta' & \sqrt{2}\kappa' \\ \sqrt{2}\eta' & -1+\epsilon' & \kappa' \\ \sqrt{2}\kappa' & \kappa' & 1+\eta' \end{pmatrix} a, \quad (19)$$

where $\epsilon', \eta', \kappa'$ are all assumed to be small compared with one. This turns out to have the same approximate solution as Eq. (6) with the following substitutions:

$$a = \frac{-3b}{2}, \qquad \eta' = \frac{\zeta}{6}, \qquad \kappa' = \frac{\kappa}{2\sqrt{6}}, \qquad \epsilon' = \frac{-4\epsilon}{3}. \quad (20)$$

The predicted m_{ee} is also approximately the same. Thus the physical manifestations of this second model are indistinguishable from those of the first to a good approximation.

Acknowledgements

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