Geometric Analysis of Singularity for Single Gimbal Control Moment Gyro Systems

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Abstract: This research is focused on the singularity analysis for single gimbal control moment gyro systems (SCMGs) which include two types, with constant speed (CSCMG) or variable speed (VSCMG) rotors. Through angular momentum hypersurfaces of singular states, the passable and in-passable singular points are discriminated easily, meanwhile the information about how much the angular momentum workspace as well as the steering capability available is provided directly. It is obvious that the null motions of steering laws are more effective for the five pyramid configuration (FPC) than for the pyramid configuration (PC) from the singular plots. The possible degenerate hyperbolic singular points of the preceding configurations are calculated and the distinctness of them is denoted by the Gaussian curvature. Furthermore, failure problems to steer integrated power and attitude control system (IPACS) are also analyzed. A sufficient condition of choosing configurations of VSCMGs to guarantee the IPACS steering is given. The angular momentum envelopes of VSCMGs, in a given energy and a limited range of rotor speeds, are plotted. The connection and distinctness between CSCMGs and VSCMGs are obtained from the point of view of envelopes.

Key words: attitude control; single gimbal control moment gyro; singularity; geometric analysis; angular momentum hypersurfaces; store energy

Single-gimbal control moment gyro systems (SCMGs) include two types, i.e., CSCMGs (with constant speed rotors) and VSCMGs (with variable speed rotors). They all have the advantage of torque amplification. An obstacle of using CSCMGs in engineering practice is the existence of singular gimbal configuration, because the CSCMGs can not generate a torque in arbitrary directions when the gimbal configuration appears with singularity. Many researchers have studied the configuration singularity of CSCMGs by means of geometric approaches [15]. Geometric theory on
singular surfaces was first formulated by Margulis\textsuperscript{[1]}. Two-dimensional angular momentum slices of singular states were presented by Dominguez\textsuperscript{[2]}. Research results on computation and visualization of singularity were summarized by Wie\textsuperscript{[3]}. A discrimination method using the surface curvature, as well as a way of understanding the steering motion using inverse kinematics and manifold theory, was proposed by Kunokawa\textsuperscript{[4]}. Here the symmetric type of configuration defined by Ref.\textsuperscript{[4]} is researched, for which all admissible torque directions lie in a two-dimensional surface at each singular state. To avoid singular states, a great variety of steering laws of CSCMGs have been proposed\textsuperscript{[6,7]}. The scheme using VSCMGs to avoid the singularity has been also considered\textsuperscript{[5,8,11]}. However, the singularity problem is far from being solved.

In fact, one of the most effective ways for analyzing singularity of SCMGs is to plot angular momentum hypersurfaces of singular states. Through the hypersurfaces, the passable and impassable singular points can be discriminated easily, meanwhile the information about how much angular momentum workspace available as well as the steering capability can be provided directly.

This paper deals with the geometric analysis of singularity for SCMGs. First, the angular momentum hypersurfaces for the five pyramid configuration (FPC) and the pyramid configuration (PC) are plotted. These singular plots show that the null motions in steering laws are more effective to FPC than to PC. Furthermore, the minimum angular momentums in FPC’s elliptic singularities are obtained. Hereafter, the possible degenerate hyperbolic points are calculated and denoted in hyperbolic singular surfaces, which is an expansion of Bednossian’s work\textsuperscript{[12]}. Next, for the aforementioned two types of configurations, the significant distinctness of Gaussian curvature on the degenerate points is found, and the number of degenerate points is given. Finally, the failure problem for steering IPACS is analyzed, which is a complementarity for the work in Ref.\textsuperscript{[5]}. A sufficient condition for IPACS with VSCMGs doing not encounter steering singularities is given. The angular momentum envelopes of VSCMGs, which are based on the specifically instantaneous energy and the limited range of rotor speed, are plotted.

1 General Equation of SCMGs

Define the \textit{i}th SCMGs reference frame as \{\textit{s}i, \textit{ti}, \textit{gi}\}, in which \textit{gi} is the unit vector of gimbal axis; \textit{si} is the unit vector of spin axis; \textit{ti} = \textit{gi} × \textit{si} is the unit vector of transverse axis in the reverse direction of the output torque. The derivatives of these unit vectors have the following relationships:

\[
\begin{align*}
\dot{s}_i &= \delta \dot{t}_i \\
\dot{t}_i &= -\delta \dot{s}_i \\
\dot{g}_i &= \mathbf{0} \\
\end{align*}
\]

(1)

In fact, the gimbal rates \(\dot{\delta}\) are much smaller than the angular speeds of rotors \(\Omega\), so that the angular momentum change resulting from \(\dot{\delta}\) can be neglected, then

\[
\begin{align}
\dot{H}_{sg} &= A_i I_{ws} \Omega \\
\dot{H}_{sg} &= A_i I_{ws} [\Omega \times \delta] + A_i I_{ws} \dot{\Omega} = C(\Omega, \delta) \delta + D(\delta) \dot{\delta}
\end{align}
\]

where \(\dot{H}_{sg}\) is the angular momentum of SCMGs, \(A_i = [s_1, \ldots, s_n] I_{ws} = \text{diag}(I_{ws1}, \ldots, I_{wsn})\), \(I_{wsi}\) is the moment of inertia of the \textit{i}th SCMG wheel about its spin axis, \(A_i = [t_1, \ldots, t_0]\), \([\Omega] z = \text{diag}(\Omega_1, \ldots, \Omega_n)\).

2 Singularity Analysis of CSCMGs

For the CSCMGs, the applied torque coming from the spacecraft is written as

\[
T_{cscmg} = \dot{H}_{cscmg} = C(\delta) \dot{\delta}
\]

(4)

Matrix \(C\) has maximal rank 3 and minimal rank 2 because of the near-coplanarity of the gimbal axis \(g_i\). When rank(\(C\)) = 2, its columns \(C_i(\delta)\) are coplanar. Vector \(u\) is called a singular direction if it satisfies

\[
u \cdot C_i(\delta) = 0 \\ i = 1, \ldots, n
\]

(5)

The relevant combinations of gimbal angles \(\delta_i = [\delta_1, \delta_2, \ldots, \delta_n]^T\) are called singular states. For a given singular vector \(u \neq \pm g_i\), the transverse axis vector and the spin axis vector at a singular state can be expressed as
The total angular momentum at singular states corresponding to a singular direction $u$ is expressed as

$$H_{\text{sing}} = \sum_{i=1}^{n} s_i = \sum_{i=1}^{n} \varepsilon(g_i \times u) \times g_i / |g_i \times u|,$$

(7)

Thus there are $2^n$ combinations of gimbal angles for the direction $u$. The domain of $u$ is a unit sphere except $\pm g_i$ directions. When $u$ is obtained from the domain in sequence, and all $\varepsilon$ fixed as parameters, the singular surfaces $S_{E_i}$ of angular momentums can be plotted. If all $\varepsilon$ are reversed simultaneously and the vector $u$ is changed to $-u$, $H_{\text{sing}}$ remains invariance. Therefore, the number of different surfaces $S_{E_i}$ is $2^{n-1}$.

Let $\delta_N$ be a solution of the following equation

$$C(\delta_N) \dot{\delta}_N = 0_{n \times 1}$$

(9)

It implies that no torque is generated by the motion of $\delta_N$, so this motion is called a null motion and $\dot{\delta}_N$ a null displacement. Escape singularity by null motion includes two meanings. First, the null motion exists in a singular configuration. Second, the rank of the Jacobian matrix $C$ can be changed to $\text{rank}(C) = 3$ by the null motion.

The singular states determine a mapping from the punctured sphere $S_0(u \neq \pm g_i)$ to the space $S_5$ of the gimbal angle combination $\delta_5$, and a mapping from $S_5$ to the singular angular momentum space $S_{10}$. A type of singularity is termed hyperbolic (passable) if a null motion at this point is possible, otherwise it is termed elliptic (impassable). To test whether or not a null motion is possible at a singular point, the total CSCMGs angular momentum is expanded by using Taylor series about a singular configuration $\delta_5$. To obtain the final calculation result, denote $n$ as the null-space basis vectors of the Jacobian matrix $C$, and $r = \text{rank}(C)$, then

$$\lambda^T M \lambda = 0$$

(10a)

where $\lambda = [\lambda_1, \ldots, \lambda_{n-r}]^T$, $\lambda_m (m = 1, \ldots, n-r)$ is the weighting coefficient, $P = \text{diag}[u^T s_1, \ldots, u^T s_n]$, $N = [n_1, \ldots, n_{n-r}]$.

If $M$ is definite or semidefinite (i.e., $M \geq 0$ or $M \leq 0$), then the quadratic form Eq. (10a) possesses the only solution $\lambda = 0$, while a null motion is impossible at the $\delta$s. If $M$ is indefinite, then the quadratic form Eq. (10a) possesses the nonzero solution and a null motion exists. In details, $p$ is termed the number of positive signatures of the quadratic form. That is an unchanged characteristic presented by Sylvester’s law of inertia. So the equivalent condition is that when $0 < p < rm \leq n$, a null motion is possible, otherwise it is impossible at the $\delta$s. Here $rm = \text{rank}(M)$. Based on geometric theory, another discrimination approach of singularity is given. First reverse all the sign $\varepsilon$ if necessary, so that the number of negative signs is less than positive ones, denoted by $\varepsilon'$. If $\varepsilon'$ are all positive, the singularity surface is impassable by null motions. If more than two signs are negative, the singularity surface is passable. In other cases, the Gaussian curvature $K$ must be calculated. Using the two approaches all together in the program of discriminating singular types can save numerical calculation times.

The PC and FPC of CSCMGs are considered in subsequent discussion. Because of the symmetry of the configurations, singular hypersurfaces which have the same number of negative signs in $\varepsilon$ are similar except the direction of the plot body. Figs. 10 show some computing results.

- **Fig. 1**: Impassable singular surface of PC with $\varepsilon_{i+1+1+1+1}$. 

Fig. 2 Impassable singular surface of PC with $\varepsilon_i{[-+ + +]}

Fig. 3 Passable singular surface of PC with $\varepsilon_i{[-+ + +]}

Fig. 4 Impassable singular surface of PC with $\varepsilon_i{[-- + +]}

Fig. 5 Passable singular surface of PC with $\varepsilon_i{[-- + +]}

Fig. 6 Impassable singular surface of FPC with $\varepsilon_i{[+- + + +]}

Fig. 7 Impassable singular surface of FPC with $\varepsilon_i{[+- + + +]}

Fig. 8 Passable singular surface of FPC with $\varepsilon_i{[-+ + + +]}

Fig. 9 Passable singular surface of FPC with $\varepsilon_i{[-- + + +]}

It is obvious that the null motions found by gradient methods of steering laws are more effective for FPC than for PC from the singular plots. Fig. 7 shows, when the angular momentum is less than $4.27638 h_i$, that there is no elliptic singular state for FPC, and that null motions are always existent.

In the Ref. [12], the so called degenerate null motion problem is briefly mentioned without any illustrated example. In sequence, the problem whether or not the null motions affect the rank of $C$ will be studied. The degenerate hyperbolic points for PC and FPC will be plotted, and the number of the points will be given out.

The measurement of singularity is defined as

$$\kappa_i = \det(C C^T) = \sum_{k=1}^{2\tilde{n}} z_k^2$$  \hspace{1cm} (11)

where $z_k = \det(C_k)$ are the Jacobian minors of order 3, $C_k$ is $C$ with the random $n-3$ columns removed, $\tilde{n} = n(n-1)(n-2)/3!$, $n \geq 3$. If $r = 2$, then $\kappa_i = 0$ and $z_1 = 0$. Expanding $\kappa_i$ about $\delta_s$ by Taylor series yields

$$\kappa_i(\delta_s + \Delta \delta) = \kappa_i(\delta_s) + \sum_{i=1}^{n} \frac{\partial \kappa_i}{\partial \delta_i} |_{\delta_s} \Delta \delta_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j>i}^{n} \frac{\partial^2 \kappa_i}{\partial \delta_i \partial \delta_j} |_{\delta_s} \Delta \delta_i \Delta \delta_j + \ldots$$ \hspace{1cm} (12)

while

$$\frac{\partial \kappa_i}{\partial \delta_i} |_{\delta_s} = \sum_{k=1}^{\tilde{n}} 2 z_k \frac{\partial z_k}{\partial \delta_i} |_{\delta_s} = 0$$

Neglecting high order terms, Eq. (12) is reduced to

$$\kappa_i(\delta_s + \Delta \delta) = \Delta \delta^T H_i \Delta \delta / 2$$ \hspace{1cm} (13)

where $H_i$ is the Hessian matrix of $\kappa_i$ evaluated at $\delta_s$. If the null motions are active, $C \Delta \delta = 0$ and $\Delta \delta = N \lambda$, therefore,

$$\kappa_i(\delta_s + \Delta \delta) = \lambda^T W \lambda / 2 = \lambda^T N^T H_i N \lambda / 2$$ \hspace{1cm} (14)

A sufficient condition for null motions increasing the rank of $C$ is $W > 0$. Denoting $p_w$ as the number of positive signatures of the quadratic form Eq. (14), an equivalent sufficient condition is

$$p_w = r_w = n - r$$ \hspace{1cm} (15)

where $r_w = \text{rank}(W)$.

The element of $H_s$ is evaluated by

$$\frac{\partial^2 \kappa_i}{\partial \delta_i \partial \delta_j} |_{\delta_s} = \sum_{k=1}^{\tilde{n}} \frac{\partial z_k}{\partial \delta_i} \frac{\partial z_k}{\partial \delta_j} |_{\delta_s}$$ \hspace{1cm} (16)

Notice $\tilde{n} = 20$ for FPC and $\tilde{n} = 4$ for PC. Figs. 11-13 show the results of possible degenerate hyperbolic singularities. In Figs. 1, 9, 12, "•" represents a possible degenerate point of PC. Fig. 13 gives all the possible degenerate points at $\varepsilon_{i(- - + + +)}$ combination for FPC. Gauss curvature $\kappa$ is important to geometry study of the singular surfaces. In Ref. [1], the direct way to compute $\kappa$ is given. Moreover here it is verified that all $\kappa$ of possible degenerate points of PC are infinite, while all $\kappa$ for possible degenerate points of FPC are less than 1. For example, all $\kappa$ of $\varepsilon_{i(- - + + +)}$ are presented in the following bracket $\{0.003 472 8, 0.001 329 4, 0.000 869 89, - 0.023 719, - 0.023 719, 0.000 869 89, 0.002 329 4, 0.009 077 3\}$.

![Fig. 11](image-url) One of the possible degenerate points on passable singular surface of PC with $\varepsilon_{i(- - + + +)}$
is the attitude control torque, \( I \) is the
\( L \) \( C \) \( C \) \( u + D \) denote a singular direction of CMG mode, 
\( v_1, v_2 \) \( v_1, v_2 \) \( \in \{ -90^\circ; -90^\circ; -90^\circ; -90^\circ \} \) or
\( \in \{ -90^\circ; 90^\circ; 90^\circ; 90^\circ \} \)
\( \in \{ -90^\circ; -90^\circ; 90^\circ; 90^\circ \} \) or
\( \in \{ -90^\circ; -90^\circ; -90^\circ; -90^\circ \} \) or
\( \in \{ -90^\circ; 90^\circ; -90^\circ; -90^\circ \} \) or
\( \in \{ -90^\circ; 90^\circ; 90^\circ; -90^\circ \} \) or
\( \in \{ -90^\circ; 90^\circ; 90^\circ; 90^\circ \} \)

3 Singularity Analysis of VSCMGs

VSCMGs are usually used in integrated power and attitude control system (IPACS) for spacecrafts\(^9,\,10\). The torque equation for VSCMGs is expressed as

\[
T_{vscmg} = \dot{H}_{vscmg} = C(\Omega, \delta) \dot{\delta} + D(\delta) \dot{\Omega}
\]  

(17)

Here at least two gimbal axes are not parallel to each other, so the column vectors of \( [C \, D] \) always span a three-dimensional space, i.e., rank \([C \, D] = 3\). The norm of the column vectors of matrix \( C \) is much larger than that of matrix \( D \). It is therefore preferable to generate the required torque by changing gimbal angle rather than changing wheel speed. Furthermore, CMG system is more efficient than RW system from the point of view of power\(^13\). Therefore, it is desirable for VSCMGs to work in CMG mode mainly except for the cases where CMGs are near singularities.

In the follows, the work of Ref. \([5]\) is expanded. The failure problems for steering VSCMGs in IPACS are analyzed.

The power and attitude control equation in IPACS with VSCMGs can be written as\(^5\)

\[
\begin{bmatrix}
C(t) \Omega(t) \delta(t) \\
0_{1 \times n}
\end{bmatrix}
\begin{bmatrix}
\dot{\delta}(t) \\
\dot{\Omega}(t)
\end{bmatrix}
= \begin{bmatrix}
C_a \\
P
\end{bmatrix}
\]  

(18)

where \( L_a \) is the attitude control torque, \( P \) is the power.

A null motion exists for the simultaneous attitude and power tracking problem, if and only if there exist \( \delta \) and \( \Omega \) such that\(^5\)

\[
\begin{bmatrix}
C(t) \Omega(t) \delta(t) \\
0_{1 \times n}
\end{bmatrix}
\begin{bmatrix}
\dot{\delta}(t) \\
\dot{\Omega}(t)
\end{bmatrix}
= \begin{bmatrix}
\eta_1 \\
\eta_2
\end{bmatrix}
\]  

(19)

It means that \( \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \in \mathbb{R}^2[ C_a] \). For an arbitrary vector \( v \in \mathbb{R}^2[ C_a] \), the dot product of \( v \) with any column of \( C_a \) is equal to zero. Thus

\[
R^T C_a = \{ v \in \mathbb{R}^2 \mid v_1 \in \mathbb{R}^2[ C_a], v_1^T D + v_2 \Omega^T I_{w1} = 0_{1 \times n} \}
\]  

(20)

Let \( u \) denote a singular direction of CMG mode, then \( R^T C_a = \text{span}[ u] \), and

\[
u_1^T D + v_2 \Omega^T I_{w1} = 0_{1 \times n}
\]  

(21)

Denote \( \Lambda = \begin{bmatrix}
\begin{bmatrix}
u_1^T D \\
\Omega^T I_{w1}
\end{bmatrix}
\end{bmatrix} \). There does not exist a \( v_2 \) satisfying Eq. \((21)\) if \( \text{rank}(\Lambda) = 2 \), i.e., \( \text{dim} R^T C_a = 0 \). Since \( \text{dim} R^T C_a = 4 \), and \( \text{dim} R[ C_a] = 3 \), then a
null motion exists, and a solution of Eq. (18) exists. Noticing that even though rank(C) < 3, it is still possible to escape the CMGs’ singularity using null motion. If rank(A) = 1, there exists a non-zero scalar v denoted by \( a \). From \( v^T \eta = 0 \), one has

\[
[u^T \, \Omega_i^2\Omega_i^2 \, \cdots \, \Omega_i^2\Omega_i^2] = 0
\]

(22)

Here, the fact \( u^T I = 0 \) is used. From Eq. (21), it can be concluded that

\[
I_{wsi} u^T \, s_i = - \, a I_{wsi} \Omega_i \quad i = 1, \ldots, n
\]

Substituting Eq. (23) into Eq. (22) yields

\[
- \, a \left( \sum_{i=1}^n I_{wsi} \Omega_i^2 + \sum_{i=1}^n I_{wsi} \Omega_i^2 \right) = 0
\]

(24)

One can conclude that [\( \Omega \) = 0, i.e., the null motion does not exist. Furthermore, the rank deficiency of \( A \) can occur when all \( \xi_i \) are + 1 or - 1. The latter is not considered in Ref. [5].

Based on the above discussion, the following conclusions can be given. First, rank(C) is equal to 2 or 3 and rank[\( C \, D \)] = 3 for a symmetric VSCMGs configuration like PC and FPC. Second, it is sure that rank(C) = 4 when rank(C) = 3, then the null motions exist and it is possible to prevent configuration singularity using null motion. Third, when rank(C) = 2 and rank(A) = 2, rank(C) is also equal to 4, so that a null motion exists and the singularity can be escaped. Finally, when rank(C) = 2 and rank(A) = 1, rank(C) is equal to 3, and no solution of Eq. (18) exists.

It is obvious that rank(C) = 3 means rank(C) = 2. However, when rank(C) = 2, one doesn’t know if it is possible to guarantee rank(A) = 2 (namely rank(C) = 4) by means of designing VSCMGs’ configuration finely. To understand this problem, rewrite \( A \) as

\[
A = \begin{bmatrix}
I_{wsi} u^T \, s_1 & I_{wsi} u^T \, s_2 & \cdots & I_{wsi} u^T \, s_n \\
I_{wsi} \Omega_1 & I_{wsi} \Omega_2 & \cdots & I_{wsi} \Omega_n
\end{bmatrix}
\]

(25)

No loss of generality, assume \( I_{wsi} \) to be equal to each other. Before IPACS reaching the singular states of CMG mode, rates \( \Omega_i \) of rotors are continuously balanced by the steering law with wheel speed equalization, and VSCMGs work at CMG mode mainly [10]. So it can be ensured that rank(C) = 4 when all \( u^T \, s_i \), are not equal to each other, i.e., the IPACS always has a good performance. Denote \( \Omega_i^* (0 < \Omega_i \leq 180^\circ) \) as the angle between \( g \) and \( u \), then

\[
u^* \, s_i = u \cdot \left( \xi_i \frac{(g_i \times u) \times g_i}{|g_i \times u|} \right) = \xi_i \sin \Omega_i
\]

(26)

A sufficient condition for choosing configuration of VSCMGs to guarantee the IPACS steering is that \( \xi_i \) are not equal to each other. Equivalently the IPACS does not encounter steering singularities, namely rank(C) = 4, if any vector \( u \) in three-dimensional space does not have the same angle with the fixed vector \( g \).

In practice, some magnitudes of angular momentum envelop must be held for CMG mode working normally. Furthermore in consideration of hardware, for example, the maximum power for the motor of wheel and imprecision of control when the rate goes through zero, it is preferable that the wheel speed is within some suitable range. Therefore, in order to find the boundary of the maximum angular momentum workspace, one should solve the following optimal problem. For a given kinetic energy \( E \) and a singular direction \( u \), find the gimbal angles \( \xi \) and wheel speeds \( \Omega \) in the range such that the following performance index is minimized

\[
\min f(\xi, \Omega) = - \sum_{i=1}^n \theta_i I_{wsi} \, \Omega_i
\]

under the restrictions

\[
J_{ui}(\theta) = \tilde{\theta}^2_{\text{max}} - \tilde{\theta}^2_i \geq 0, \quad (i = 1, \ldots, n)
\]

\[
J_{hi}(\Omega) = \Omega_{\text{max}} - \Omega_i \geq 0
\]

\[
J_{ci}(\Omega) = \Omega_i - \Omega_{\text{min}} \geq 0, \quad \Omega_{\text{max}} > 0
\]

\[
l(\Omega) = \sum_{i=1}^n I_{wsi} \Omega_i^2 - 2E = 0
\]

where \( \tilde{\theta}_i = u^T \, s_i \), \( \Omega_{\text{max}} = |g_i \times u| \). Introduce the Lagrangian multipliers \( \lambda_0, \lambda_1, \lambda_0, \lambda_1 \) while define the Lagrangian function \( L \) as

\[
L(\xi, \Omega, \lambda, \Omega_i) = f - \lambda_0 l - \sum_{i=1}^n \lambda_i J_{ui} + \lambda_h J_{hi} + \lambda_c J_{ci}
\]

(28)

The necessary condition for local optimal solution in
every feasible domain can be obtained by the Kuhn-Tucker condition in nonlinear programming. Here consider a numerical example\textsuperscript{[10]}.

Fig. 14 shows its angular momentum envelope with \( E = 187.275 \) J and a wheel speed range 40 000–100 000 r/min. Comparing Fig. 14 with surface \( C \) in Fig. 6 of Ref. \textsuperscript{[5]}, one can see the difference between the two figures. That is, holes and cracks exist only in Fig. 14. The reason shown in Fig. 15 is that \( h_4 \) can not reach zero and \( h_1 \) and \( h_3 \) have upper bounds because of the limit of wheel speed range. It is also proved by Fig. 15 that the holes in Fig. 1 and Fig. 6 are unit circles which connect different singular surfaces. Moreover it can be similarly analyzed and concluded like the Ref. \textsuperscript{[5]}, especially including the situation of all \( \xi = -1 \). For a practical satellite, the angular momentum envelope and the configuration of VSCMGs must be designed in detail based on the requirement of attitude control system.

The failure problems of steering IPACS are also analyzed. A sufficient condition for choosing VSCMGs configurations to guarantee the steering of IPACS is given. The angular momentum envelop of VSCMGs, in a specific energy and a limited range of rotors' speeds, is plotted.

The obtained results are useful to the system analysis and the desig of spacecraft with SCMGs.

**Conclusions**

In this paper, a geometric analysis of singularity for SCMGs is carried out. Through angular momentum hypersurfaces of singular states, the passable and impassable singular points are discriminated, meanwhile the information about how much angular momentum workspace as well as the steering capability available is provided directly. The singular plots show that the null motions of steering laws are more effective for FPC than for PC. The possible degenerate hyperbolic singular points of the preceding configurations are calculated and the distinctness of them is denoted by the Gaussian curvature.

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