Physics Letters B 687 (2010) 405–409

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Large-x structure of physical evolution kernels in deep inelastic scattering

G. Grunberg

Centre de Physique Théorique, École Polytechnique, 91128 Palaiseau Cedex, France

ARTICLE INFO

Article history:
Received 30 November 2009
Received in revised form 10 February 2010
Accepted 9 March 2010
Available online 16 March 2010
Editor: B. Grinstein

Keywords:
QCD
Resummation

ABSTRACT

The modified evolution equation for parton distributions of Dokshitzer, Marchesini and Salam is extended to non-singlet deep inelastic scattering coefficient functions and the physical evolution kernels which govern their scaling violation. Considering the $x \to 1$ limit, it is found that the leading next-to-eikonal logarithmic contributions to the physical kernels at any loop order can be expressed in term of the one-loop cusp anomalous dimension, a result which can presumably be extended to all orders in $(1-x)$, and has eluded so far threshold resummation. Similar results are shown to hold for fragmentation functions in semi-inclusive e^+e^- annihilation. Gribov–Lipatov relation is found to be satisfied by the leading logarithmic part of the modified physical evolution kernels.

© 2010 Elsevier B.V. All rights reserved.

1. Introduction

There has been recently renewed interest [1–9] in threshold resummation of "next-to-eikonal" logarithmically enhanced terms which are suppressed by some power of the gluon energy $(1-x)$ for $x \to 1$ in momentum space (or by some power of $1/N$, $N \to \infty$ in moment space). In particular, in [2,5–9] this question has been investigated at the level of "physical evolution kernels" which control the scaling violation of (non-singlet) structure functions. The scale-dependence of the deep inelastic coefficient function $\mathcal{C}_2(x, Q^2, \mu_F^2)$ corresponding to the flavor non-singlet $F_2(x, Q^2)$ structure function $(F_2(x, Q^2)/x = \mathcal{C}_2(x, Q^2, \mu_F^2) \otimes q_{2,ns}(x, \mu_F^2)$, where $q_{2,ns}(x, \mu_F^2)$ is the corresponding quark distribution) can be expressed in terms of $\mathcal{C}_2(x, Q^2, \mu_F^2)$ itself, yielding the following "physical" evolution equation (see e.g. Refs. [10–12]):

$$\frac{\partial \mathcal{C}_2(x, Q^2, \mu_F^2)}{\partial \ln Q^2} = \int_x^1 \frac{dz}{z} K\big(z, a_s(Q^2)\big)\mathcal{C}_2\big(x/z, Q^2, \mu_F^2\big) \equiv K\big(x, a_s(Q^2)\big) \otimes \mathcal{C}_2\big(x, Q^2, \mu_F^2\big), \tag{1}$$

where μ_F is the factorization scale (I assume for definiteness the $\overline{MS}$ factorization scheme is used). $K(x, a_s(Q^2))$ is the momentum space *physical evolution kernel*, or *physical anomalous dimension*; it is independent of the factorization scale and renormalization scheme invariant.

In [13], the result for the leading contribution to this quantity in the $x \to 1$ limit was derived, which resumes all logarithms at the leading eikonal level, and nicely summarizes analytically in *momentum* space the standard results [14,15] of threshold resummation:

$$K\big(x, a_s(Q^2)\big) \sim \frac{\mathcal{J}(rQ^2)}{r} + B_\delta^{\mathrm{DIS}}\big(a_s(Q^2)\big)\delta(1-x), \tag{2}$$

where $r = \frac{1-x}{x}$ (with $rQ^2 \equiv W^2$ the final state "jet" mass),

$$B_\delta^{\mathrm{DIS}}(a_s) = \sum_{i=1}^{\infty} \Delta_i a_s^i \tag{3}$$

is related to the quark form factor, and $\mathcal{J}(Q^2)$, the "physical Sudakov anomalous dimension" (a renormalization scheme invariant quantity), is given by:

$$\mathcal{J}(Q^2) = A\big(a_s(Q^2)\big) + \beta\big(a_s(Q^2)\big)\frac{dB(a_s(Q^2))}{da_s} \equiv \sum_{i=1}^{\infty} j_i a_s^i(Q^2). \tag{4}$$

In Eq. (4),

$$A(a_s) = \sum_{i=1}^{\infty} A_i a_s^i \tag{5}$$

is the universal "cusp" anomalous dimension [16] (see also [17]), with $a_s \equiv \frac{\alpha_s}{4\pi}$ the $\overline{MS}$ coupling,

$$\beta(a_s) = \frac{da_s}{d\ln Q^2} = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 + \cdots \tag{6}$$

is the beta function (with $\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f$) and

E-mail address: georges.grunberg@cpht.polytechnique.fr.

0370-2693/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.physletb.2010.03.036

$$B(a_s) = \sum_{i=1}^{\infty} B_i a_s^i \tag{7}$$

is the usual final state "jet function" anomalous dimension. It should be noted that $j_1 = A_1$ (the one loop cusp anomalous dimension), and also that both $A(a_s)$ and $B(a_s)$ (in contrast to $\mathcal{J}(Q^2)$) are renormalization scheme-dependent quantities. The renormalization group invariance of $\mathcal{J}(Q^2)$ yields the standard relation:

$$\begin{aligned}\mathcal{J}\big((1-x)Q^2\big) &= j_1 a_s + a_s^2[-j_1\beta_0 L_x + j_2] \\ &\quad + a_s^3\big[j_1\beta_0^2 L_x^2 - (j_1\beta_1 + 2j_2\beta_0)L_x + j_3\big] + \cdots,\end{aligned} \tag{8}$$

where $L_x \equiv \ln(1-x)$ and $a_s = a_s(Q^2)$, from which the structure of all the eikonal logarithms in $K(x, a_s(Q^2))$ can be derived. A term like $\frac{L_x^p}{1-x}$ arising from $\frac{\mathcal{J}(rQ^2)}{r}$ in Eq. (2) must be interpreted as usual as a standard $+$-distribution. *All* the eikonal logarithms are thus absorbed into the *single* scale $(1-x)Q^2$ (see also [18]).

However, no analogous result holds [6] at the next-to-eikonal level. In this Letter, I show that the *leading* next-to-eikonal logarithmic contributions to the physical evolution kernel at a given order in a_s can actually be determined in term of lower order *leading* eikonal coefficients, representing the first step towards threshold resummation at the next-to-eikonal level. This result is obtained by extending the approach of [19,20] (which deals with parton distributions) to the DIS coefficient functions themselves.

2. The modified physical kernel

I consider the class of modified physical evolution equations:

$$\begin{aligned}&\frac{\partial \mathcal{C}_2(x, Q^2, \mu_F^2)}{\partial \ln Q^2} \\ &= \int_x^1 \frac{dz}{z} K\big(z, a_s(Q^2), \lambda\big)\mathcal{C}_2\big(x/z, Q^2/z^\lambda, \mu_F^2\big),\end{aligned} \tag{9}$$

where for book-keeping purposes I introduced the parameter λ, which shall eventually be set to its physically meaningful value $\lambda = 1$, in straightforward analogy to the modified evolution equation for parton distributions of [20]. I note that $K(x, a_s, \lambda = 0) \equiv K(x, a_s)$, the "standard" physical evolution kernel. Eq. (9) allows to determine $K(x, a_s, \lambda)$ given $K(x, a_s)$ (or vice-versa). Indeed, expanding $\mathcal{C}_2(y, Q^2/z^\lambda, \mu_F^2)$ around $z = 1$, keeping the other two variables fixed, and reporting into Eq. (9), one easily derives the following relation between $K(x, a_s, \lambda)$ and $K(x, a_s)$:

$$\begin{aligned}K(x, a_s) &= K(x, a_s, \lambda) - \lambda\big[\ln x\, K(x, a_s, \lambda)\big] \otimes K(x, a_s) \\ &\quad + \frac{\lambda^2}{2}\big[\ln^2 x\, K(x, a_s, \lambda)\big] \\ &\quad \otimes \left[\beta(a_s)\frac{\partial K(x, a_s)}{\partial a_s} + K(x, a_s) \otimes K(x, a_s)\right] + \cdots,\end{aligned} \tag{10}$$

where only terms with a single overall factor of λ need actually to be kept up to next-to-eikonal order, since one can check terms with more factors of λ, which are associated to more factors of $\ln x$, are not relevant to determine the next-to-eikonal logarithms in the physical kernel. Eq. (10) can be solved perturbatively. Setting:

$$\begin{aligned}K(x, a_s, \lambda) &= K_0(x, \lambda)a_s + K_1(x, \lambda)a_s^2 + K_2(x, \lambda)a_s^3 \\ &\quad + K_3(x, \lambda)a_s^3 + \cdots\end{aligned} \tag{11}$$

(and similarly for $K(x, a_s)$), one gets:

$$\begin{aligned}K_0(x, \lambda) &= K_0(x), \\ K_1(x, \lambda) &= K_1(x) + \lambda\big[\ln x\, K_0(x)\big] \otimes K_0(x), \\ K_2(x, \lambda) &= K_2(x) + \lambda\big\{\big[\ln x\, K_1(x)\big] \otimes K_0(x) \\ &\quad + \big[\ln x\, K_0(x)\big] \otimes K_1(x)\big\} + \cdots, \\ K_3(x, \lambda) &= K_3(x) + \lambda\big\{\big[\ln x\, K_2(x)\big] \otimes K_0(x) + \big[\ln x\, K_1(x)\big] \otimes K_1(x) \\ &\quad + \big[\ln x\, K_0(x)\big] \otimes K_2(x)\big\} + \cdots.\end{aligned} \tag{12}$$

The $K_i(x)$'s are determined in term of splitting functions and coefficient functions as follows [12]:

$$\begin{aligned}K_0(x) &= P_0(x), \\ K_1(x) &= P_1(x) - \beta_0 c_1(x), \\ K_2(x) &= P_2(x) - \beta_1 c_1(x) - \beta_0\big(2c_2(x) - c_1^{\otimes 2}(x)\big), \\ K_3(x) &= P_3(x) - \beta_2 c_1(x) - \beta_1\big(2c_2(x) - c_1^{\otimes 2}(x)\big) \\ &\quad - \beta_0\big(3c_3(x) - 3c_2(x) \otimes c_1(x) + c_1^{\otimes 3}(x)\big),\end{aligned} \tag{13}$$

where $P_i(x)$ are the standard $(i+1)$-loop splitting functions, $c_i(x)$ are the i-loop coefficient functions, and $c_1^{\otimes 2}(x) \equiv c_1(x) \otimes c_1(x)$, etc.

Consider now the $x \to 1$ limit. The one-loop splitting function is given by:

$$P_0(x) = A_1 \frac{1}{2} p_{qq}(x) + B_1^\delta \delta(1-x), \tag{14}$$

with $A_1 = 4C_F$, $B_1^\delta = 3C_F$, and (using the notation of [7]):

$$\begin{aligned}\frac{1}{2}p_{qq}(x) &= \frac{1}{1-x} - 1 + \frac{1}{2}(1-x) = \frac{x}{1-x} + \frac{1}{2}(1-x) \\ &= \frac{1}{r} + \frac{1}{2}(1-x).\end{aligned} \tag{15}$$

Moreover [21–23]:

$$P_1(x) = \frac{A_2}{r} + B_2^\delta \delta(1-x) + C_2 L_x + D_2 + \cdots, \tag{16}$$

where [23]:

$$C_2 = A_1^2, \tag{17}$$

and [20,24]:

$$D_2 = A_1\big(B_1^\delta - \beta_0\big). \tag{18}$$

Also:

$$c_1(x) = \frac{c_{11}L_x + c_{10}}{r} + c_1^\delta \delta(1-x) + b_{11}L_x + b_{10} + \cdots \tag{19}$$

with $c_{11} = A_1 = 4C_F$, $c_{10} = -B_1^\delta = -3C_F$, $b_{11} = 0$, $b_{10} = 11C_F$. From Eq. (13) one can derive [6,7] the following expansions for $x \to 1$:

$$\begin{aligned}K_0(x) &= P_0(x) = \frac{k_{10}}{r} + \Delta_1\delta(1-x) + h_{10} + \cdots, \\ K_1(x) &= \frac{k_{21}L_x + k_{20}}{r} + \Delta_2\delta(1-x) + h_{21}L_x + h_{20} + \cdots, \\ K_2(x) &= \frac{k_{32}L_x^2 + k_{31}L_x + k_{30}}{r} + \Delta_3\delta(1-x) + h_{32}L_x^2 + h_{31}L_x \\ &\quad + h_{30} + \cdots, \\ K_3(x) &= \frac{k_{43}L_x^3 + k_{42}L_x^2 + k_{41}L_x + k_{40}}{r} + \Delta_4\delta(1-x) + h_{43}L_x^3 \\ &\quad + h_{42}L_x^2 + h_{41}L_x + h_{40} + \cdots.\end{aligned} \tag{20}$$

3. Leading next-to-eikonal logarithms

3.1. Two loop kernel

From Eqs. (13), (14), (16) and (19) one deduces: $k_{10} = A_1$, $h_{10} = 0$, $\Delta_1 = B_1^\delta = 3C_F$, and $k_{21} = -\beta_0 A_1$, $h_{21} = C_2$, $k_{20} = A_2 - \beta_0 c_{10}$, $h_{20} = D_2 - \beta_0 b_{10}$. Then Eq. (12) yields for $x \to 1$:

$$K_0(x,\lambda) = P_0(x),$$
$$K_1(x,\lambda) = \frac{k_{21}L_x + k_{20}}{r} + \Delta_2\delta(1-x) + \big(h_{21} - \lambda\, k_{10}^2\big)L_x + \mathcal{O}\big(L_x^0\big). \tag{21}$$

Now

$$h_{21}(\lambda) = h_{21} - \lambda k_{10}^2 = C_2 - \lambda A_1^2 = (1-\lambda)A_1^2. \tag{22}$$

Thus, setting $\lambda = 1$, one finds that the leading next-to-eikonal logarithm in $K_1(x,\lambda=1)$ vanishes, yielding the relation:

$$h_{21} = k_{10}^2 = 16C_F^2, \tag{23}$$

which is correct [6,7]. This finding is not surprising: up to two loop, the leading next-to-eikonal logarithm is contributed only by the splitting function, since $b_{11} = 0$ (e.g. $h_{21} = C_2$), and one effectively recovers the results holding [20] for the two loop splitting function. The situation however changes drastically at three loop, where the leading next-to-eikonal logarithm is contributed by the coefficient function rather then the splitting function, and the crucial question is whether the leading next-to-eikonal logarithm still vanishes for $\lambda = 1$.

3.2. Three loop kernel

Eq. (12) yields for $x \to 1$:

$$K_2(x,\lambda) = \frac{k_{32}L_x^2 + k_{31}L_x + k_{30}}{r} + \Delta_3\delta(1-x) + \left(h_{32} - \lambda\frac{3}{2}k_{21}k_{10}\right)L_x^2 + \mathcal{O}(L_x). \tag{24}$$

Requiring the coefficient of the $\mathcal{O}(L_x^2)$ term to vanish for $\lambda = 1$ predicts:

$$h_{32} = \frac{3}{2}k_{21}k_{10} = -\frac{3}{2}\beta_0 A_1^2 = -24\beta_0 C_F^2, \tag{25}$$

which is indeed the correct [6,7] value. I stress that this result is *not* a consequence of the relation [25,20,24] $C_3 = 2A_1A_2$ for $P_2(x)$. Indeed it is well known [26] that the $P_i(x)$'s, and in particular $P_2(x)$, have only a *single* next-to-eikonal logarithm:

$$P_2(x) = \frac{A_3}{r} + B_3^\delta\delta(1-x) + C_3L_x + D_3 + \cdots, \tag{26}$$

and thus $P_2(x)$ cannot contribute to the *double* logarithm in $K_2(x)$. Rather, h_{32} is contributed by the coefficient functions in Eq. (13), and Eq. (25) yields a prediction for the $\mathcal{O}(L_x^2)$ term in $c_2(x)$.

3.3. Four loop kernel

Eq. (12) yields for $x \to 1$:

$$K_3(x,\lambda) = \frac{k_{43}L_x^3 + k_{42}L_x^2 + k_{41}L_x + k_{40}}{r} + \Delta_4\delta(1-x) + \left[h_{43} - \lambda\left(\frac{4}{3}k_{10}k_{32} + \frac{1}{2}k_{21}^2\right)\right]L_x^3 + \mathcal{O}\big(L_x^2\big), \tag{27}$$

where $k_{32} = A_1\beta_0^2$ (consistently with Eq. (8)). Requiring the coefficient of the $\mathcal{O}(L_x^3)$ term to vanish for $\lambda = 1$ predicts:

$$h_{43} = \frac{4}{3}k_{10}k_{32} + \frac{1}{2}k_{21}^2 = \frac{11}{6}\beta_0^2A_1^2 = \frac{88}{3}\beta_0^2C_F^2, \tag{28}$$

which is again the correct [6,7] value.

3.4. Five loop kernel

Finally, one can similarly predict the leading next-to-eikonal logarithm in the five loop physical kernel (which depends on the four loop coefficient function). The coefficient of the $\mathcal{O}(L_x^4)$ term in $K_4(x,\lambda)$ is found to be given by:

$$h_{54}(\lambda) = h_{54} - \lambda\left(\frac{5}{4}k_{10}k_{43} + \frac{5}{6}k_{21}k_{32}\right), \tag{29}$$

where $k_{43} = -A_1\beta_0^3$. Requiring this coefficient to vanish for $\lambda = 1$ predicts:

$$h_{54} = \frac{5}{4}k_{10}k_{43} + \frac{5}{6}k_{21}k_{32} = -\frac{25}{12}\beta_0^3A_1^2 = -\frac{100}{3}\beta_0^3C_F^2. \tag{30}$$

One can further show [27], going to Mellin space, that the previous results can be derived from the resummation formula:

$$\sum_{i=0}^{\infty} h_{i+1,i}L_x^i a_s^{i+1} = \frac{A_1^2}{\beta_0}\frac{a_s}{1+a_s\beta_0L_x}\ln(1+a_s\beta_0L_x). \tag{31}$$

4. Leading next-to-next-to-eikonal logarithms

It can be checked [27] that similar methods allow to predict using Eq. (10) the *leading* logarithmic contributions at the next-to-next-to-eikonal level, i.e. the coefficient of the $(1-x)L_x^i$ term in $K_i(x)$. The crucial new point, however, is that the leading term in the eikonal expansion has to be defined in term of the one-loop splitting function prefactor $p_{qq}(x)$ (Eq. (15)), instead of $1/r$ as in Eq. (20). Namely, keeping only leading logarithms at each eikonal order, the predicted $f_{i+1,i}$ coefficients ($i \geqslant 0$) are defined by:

$$K_i(x)\big|_{\rm LL} = L_x^i\left[\frac{1}{2}p_{qq}(x)k_{i+1,i} + h_{i+1,i} + (1-x)f_{i+1,i} + \mathcal{O}\big((1-x)^2\big)\right]. \tag{32}$$

Then, assuming the corresponding $f_{i+1,i}(\lambda)$ coefficients in $K_i(x,\lambda)$ vanish for $\lambda = 1$, one derives from Eq. (10) the relations:

$$f_{10} = 0,$$
$$f_{21} = h_{10}k_{10} - \frac{1}{2}k_{10}^2 = -8C_F^2,$$
$$f_{32} = \frac{1}{2}h_{10}k_{21} - \frac{3}{4}k_{10}k_{21} + k_{10}h_{21} - \frac{1}{2}k_{10}^3 = 12C_F^2\beta_0 + 32C_F^3,$$
$$f_{43} = \frac{1}{3}h_{10}k_{32} - \frac{2}{3}k_{10}k_{32} + \frac{1}{2}\left(h_{21} - \frac{1}{2}k_{21}\right)k_{21} + k_{10}h_{32} - k_{10}^2k_{21} = -\frac{44}{3}C_F^2\beta_0^2 - 64C_F^3\beta_0, \tag{33}$$

which are seen to be correct using Eq. (3.26) in [7]. The latter equation also makes it likely that similar leading logarithmic predictions can be obtained to any order in $(1-x)$, using the *same* prefactor $\frac{1}{2}p_{qq}(x)$ as in Eq. (32) to define the leading term in the eikonal expansion.

5. Fragmentation functions in e^+e^- annihilation

Similar results hold for physical evolution kernels associated to fragmentation functions in semi-inclusive e^+e^- annihilation (SIA), provided one sets $\lambda = -1$ in the analogue of Eq. (9):

$$\frac{\partial \mathcal{C}_T(x, Q^2, \mu_F^2)}{\partial \ln Q^2} = \int_x^1 \frac{dz}{z} K_T\big(z, a_s(Q^2), \lambda\big)\mathcal{C}_T\big(x/z, Q^2/z^\lambda, \mu_F^2\big), \tag{34}$$

where $\mathcal{C}_T$ denotes a generic non-singlet SIA coefficient function.

I first note that threshold resummation in this case [29] leads at the leading eikonal level to an equation similar to Eq. (2):

$$K_T\big(x, a_s(Q^2)\big) \sim \frac{\mathcal{J}(rQ^2)}{r} + B_\delta^{\rm SIA}\big(a_s(Q^2)\big)\delta(1-x), \tag{35}$$

where $r = \frac{1-x}{x}$ (with x now being identified to Feynman-x rather then Bjorken-x), and I used the results of [30] which imply that the "physical Sudakov anomalous dimension" $\mathcal{J}(Q^2)$ is *the same* for structure and fragmentation functions. The statement above Eq. (34) then follows from the following two observations:

(i) The predictions in Eqs. (23), (25), (28) and (30) depend only upon coefficients of *leading* eikonal logarithms in the physical evolution kernels.

(ii) Eq. (3.26) in [7] shows that the latter coefficients are *identical* for deep-inelastic structure functions and for e^+e^- fragmentation functions (consistently with the remark below Eq. (35)), but that the coefficients of the leading *next-to-eikonal* logarithms are equal only up to a sign change (in an expansion in $1/r$) between deep-inelastic structure functions and fragmentation functions.

One deduces the following prediction for the five loop physical evolution kernel of semi-inclusive e^+e^- annihilation (SIA) (which involves four loop coefficient functions):

$$h_{54}|_{\rm SIA} = -\left(\frac{5}{4}k_{10}k_{43} + \frac{5}{6}k_{21}k_{32}\right)\Big|_{\rm DIS} = +\frac{100}{3}\beta_0^3 C_F^2. \tag{36}$$

6. Conclusion

A modified[1] evolution equation for DIS non-singlet structure functions, analogous to the one used in [20] for parton distributions, but which deals with the *physical* scaling violation and coefficient functions, has been proposed. It allows to relate the leading next-to-eikonal logarithmic contributions in the momentum space physical evolution kernel to coefficients of leading eikonal logarithms at lower loop order (depending only upon the one-loop cusp anomalous dimension A_1), which represents the first step towards threshold resummation at the next-to-eikonal level. This result also explains the observed [6,7] *universality* of the leading next-to-eikonal logarithmic contributions to the physical kernels of the various non-singlet structure functions, linking them to the known [31] universality of the eikonal contributions. Similar results hold at the next-to-next-to-eikonal level with a proper definition of the leading eikonal piece, and can presumably be extended to leading logarithmic contributions at all orders in $(1-x)$. Analogous results are obtained for fragmentation functions in semi-inclusive e^+e^- annihilation.

[1] Evolution equations involving similar kinematical rescaling factors have been suggested in the past: see e.g. Eq. (3.2) in [18].

One may ask to what extent the success of the present approach may be attributed, as in the parton distribution case [20,28], to the *classical nature* [32] of soft radiation. In fact, the main result of this Letter for the (modified) DIS physical evolution kernel can be summarized (barring the δ-function contribution) by the following equation:

$$K(x, a_s, \lambda = 1) \sim \left[\frac{x}{1-x} + \frac{1}{2}(1-x)\right]\mathcal{J}\big((1-x)Q^2\big) + \text{subleading logarithms}, \tag{37}$$

where the second term (the "subleading logarithms") is contributed by all powers in $(1-x)$ except the leading eikonal one. The first term in Eq. (37) accounts for the leading logarithmic contributions to the modified kernel (together with some subleading logarithms) to *all* powers in $(1-x)$ at any given loop order, and implies leading logarithmic contributions are actually absent beyond $\mathcal{O}(1-x)$ power. This term has the remarkable effective one-loop splitting function form $4C_F a_{\rm eff}((1-x)Q^2)\frac{1}{2}p_{qq}(x)$, with $a_{\rm eff}(Q^2) \equiv \frac{1}{4C_F}\mathcal{J}(Q^2)$.

As pointed out in [28], the $\frac{x}{1-x}$ part of the one-loop prefactor (Eq. (15)) should be interpreted as corresponding to universal classical radiation, a QCD manifestation of the Low–Burnett–Kroll theorem, while the $1-x$ part represents a genuine quantum contribution. Now, it is clear that at the next-to-eikonal level, the $1-x$ part of the prefactor is irrelevant: only the "classical" $1/r$ part is required to separate those leading logarithms in the standard ($\lambda = 0$) physical evolution kernel which are correctly predicted in the present approach (the $h_{i+1,i}$ in Eq. (32)), hence "inherited" in the sense of [28], from the "primordial" ones (those which at each loop order carry the *same* color factors as the leading $\mathcal{O}(1/(1-x))$ eikonal logarithms, and can thus be absorbed into the definition of the leading term). However, it appears from the results of Section 4 that, starting at next-to-next-to-eikonal level, the full one-loop prefactor has to be used into the definition of the leading term, and thus both the "classical" and the "quantum" parts of the prefactor are on an equal footing to properly isolate the "inherited" next-to-next-to-eikonal logarithms (the $f_{i+1,i}$ in Eq. (32)).

It can be further checked [27] that the very same first term in Eq. (37) also accounts for the leading logarithmic contributions to the $\lambda = -1$ modified SIA physical evolution kernel to *all* powers in $(1-x)$, which implies that the *leading logarithmic parts* of the *modified* DIS and SIA physical evolution kernels satisfy Gribov–Lipatov relation [33], namely we have:

$$K(x, a_s, \lambda = 1)|_{\rm LL} = K_T(x, a_s, \lambda = -1)|_{\rm LL} = \frac{1}{2}p_{qq}(x)\mathcal{J}\big((1-x)Q^2\big)\big|_{\rm LL}, \tag{38}$$

where $\mathcal{J}((1-x)Q^2)|_{\rm LL} = A(a_s((1-x)Q^2))|_{\rm LL} = \frac{A_1 a_s(Q^2)}{1+a_s(Q^2)\beta_0 L_x}$ is the leading logarithmic contribution to Eq. (8). Indeed, once transformed back to the standard ($\lambda = 0$) physical kernels, Eq. (38) is consistent with Eq. (3.26) in [7] at least to next-to-next-to-eikonal order, and is probably correct to all orders in $(1-x)$ (with identically vanishing contributions beyond $\mathcal{O}(1-x)$ order). On the other hand, contrary to the splitting functions case [34,28], a full Gribov–Lipatov relation $K(x, a_s, \lambda = 1) = K_T(x, a_s, \lambda = -1)$ does not seem to hold beyond the leading eikonal level.

The resummation of the subleading logarithmic contributions at next-to-eikonal order in Eq. (37), not addressed here, remains an open issue.

Acknowledgements

I thank G. Marchesini for stimulating discussions and hospitality at the University of Milano–Bicocca where part of this Letter has been written. I also wish to thank the referees for constructive suggestions.

References

[1] M. Kramer, E. Laenen, M. Spira, Nucl. Phys. B 511 (1998) 523, arXiv:hep-ph/9611272.
[2] G. Grunberg, arXiv:0710.5693 [hep-ph].
[3] E. Laenen, L. Magnea, G. Stavenga, Phys. Lett. B 669 (2008) 173, arXiv:0807.4412 [hep-ph].
[4] E. Laenen, G. Stavenga, C.D. White, JHEP 0903 (2009) 054, arXiv:0811.2067 [hep-ph].
[5] S. Moch, A. Vogt, JHEP 0904 (2009) 081, arXiv:0902.2342 [hep-ph].
[6] G. Grunberg, V. Ravindran, JHEP 0910 (2009) 055, arXiv:0902.2702 [hep-ph].
[7] S. Moch, A. Vogt, JHEP 0911 (2009) 099, arXiv:0909.2124 [hep-ph].
[8] G. Grunberg, arXiv:0910.3894 [hep-ph].
[9] G. Soar, A. Vogt, S. Moch, J. Vermaseren, arXiv:0912.0369 [hep-ph].
[10] W. Furmanski, R. Petronzio, Z. Phys. C 11 (1982) 293.
[11] S. Catani, Z. Phys. C 75 (1997) 665, hep-ph/9609263.
[12] W.L. van Neerven, A. Vogt, Nucl. Phys. B 603 (2001) 42, arXiv:hep-ph/0103123.
[13] E. Gardi, G. Grunberg, Nucl. Phys. B 794 (2008) 61, arXiv:0709.2877 [hep-ph].
[14] G. Sterman, Nucl. Phys. B 281 (1987) 310.
[15] S. Catani, L. Trentadue, Nucl. Phys. B 327 (1989) 323.
[16] G.P. Korchemsky, G. Marchesini, Phys. Lett. B 313 (1993) 433.
[17] S. Catani, B.R. Webber, G. Marchesini, Nucl. Phys. B 349 (1991) 635.
[18] D. Amati, A. Bassetto, M. Ciafaloni, G. Marchesini, G. Veneziano, Nucl. Phys. B 173 (1980) 429.
[19] Y.L. Dokshitzer, V.A. Khoze, S.I. Troian, Phys. Rev. D 53 (1996) 89, arXiv:hep-ph/9506425.
[20] Yu.L. Dokshitzer, G. Marchesini, G.P. Salam, Phys. Lett. B 634 (2006) 504, arXiv:hep-ph/0511302.
[21] J. Kodaira, L. Trentadue, Phys. Lett. B 112 (1982) 66.
[22] E.G. Floratos, D.A. Ross, C.T. Sachrajda, Nucl. Phys. B 129 (1977) 66;
E.G. Floratos, D.A. Ross, C.T. Sachrajda, Nucl. Phys. B 139 (1978) 545, Erratum.
[23] G. Curci, W. Furmanski, R. Petronzio, Nucl. Phys. B 175 (1980) 27.
[24] B. Basso, G.P. Korchemsky, Nucl. Phys. B 775 (2007) 1, arXiv:hep-th/0612247.
[25] S. Moch, J.A.M. Vermaseren, A. Vogt, Nucl. Phys. B 688 (2004) 101, hep-ph/0403192.
[26] G.P. Korchemsky, Mod. Phys. Lett. A 4 (1989) 1257.
[27] G. Grunberg, On the large-x structure of physical evolution kernels, in preparation.
[28] Yu.L. Dokshitzer, G. Marchesini, Phys. Lett. B 646 (2007) 189, arXiv:hep-th/0612248.
[29] M. Cacciari, S. Catani, Nucl. Phys. B 617 (2001) 253, arXiv:hep-ph/0107138.
[30] S. Moch, A. Vogt, Phys. Lett. B 680 (2009) 239, arXiv:0908.2746 [hep-ph].
[31] S. Moch, J.A.M. Vermaseren, A. Vogt, Nucl. Phys. B 813 (2009) 220, arXiv:0812.4168 [hep-ph].
[32] F.E. Low, Phys. Rev. 110 (1958) 974;
T.H. Burnett, N.M. Kroll, Phys. Rev. Lett. 20 (1968) 86.
[33] V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438;
V.N. Gribov, L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 675.
[34] A. Mitov, S. Moch, A. Vogt, Phys. Lett. B 638 (2006) 61, arXiv:hep-ph/0604053.