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Parity violation in hot QCD: Why it can happen, and how to look for it

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Abstract

The arguments for the possibility of violation of $\mathcal{P}$ and $\mathcal{CP}$ symmetries of strong interactions at finite temperature are presented. A new way of observing these effects in heavy ion collisions is proposed—it is shown that parity violation should manifest itself in the asymmetry between positive and negative pions with respect to the reaction plane. Basing on topological considerations, we derive a *lower* bound on the magnitude of the expected asymmetry, which may appear within the reach of the current and/or future heavy ion experiments.

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The strong $\mathcal{CP}$ problem remains one of the most outstanding puzzles of the Standard Model. Even though several possible solutions have been put forward (for example, the axion scenario [1]), at present it is still not clear why $\mathcal{P}$ and $\mathcal{CP}$ invariances are respected by strong interactions.

A few years ago, it was proposed that in the vicinity of the deconfinement phase transition QCD vacuum can possess metastable domains leading to $\mathcal{P}$ and $\mathcal{CP}$ violation [2]. It was also suggested that this phenomenon would manifest itself in specific correlations of pion momenta [2,3]. Such "$\mathcal{P}$-odd bubbles" are a particular realization of an excited vacuum domain which may be produced in heavy ion collisions [4], and several other realizations have been proposed before [5,6]. (For related studies of metastable vacuum states, especially in supersymmetric theories, see [7–9].) However the peculiar pattern of $\mathcal{P}$ and $\mathcal{CP}$ breaking possessed by $\mathcal{P}$-odd bubbles may make them amenable to observation, as we will discuss in this letter.

The existence of metastable $\mathcal{P}$-odd bubbles does not contradict the Vafa–Witten theorem [10] stating that $\mathcal{P}$ and $\mathcal{CP}$ cannot be broken in the true ground state of QCD for $\theta = 0$. Moreover, this theorem does not apply to QCD matter at finite isospin density [11] and finite temperature [12], where Lorentz-noninvariant $\mathcal{P}$-odd operators are allowed to have nonzero expectation values. Degenerate vacuum states with opposite parity were found [13] in the superconducting phase of QCD. Parity broken phase also exists in lattice QCD with Wilson fermions [14], but this phenomenon has been recognized as a lattice artifact for the case of mass-degenerate quarks; spontaneous $\mathcal{P}$ and $\mathcal{CP}$ breaking similar to the Dashen's phenomenon [15] can however occur for nonphysical values of quark masses [16]. $\mathcal{P}$-even, but $\mathcal{C}$-odd metastable states have also been argued to exist in hot gauge theories [17]. The conditions for the applicability of Vafa–Witten theorem have been repeatedly re–examined in recent years [18].

Several dynamical scenarios for the decay of $\mathcal{P}$-odd bubbles have been considered [19], and a numerical lattice calculation of the fluctuations of topological charge in classical Yang–Mills fields has been performed [20]. The studies of $\mathcal{P}$- and $\mathcal{CP}$-odd correlations of pion momenta [21,22], including those proposed in Ref. [23], have shown that such measurements are in principle feasible but would require large event samples. In addition, the magnitude of the expected effect despite the estimates done using the chiral Lagrangian approach [3] and a quasi-classical color field model [24] remained somewhat uncertain.

In this Letter, we will give additional arguments in favor of $\mathcal{P}$- and $\mathcal{CP}$-breaking in a domain of a highly excited vacuum state. A new way of observing $\mathcal{P}$-odd effects in experiment through the asymmetry in the production of charged pions with respect to the reaction plane will then be proposed. It appears

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that the magnitude of the expected asymmetry can be estimated on the basis of topological considerations alone, and that the effect may be amenable to observation in the existing and/or future heavy ion experiments.

Let us begin with a brief introduction to the strong $\mathcal{CP}$ problem. Strong interactions within the Standard Model are described by quantum chromo-dynamics, with the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_\alpha^{\mu\nu} F_{\alpha\mu\nu} + \sum_f \bar\psi_f \big[i\gamma^\mu(\partial_\mu - igA_{\alpha\mu}t_\alpha) - m_f \big]\psi_f, \tag{1}$$

where $F_\alpha^{\mu\nu}$ and $A_{\alpha\mu}$ are the color field strength tensor and vector potential, respectively, g is the strong coupling constant, ψ_f are the quark fields of different flavors f with masses m_f, and t_α the generators of the color $SU(3)$ group in the fundamental representation. The Lagrangian (1) is symmetrical with respect to space parity $\mathcal{P}$ and charge conjugation parity $\mathcal{C}$ transformations.

However, these classical symmetries of QCD become questionable due to the interplay of quantum axial anomaly [25] and classical topologically nontrivial solutions—the instantons [26]. The axial anomaly arises due to the fact that the renormalization of the theory (1) cannot be performed in a chirally invariant way. As a result the flavor-singlet axial current $J_{\mu 5} = \bar\psi_f \gamma_\mu \gamma_5 \psi_f$ is no longer conserved even in the $m \to 0$ limit:

$$\partial^\mu J_{\mu 5} = 2m_f i \bar\psi_f \gamma_5 \psi_f - \frac{N_f g^2}{16\pi^2} F_\alpha^{\mu\nu} \tilde F_{\alpha\mu\nu}, \tag{2}$$

where $\tilde F_{\alpha\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F^{\alpha\rho\sigma}$. The last term in (2) is seemingly irrelevant since it can be written down as a full divergence, $F_\alpha^{\mu\nu}\tilde F_{\alpha\mu\nu} = \partial_\mu K^\mu$, of the (gauge-dependent) topological gluon current $K^\mu = \epsilon^{\mu\nu\rho\sigma} A_{\alpha\nu}[F_{\alpha\rho\sigma} - \frac{g}{3} f_{\alpha\beta\gamma} A_{\beta\rho} A_{\gamma\sigma}]$. However this conclusion is premature due to the existence of instantons which induce a change in the value of the chiral charge $Q_5 = \int d^3x\, K^0$ associated with the topological current between $t = -\infty$ and $t = +\infty$: $\nu = \int_{-\infty}^{+\infty} dt\, \frac{dQ_5}{dt} = 2N_f q[F]$, where $q[F] = \frac{g^2}{32\pi^2} \int d^4x\, F_\alpha^{\mu\nu} \tilde F_{\alpha\mu\nu}$ is the topological charge; for a one-instanton solution, $q = +1$.

In the presence of degenerate topological vacuum sectors, an expectation value of an observable $\mathcal{O}$ has to be evaluated by first computing an average $\int_q D[\psi]D[\bar\psi]D[A] \exp(iS_{\mathrm{QCD}}) \times \mathcal{O}(\psi, \bar\psi, A)$ over a sector with a fixed topological charge q, and then by summing over all sectors with the weight $f(q)$ [27]. The additivity constraint $f(q_1 + q_2) = f(q_1) f(q_2)$ restricts the weight to the form $f(q) = \exp(i\theta q)$, where θ is a free parameter. Recalling an explicit expression $q[F] = \frac{g^2}{32\pi^2} \int d^4x\, F_\alpha^{\mu\nu} \tilde F_{\alpha\mu\nu}$ one can see that this procedure is equivalent to adding to the QCD Lagrangian (1) $S_{\mathrm{QCD}} = \int d^4x\, \mathcal{L}_{\mathrm{QCD}}$ a new term

$$\mathcal{L}_\theta = -\frac{\theta}{32\pi^2} g^2 F_\alpha^{\mu\nu} \tilde F_{\alpha\mu\nu}. \tag{3}$$

Unless θ is identically equal to zero, $\mathcal{P}$ and $\mathcal{CP}$ invariances of QCD are lost.

One can eliminate the "θ-term" (3) (but not $\mathcal{CP}$ violation itself) by a redefinition of the quark fields through the chiral rotation $\psi_f \to \exp(i\gamma_5\theta_f/2)\, \psi_f$ with real phases $\theta = \sum_f \theta_f$. Indeed, because of the axial anomaly (2), this is equivalent to the replacement

$$\theta \to \theta + \sum_f \theta_f \tag{4}$$

so that the term (3) can be eliminated at the cost of introducing complex quark masses. Introducing the left- and right-handed quark fields $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$, $\psi_R = \frac{1}{2}(1+\gamma_5)\psi$, we can write the quark mass term of (1) in the following form

$$\mathcal{L}_{\mathrm{quark}} = -\sum_f \big(\hat m_f \bar\psi_{L,f} \psi_{R,f} + \hat m_f^* \bar\psi_{R,f} \psi_{L,f} \big), \tag{5}$$

where the real masses m_f from the Lagrangian (1) have been replaced by complex mass parameters $\hat m_f = m_f \exp(i\theta_f)$. Because of (4), all $\mathcal{CP}$-violating phase can be attributed to a single quark flavor, say u, so that $\theta = \theta_u$, $\theta_d = \theta_s = 0$. Therefore if at least one of the quarks is massless, the $\mathcal{CP}$-violating phase would not have any observable effect. From now on, we will rotate for simplicity all $\mathcal{CP}$ violating phase into the "up" quark of mass $m \equiv m_u$; this does not lead to any loss of generality. We would like to emphasize again that quark masses are absolutely essential in the strong $\mathcal{CP}$ violation—this will be important in what follows.

The complex mass parameters in (5) can be treated as "spurion" fields [28], with an insertion of $\hat m$ flipping left quarks into right, and vice versa for $\hat m^*$. This "spurion" field is associated with a canonical chiral charge operator

$$\Delta Q_5 = 2\left(\hat m^* \frac{\partial}{\partial \hat m^*} - \hat m \frac{\partial}{\partial \hat m} \right) = 2i \frac{\partial}{\partial \theta}. \tag{6}$$

The parity-odd effect of the complex mass parameters inducing the difference between the left- and right-handed fermions can be made completely manifest by re-writing the u quark part of (5) as

$$\mathcal{L}_\theta = -m\cos\theta (\bar u_L u_R + \bar u_R u_L) - im \sin\theta (\bar u_L u_R - \bar u_R u_L). \tag{7}$$

Parity violation in strong interactions has been never detected, and stringent limits exist on the value of $\mathcal{CP}$ violating phase $\theta < 3 \times 10^{-10}$. This means that in the physical vacuum the "spurion" field $\hat m = m\exp(i\theta)$ has a real expectation value determined by the quark masses $\langle \hat m \rangle = m$. Because $\hat m$ and θ cannot have any space–time dependence in the physical vacuum, the "spurion" field does not carry any energy or momentum.

The metastable $\mathcal{P}$ and $\mathcal{CP}$ odd state of Ref. [2] acts as a localized in space and time vacuum domain with $\theta = \theta(\mathbf{x}, t) \neq 0$; the space–time dependence of θ and thus of $\hat m(\mathbf{x}, t) = m\exp(i\theta(\mathbf{x}, t))$ implies that the chiral charge operator (6) no longer commutes with the operator of momentum and the Hamiltonian. Therefore the field $\hat m$ can now scatter quarks and create quark–antiquark pairs with nonzero chirality. What is the definition of chirality in this situation? This question is not trivial since as we have seen above parity violation in QCD is possible only if all quark masses are different from zero, and the definition of chirality for a massive fermion is not Lorentz invariant and depends on the frame.

Let us discuss this in more detail. Consider the second term in (7) which is responsible for parity violation; in terms of the two-component spinors χ and Pauli spin matrices σ it involves

$$\chi^{+}\sigma(\mathbf{n}-\mathbf{n}')\chi, \tag{8}$$

where $\mathbf{n}=\mathbf{p}/p$ is the unit vector in the direction of the quark momentum $\mathbf{p}$, and we have assumed that the quark energy $E \gg m$. In the vacuum, the "spurion" field $\hat{m}$ carries no energy or momentum, so the interaction of quarks with spurions leaves $\mathbf{p}=\mathbf{p}'$, $\mathbf{n}=\mathbf{n}'$. This means that the chirality change is possible only through the flip of the spin of the quark, which changes the sign of the spin projection on the momentum, so that $\langle\sigma\mathbf{n}\rangle_i = -\langle\sigma\mathbf{n}\rangle_f$.

Consider now a domain of excited QCD vacuum with $\theta = \theta(\mathbf{x},t)$; the "spurion" field associated with it can now transfer energy and momentum to the quarks, so that $\mathbf{p} \neq \mathbf{p}'$ in the quark–spurion interaction vertex. Moreover, the rest frame of the domain defines a preferred reference frame in which the chirality of the massive quark is to be measured. If the domain is axially symmetric, and $\theta = \theta(r,\Omega)$ depends only on the polar angle Ω and not on the azimuthal angle ϕ (which as we will soon see is the case for QCD matter produced in heavy ion collisions), this symmetry by Wigner–Eckart theorem defines the appropriate quantization axis for the quark spin σ. Such a domain can generate chirality not by flipping the spins of the quarks, but by inducing up–down asymmetry (as measured with respect to the symmetry axis) in the production of quarks and antiquarks.

Formally, this happens because the operator of chiral charge (6), corresponding to the rotation in the θ space, in this case commutes with the operator of rotations $-i\frac{\partial}{\partial\phi}$ in azimuthal angle, but not with rotations in polar angle Ω. If the spins of the quark and antiquark are aligned parallel to the symmetry axis of the domain, "right" quark would refer to the quark emitted in the upper hemisphere (along the direction of the symmetry axis, with $\sigma\mathbf{n} > 0$), and vice versa for the "left" antiquark. Therefore, a domain with $\theta = \theta(\mathbf{x},t)$ can generate *spatial* asymmetry in the production of $\bar{u}u$ and other quark pairs. In terms of the observable charged pions, this would mean that positive and negative pions will be produced asymmetrically with respect to the symmetry axis. Because of the overall charge conservation, this implies that there will be more positive than negative pions in the upper hemisphere, and more negative than positive pions in the lower hemisphere (the sign of the asymmetry is of course determined by the sign of the topological chiral charge of the domain).

The spatial separation of positive and negative charges will induce an electric dipole moment (e.d.m.) in the system, which is a clear signature of $\mathcal{CP}$ violation. Searching for the fluctuations of θ angle through the spatial separation of electric charges in the hot quark–gluon fireball is analogous to the proposal of constraining the value of θ in the vacuum by measuring the e.d.m. of the neutron [29]. In the framework of the chiral Lagrangian description [29], the spatial asymmetry of the pion cloud around the neutron is caused by the $\mathcal{P}$-odd πN coupling. Recently, the phenomenon of the spatial separation of quarks with different electric charges at finite θ has also been demonstrated in the framework of the instanton liquid model [30].

Would a θ domain produced in a heavy ion collision have a symmetry axis? Consider two symmetrical heavy ions with mass number A colliding with the center-of-mass energy $\sqrt{s}$ per nucleon pair, at an impact parameter b. In the c.m.s. frame the initial angular momentum of this system is $L \approx A|[\mathbf{b}\times\mathbf{p}]| \simeq Ab\sqrt{s}/2$. With $\sqrt{s} = 200$ GeV (the energy of the RHIC collider), we have $L \simeq A/2b[\mathrm{fm}] \times 10^3$ units of angular momentum in the system. After the collision, part of this angular momentum is carried away from the produced fireball by the "spectator" nucleons, but it is clear that the produced matter must have thousands of units of angular momentum. This angular momentum is pointing perpendicular to the reaction plane, which can be reconstructed both by detecting the directions of forward fragments in the fragmentation regions on both sides, and by studying the particle correlations at mid-rapidity region. The angular momentum vector provides us with the symmetry axis discussed above. Moreover, we can now supplement our arguments with a simple semi-classical picture: rotating deconfined color charges generate chromo-magnetic field $\mathbf{H}$ parallel to the angular momentum vector, and the quarks spins align along $\mathbf{H}$.

What is the magnitude of the expected effect? Fortunately we can estimate it without invoking any models for the $\mathcal{CP}$-odd domain structure. Let us choose the polar axis along the vector of angular momentum; the distribution N_+ (N_-) of the produced u ($\bar{u}$) quarks in the polar angle Ω according to (7), (8) will then be given by

$$\frac{dN_\pm}{d\Omega} = \mathrm{const}\cdot(1 \pm \kappa\cos\Omega)\sin\Omega. \tag{9}$$

As usual, the $\mathcal{CP}$-odd term in (9) appears due to the interference of $\mathcal{CP}$ breaking term (8) with the $\mathcal{CP}$ even terms. Because of this, and because most of the quarks will be produced by parity-conserving interactions, one cannot evaluate the constant κ in (9) from (7) alone. Moreover, the dynamics of the collision will severely affect the shape of the distribution, adding parity–even harmonics to (9). Nevertheless, since (7) is the only source of parity violation and all other interactions conserve parity, the up–down asymmetry in the production of u quarks defined as

$$\begin{aligned} A_u &= \frac{N_R - N_L}{N_R + N_L} \\ &= \left(\int_0^{\pi} \frac{dN_+}{d\Omega}\right)^{-1} \left(\int_0^{\pi/2} \frac{dN_+}{d\Omega} - \int_{\pi/2}^{\pi} \frac{dN_+}{d\Omega}\right), \end{aligned} \tag{10}$$

will be preserved in the subsequent evolution of the system. Obviously, the asymmetry for $\bar{u}$ antiquarks will be $A_{\bar{u}} = -A_u = -\kappa/2$. The asymmetry between u and $\bar{u}$ quarks (10) is not directly observable; however if the hadronization process preserves $\mathcal{P}$ and $\mathcal{CP}$, it should translate into the observable asymmetry in the production of charged pions; we will thus assume that $A_{\pi^+} = -A_{\pi^-} = A_u$.

Let us consider a $\mathcal{P}$-odd domain with a topological charge $Q \geqslant 1$. Then $N_R - N_L = Q$ in (10); if the total multiplicity of positive pions is $N_R + N_L = N_{\pi^+}$ we get for the asymmetry an estimate

$$A_{\pi^+} = -A_{\pi^-} \simeq \frac{Q}{N_{\pi^+}}, \tag{11}$$

where $Q \geqslant 1$. It is important to note that topological charge Q of the domain is a conserved quantity, whereas the multiplicity of final state pions N_π strongly fluctuates. In the deconfined phase, the probability of forming topologically charged domains is not suppressed so one may expect the $\mathcal{CP}$-odd effects in almost every heavy ion collision event at sufficiently high energy.

Soft particles produced in high-energy collisions are known to be correlated over about one unit of rapidity, which would most likely be a typical extent of a $\mathcal{P}$-odd bubble in rapidity space, so one can take $N_{\pi^+} = dn_{\pi^+}/dy$. Even in the central rapidity region of heavy ion collisions the multiplicity of positive pions can slightly exceed the one for negative pions because the colliding nuclei are positively charged; however the normalized asymmetries (10) of course should still be equal and opposite in sign. (If the temperature is low and the isospin asymmetry is large, $\mathcal{P}$-odd condensates can form in the system [11], but these conditions are not met in heavy ion collisions).

The multiplicity dn_{π^+}/dy depends on the centrality of the collision (apart from the energy and the mass number of the colliding ions); very peripheral collisions are most likely incapable of producing a sufficiently extended volume of hot matter, so excluding them the multiplicity per unit of rapidity in RHIC $Au - Au$ events typically varies within the limits $100 \leqslant N_{\pi^+} \leqslant 300$. The expected magnitude of the asymmetry (11) is thus $A_{\pi^+} \sim 10^{-2}$. It may be possible to detect asymmetry of this magnitude by studying $\pi^+\pi^+$ and $\pi^-\pi^-$ correlations with respect to the reaction plane of the collision. The average angle $\delta\chi = \pi/2 - \Omega$ of π^+ meson with respect to the reaction plane according to (9) is $\langle \delta\chi_{\pi^+} \rangle = 2\kappa/3 = 4A_{\pi^+}/3 \sim 10^{-2}$. While the parity violation of that magnitude may well be amenable to observation, an experimental study of the effect will require an ingenious high-precision method of correlating pion momentum asymmetries with the reaction plane, reconstructed from the elliptic flow and/or from the directions of the forward fragments.

The ideas of using a decay of an oriented system to test fundamental symmetries date back to the work [31] which led to the discovery of parity violation in weak interactions. The spatial separation of positive u quarks and negative $\bar{u}$ antiquarks in hot QCD matter (and the resulting spatial asymmetry for π^+ and π^- production) induces an electric dipole moment of the system.

An observation of such an asymmetry in heavy ion collisions would signal for the first time the possibility of $\mathcal{P}$ and $\mathcal{CP}$-odd effects in strong interactions. Moreover, since the QCD vacuum is known to conserve parity, such an observation would establish unambiguously the creation of a different phase of quark–gluon matter.

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