Pentaquark and diquark–diquark clustering: a QCD sum rule approach

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Abstract

In this work we study the $\Theta^+(1540)$ in the framework of QCD sum rules based on $(ud)^2\bar{s}$ diquark clustering as suggested by Jaffe and Wilczek. Within errors, the mass of the pentaquark is compatible with the experimentally measured value. The mass difference between the Θ^+ and the pentaquark with the quantum numbers of the nucleon amounts to 70 MeV, consistent with the interpretation of the $N(1440)$ as a pentaquark.

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PACS: 12.38.Lg; 12.90.+b

Keywords: Pentaquark; QCD sum rules

Recently, several experiments [1–10] have observed a new baryon resonance $\Theta^+(1540)$ with positive strangeness. Therefore it requires an $\bar{s}$ and has a minimal quark content of five quarks. This discovery has triggered an intense experimental and theoretical activity to clarify the quantum numbers and to understand the structure of the pentaquark state. The Θ has the third component of isospin zero and the absence of isospin partners suggests strongly that the Θ is an isosinglet what we also assume in this work. A puzzling characteristics of the Θ is its narrow width below 15 MeV. A suggestive way to explain the small width is by the assumption of diquark clustering. The formation of diquarks presents an important concept and has direct phenomenological impact [11]. Two models have been proposed based on the strong attraction of the (ud)-diquarks: one by Karliner and Lipkin [12] where the pentaquark is described as diquark–triquark system in a non-standard colour representation. The other one is due to Jaffe and Wilczek [13,14] and describes the Θ as bound state of an $\bar{s}$ with two highly correlated (ud)-diquarks. In this work we investigate the second approach by Jaffe and Wilczek in the framework of QCD sum rules. In principle, as was discussed in [15], even a mixing between the two states could be possible. However,

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an estimation of such a potential mixing would require a detailed investigation of the model by Karliner and Lipkin. The basis of the sum rules was laid in [16] and their extension to baryons was developed in [17]. The assumptions of the model are incorporated by an appropriate current. Since the sum rules are directly based on QCD and keep the analytic dependence on the input parameters, they can help to differentiate between the models and to test their features. The relevance of the diquark picture within the context of the sum rules was shown in [18]. Several sum rule investigations for the pentaquark already exist [19–22] which, however, are based on different models or currents. The diquark models for the pentaquark have also been investigated within other approaches [23].

In the model by Jaffe and Wilczek the (ud)-diquarks have zero spin and are in a $\bar{3}_c$ and $\bar{3}_f$ representation of colour and flavour. In order to combine with the antiquark into a colour singlet, the two diquarks must combine into a colour 3. The diquark–diquark wavefunction is antisymmetric and has angular momentum one. This combines with the spin of the $\bar{s}$ to total angular momentum $1/2$ and results in positive parity. In [13] it was suggested to interpret the Roper resonance $N(1440)$ as $(ud)^2\bar{d}$ pentaquark state and we will study this resonance at the end of our analysis.

The basic object in our sum rule analysis is the two-point correlation function

$$\Pi(p) = i \int d^4x \, e^{ipx} \langle 0| T\{\eta(x)\bar{\eta}(0)\}|0\rangle, \tag{1}$$

where $\eta(x)$ represents the interpolating field of the pentaquark under investigation.

The diquarks have a particularly strong attraction in the flavour antisymmetric $J^P = 0^+$ channel. Thus the current contains two diquarks of the form

$$\mathcal{Q}^c(x) = \epsilon^{abc} \, Q_{ab}(x) = \epsilon^{abc}\big[u_a^T C\gamma_5 d_b\big](x). \tag{2}$$

C denotes the charge conjugation matrix. The two diquarks must be in a p-wave to satisfy Bose statistics. Therefore the current contains a derivative to generate one unit of angular momentum. The diquarks couple to a 3_c in colour to form the current

$$\eta(x) = \big(\epsilon^{abd}\delta^{ce} - \epsilon^{abc}\delta^{de}\big)\big[Q_{ab}\big(D^\mu Q_{cd}\big) - \big(D^\mu Q_{ab}\big)Q_{cd}\big]\gamma_5\gamma_\mu C\bar{s}_e^T, \tag{3}$$

where the covariant derivative for the $\bar{3}_c$ is given by $D^\mu = \partial^\mu - ig\lambda_l^\dagger A^{\mu l}$ [14]. The parity is positive. This current has a different structure than the current in [21] which contains no derivative to produce the angular momentum between the diquarks. Inserting the current and neglecting higher orders in the strong coupling constant the correlator is given by

$$\begin{aligned}
\Pi(x) &= \langle 0| T\{\eta(x)\bar{\eta}(0)\}|0\rangle = \big[\gamma_5\gamma^\mu C S_{e'e}^{(s)\,T}(-x)C\gamma^\nu\gamma_5\big]T_{\mu\nu}^{ee'}(x), \\
T_{\mu\nu}^{ee'}(x) &= \big(\epsilon^{abd}\delta^{ce} - \epsilon^{abc}\delta^{de}\big)\big(\epsilon^{a'b'd'}\delta^{c'e'} - \epsilon^{a'b'c'}\delta^{d'e'}\big) \\
&\quad \times \big[-\partial_\mu^{(cd)}\partial_\nu^{(c'd')} + \partial_\mu^{(cd)}\partial_\nu^{(a'b')} + \partial_\mu^{(ab)}\partial_\nu^{(c'd')} - \partial_\mu^{(ab)}\partial_\nu^{(a'b')}\big] \\
&\quad \times \big\{\langle\gamma_5 S_{bb'}(x)\gamma_5 C S_{aa'}^T(x)C\rangle\langle\gamma_5 S_{dd'}(x)\gamma_5 C S_{cc'}^T(x)C\rangle \\
&\quad + \langle\gamma_5 S_{bd'}(x)\gamma_5 C S_{ac'}^T(x)C\rangle\langle\gamma_5 S_{db'}(x)\gamma_5 C S_{ca'}^T(x)C\rangle \\
&\quad - \langle\gamma_5 S_{bd'}(x)\gamma_5 C S_{cc'}^T(x)C\gamma_5 S_{db'}(x)\gamma_5 C S_{aa'}^T(x)C\rangle \\
&\quad - \langle\gamma_5 S_{bb'}(x)\gamma_5 C S_{ca'}^T(x)C\gamma_5 S_{dd'}(x)\gamma_5 C S_{ac'}^T(x)C\rangle\big\},
\end{aligned} \tag{4}$$

with $\partial_\mu^{(ab)} = \partial^{(a)}/\partial x^\mu + \partial^{(b)}/\partial x^\mu$ and the upper colour index indicates the propagator on which the derivative is acting. $S(x)$ and $S^{(s)}(x)$ are the light and strange quark propagators, respectively. The quark propagator has been evaluated in the presence of quark and gluon condensates in [21,24,25], where the explicit expressions can be found. Using the following Lorentz decomposition for $T_{\mu\nu}^{ee'} = \delta^{ee'}T_{\mu\nu}/3$,

$$T_{\mu\nu} = g_{\mu\nu}f_1\big(x^2\big) + x_\mu x_\nu f_2\big(x^2\big), \tag{5}$$

the functions $f_1(x^2)$ and $f_2(x^2)$ are determined to

$$
\begin{aligned}
f_1(x^2) &= \frac{576}{\pi^8 x^{14}} - \frac{240 m_q^2}{\pi^8 x^{12}} + \frac{24 m_q^4 - 64\pi^2 m_q \langle \bar{q}q \rangle + \frac{29\pi^2}{8}\langle \frac{\alpha_s}{\pi} FF \rangle}{\pi^8 x^{10}} \\
&\quad + \frac{12 m_q^3 \langle \bar{q}q \rangle - 4 m_q \langle g_s \bar{q}\sigma Fq \rangle - 16\pi^2 \langle \bar{q}q \rangle^2}{\pi^6 x^8} + O(1/x^6),
\end{aligned}
$$

$$
\begin{aligned}
f_2(x^2) &= -\frac{1152}{\pi^8 x^{16}} + \frac{576 m_q^2}{\pi^8 x^{14}} + \frac{-48 m_q^4 + 256\pi^2 m_q \langle \bar{q}q \rangle - \frac{61\pi^2}{4}\langle \frac{\alpha_s}{\pi} FF \rangle}{\pi^8 x^{12}} \\
&\quad + \frac{-32 m_q^3 \langle \bar{q}q \rangle + 32 m_q \langle g_s \bar{q}\sigma Fq \rangle + 128\pi^2 \langle \bar{q}q \rangle^2}{\pi^6 x^{10}} + O(1/x^8).
\end{aligned}
\tag{6}
$$

The colour non-diagonal part of $T_{\mu\nu}^{ee'}$ vanishes for the considered orders. In momentum space the correlator can be parametrised as

$$
\Pi(p) = \not{p} \Pi^{(p)}(p^2) + \Pi^{(1)}(p^2).
\tag{7}
$$

To obtain the phenomenological side we insert intermediate baryon states with the corresponding quantum numbers. The matrix element of the Θ is parametrised by

$$
\langle 0|\eta(0)|\Theta(p)\rangle = f_\Theta \cdot u(p).
\tag{8}
$$

Since no experimental information on higher pentaquark states is available we make the assumption of quark–hadron duality and approximate the higher states by the perturbative spectral density above a threshold s_0. In fact, the uncertainty on s_0 will be one of the dominant errors in the sum rule analysis.

In order to suppress the higher-dimensional condensates and to reduce the influence of the higher resonances we employ a Borel transformation defined by

$$
\hat{\mathcal{B}}_M = \lim_{Q^2, n \to \infty} \frac{(-Q^2)^n}{\Gamma(n)} \left(\frac{d}{dQ^2}\right)^n, \qquad M^2 = \frac{Q^2}{n} \quad \text{fixed},
\tag{9}
$$

with $Q^2 = -p^2$. As in [20,21] we now concentrate on the chirality even part $\Pi^{(p)}$ in Eq. (7) which contains the leading order term from the operator product expansion. The spectral density $\rho(s) = 1/\pi \, \mathrm{Im} \, \Pi^{(p)}(s + i\epsilon)$ has the form

$$
\rho(s) = a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + \cdots.
\tag{10}
$$

The coefficients a_i can easily be obtained from the results of Eqs. (4) and (6) by inserting the strange quark propagator and performing a Fourier transformation. The theoretical moments are then given by

$$
\begin{aligned}
\hat{\Pi}(M^2) = \hat{\mathcal{B}}_M \Pi(Q^2) &= \int_0^\infty ds \, \frac{\rho(s)}{M^2} e^{-s/M^2} \\
&= a_6 \Gamma(7)(M^2)^6 + a_5 \Gamma(6)(M^2)^5 + a_4 \Gamma(5)(M^2)^4 + a_3 \Gamma(4)(M^2)^3 + \cdots.
\end{aligned}
\tag{11}
$$

Transferring the continuum contribution to the theoretical side and taking a logarithmic derivative with respect to $-1/M^2$, one obtains the sum rule for the mass of the pentaquark,

$$
m_\Theta^2 = \frac{\sum_{k=0}^{k=3} a_{6-k} \Gamma(8-k)(M^2)^{8-k} E_{7-k}}{\sum_{k=0}^{k=3} a_{6-k} \Gamma(7-k)(M^2)^{7-k} E_{6-k}},
\tag{12}
$$

where $E_\alpha = 1 - \Gamma(\alpha+1, s_0/M^2)/\Gamma(\alpha+1)$.

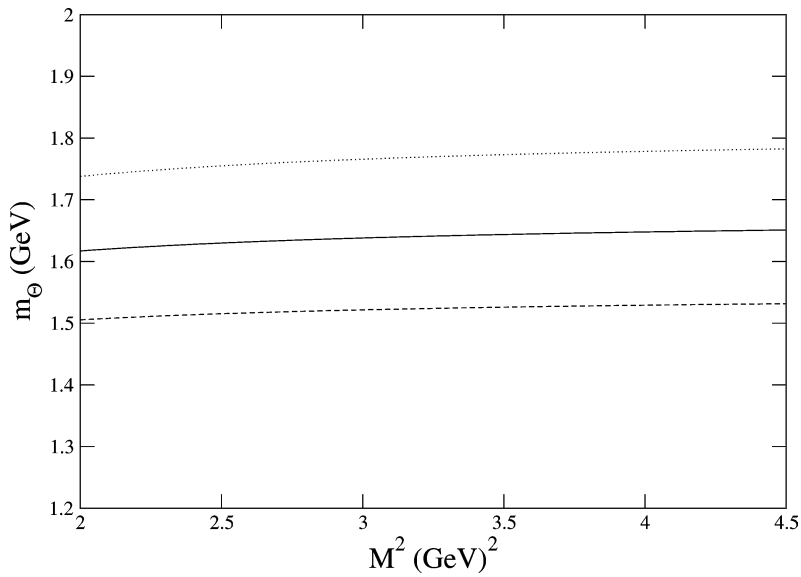Fig. 1. m_Θ as a function of the Borel parameter M^2 for different $s_0 = 3.5$ GeV^2 (solid), $s_0 = 4.1$ GeV^2 (dotted) and $s_0 = 3.0$ GeV^2 (dashed).
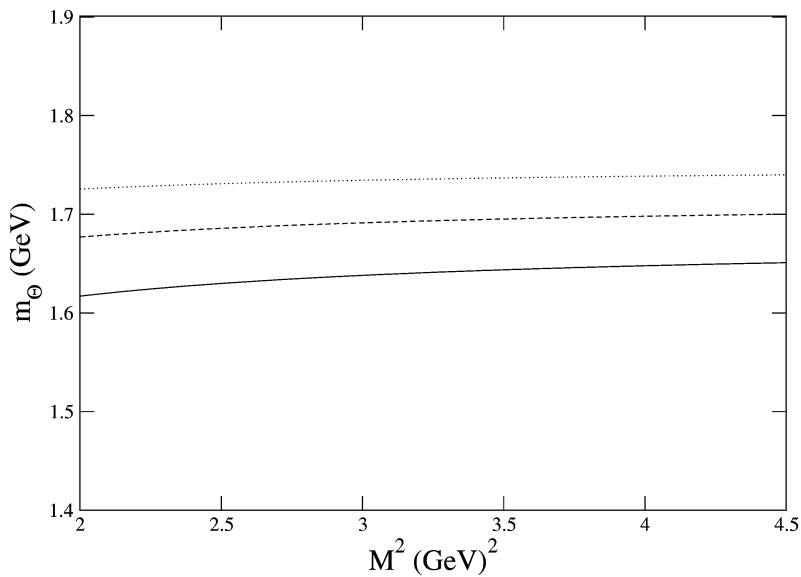

Fig. 2. m_Θ for different orders of the OPE, using only the leading order perturbative expansion (dotted), with dimension 4 condensates (dashed) and including the condensates of dimension 6 (solid).

A basic input for the sum rule analysis is the Borel parameter M. The sum rule should be stable with respect to M to allow a reliable determination of the pentaquark mass. For large values of M the operator product expansion converges well, however, for small M the expansion becomes problematic and thus we restrict the range of the Borel parameter to $M \gtrsim 1.6$ GeV. Small M suppress the phenomenological continuum part which becomes very dominant for large M. Therefore we employ a sum rule window of 2.5 GeV$^2 < M^2 < 4.0$ GeV^2.
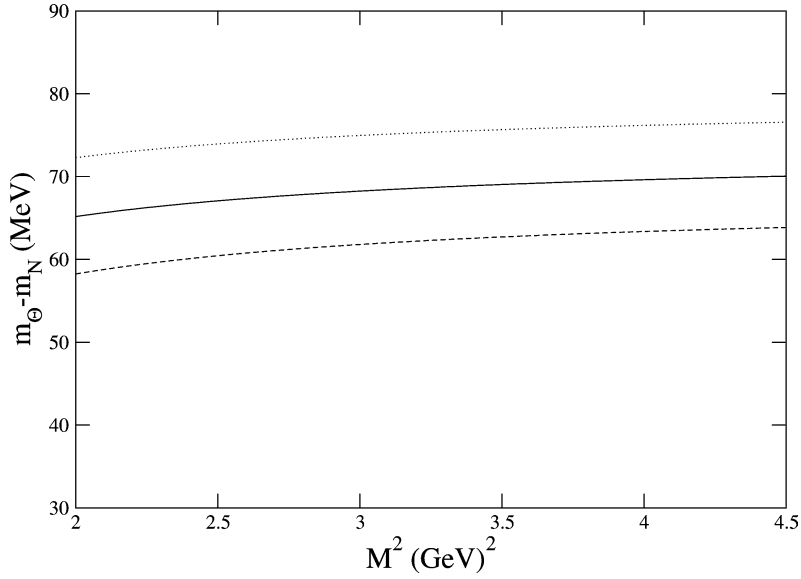
Fig. 3. Mass difference $m_\Theta - m_N$ for different values of the continuum threshold, the solid, dashed and dotted lines are for $s_{0\Theta} = 3.5$ GeV^2 and $s_{0N} = 3.2$ GeV^2, $s_{0\Theta} = 4.1$ GeV^2 and $s_{0N} = 3.8$ GeV^2 and $s_{0\Theta} = 3.0$ GeV^2 and $s_{0N} = 2.7$ GeV^2, respectively.

As input parameters in our analysis we use $m_s = 0.15$ GeV, $\langle \bar{q}q \rangle = -(0.267 \pm 0.018$ GeV$)^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.2)\langle \bar{q}q \rangle$, $\langle g_s \bar{s}\sigma Fs \rangle = M_0^2 \langle \bar{s}s \rangle$ with $M_0^2 = (0.8 \pm 0.2)$ GeV^2, and $\langle \frac{\alpha_s}{\pi} FF \rangle = 0.024 \pm 0.012$ GeV^4 [26]. For the continuum threshold we use a central value of $s_0 = (1.54 + 0.35$ GeV$)^2$. Thus the continuum starts 350 MeV above the measured pentaquark mass. This difference should roughly correspond to one radial excitation [20] and represents a typical value for sum rule analyses with light quarks as degrees of freedom [16]. Fig. 1 shows the mass as a function of the Borel parameter M^2. The sum rule has a good stability with respect to M. As central value for the pentaquark mass we obtain $m_\Theta = 1.64$ GeV. The two most important sources of the error are the choice of the continuum threshold and the convergence of the operator product expansion. Since we have substituted the phenomenological spectral density, using the assumption of quark hadron duality, by the perturbative expansion, the uncertainty on s_0 reflects the missing knowledge of the experimental cross section for higher energies. To estimate the error on m_Θ we vary s_0 between $3.0 < s_0 < 4.1$ GeV^2. In Fig. 1 we have also plotted the change of m_Θ with the continuum threshold from which we obtain an error of $\Delta m_\Theta \approx 125$ MeV. More phenomenological information would be essential to reduce this kind of error. To estimate the dependence of the sum rules on the OPE we successively remove the different orders. Fig. 2 shows the convergence of the pentaquark mass including the condensate contributions up to a specific power. The inclusion of the higher condensates lowers the mass. Using only the leading order perturbative result the central value is about 100 MeV larger than the full result. We have not included an extra graph for the term $\propto a_5$ since this contribution is proportional to the light quark masses and their influence on the analysis can be neglected. The four-dimensional condensates lower the leading order result by about 50 MeV and the condensates of dimension 6 by another 50 MeV. We assume that a reasonable error estimate from the OPE would be $\Delta m_\Theta \approx 75$ MeV. Furthermore, contributions to the error also arise from the other input parameters which we vary in the ranges presented above. As it turns out, their influence on the value of m_Θ is small compared to the errors from the continuum threshold and the convergence of the OPE. Adding the errors quadratically our final result reads

$$m_\Theta = 1.64 \pm 0.15 \text{ GeV}. \tag{13}$$

In [13] Jaffe and Wilczek suggested to interpret the Roper resonance as $(ud)^2\bar{d}$ pentaquark state. One can then perform a similar analysis for the $N(1440)$ as has been done for the Θ by substituting the $\bar{s}$ antiquark by a $\bar{d}$ antiquark. As central value for the continuum threshold we choose, as in the Θ^+ case, a value of 350 MeV above the ground state mass. For the error range we use $2.7 < s_{0N} < 3.8$ GeV^2. Performing a sum rule analysis for the N with the above given parameters, we obtain a mass of $m_N = 1.57 \pm 0.15$ GeV. Similar as it has been done in [21], in Fig. 3 we plot the mass difference $m_\Theta - m_N$ for different values of the continuum thresholds. The mass splitting between the pentaquark states comes out to be about 70 MeV. The error represented in Fig. 3 is based on the assumption that the continuum thresholds have the same offset for both pentaquark states. Phenomenologically, these values can be different and one should add to the error a part of the uncertainty from s_0 given in Fig. 1. Thus the error can easily amount to 50 MeV. Though the mass difference is consistent with the interpretation of the $N(1440)$ as a pentaquark, the uncertainty remains large and a reduction of the error would be essential to clarify the situation.

Recently, in [27] it has been argued that one should subtract all possible colour-singlet meson–baryon contributions from the pentaquark current. We believe that this claim is not correct. Nothing is wrong to use the current of Eq. (3). This current contains also 2-particle intermediate states which have to be added to the phenomenological side. However, at energies around the pentaquark mass we expect the pentaquark contribution to dominate the spectral density. Apart from KN production whose threshold lies somewhat below the pentaquark energy other intermediate states start at higher energy. Therefore it is expected that the baryon–meson continuum contribution only becomes important at energies much above the pentaquark mass. In this energy range the spectral density is suppressed by the exponential in Eq. (11) and the correlator should be well approximated by the assumption of quark–hadron duality. Furthermore, the current $\eta(x)$ is based on the assumption of diquark formation. Subtracting partial contributions from the OPE side changes the pentaquark current and can remove contributions relevant for the diquark formation. Thus these contributions can form an important part of the pentaquark and should not be subtracted.

To summarise, we have performed a QCD analysis based on the approach by Jaffe and Wilczek. We obtain a sum rule that is stable over the Borel parameter M and reproduces the mass of the pentaquark within errors. The error is to a large part due to the lack of experimental information above the pentaquark energy. Furthermore, a complete calculation at next-to-leading order would help to quantify the uncertainties in the theoretical expansion. However, with the complex structure of the current and given the fact that this includes a calculation of five loops, this is a difficult task. We have also performed an analysis for the pentaquark with the quantum numbers of the nucleon and have shown that the interpretation of the Roper resonance $N(1440)$ as $(ud)^2\bar{d}$ pentaquark state is consistent with the sum rules. It is important to note that the sum rules are directly based on QCD and thus, apart from the structure of the current, do not contain further model assumptions. It would be interesting to see if lattice calculations could confirm these findings. First lattice calculations exist [28] which, however, are based on different interpolating currents and whose results are not yet conclusive. Further advance in two directions seems feasible: higher lying pentaquark states with different quantum numbers and internal structure could be investigated and a QCD analysis based on the approach by Karliner and Lipkin should be done. This might help to understand the specific features of the models and to differentiate between the approaches.

Acknowledgements

References

[1] LEPS Collaboration, T. Nakano, et al., Phys. Rev. Lett. 91 (2003) 012002, hep-ex/0301020.
[2] DIANA Collaboration, V.V. Barmin, et al., Phys. At. Nucl. 66 (2003) 1715, Yad. Fiz. 66 (2003) 1763 (in Russian), hep-ex/0304040.
[3] CLAS Collaboration, S. Stepanyan, et al., Phys. Rev. Lett. 91 (2003) 252001, hep-ex/0307018.
[4] SAPHIR Collaboration, J. Barth, et al., Phys. Lett. B 572 (2003) 127.
[5] CLAS Collaboration, V. Kubarovsky, et al., Phys. Rev. Lett. 92 (2004) 032001, hep-ex/0311046;
    CLAS Collaboration, V. Kubarovsky, et al., Phys. Rev. Lett. 92 (2004) 049902, Erratum.
[6] HERMES Collaboration, A. Airapetian, et al., Phys. Lett. B 585 (2004) 213, hep-ex/0312044.
[7] A.E. Asratyan, A.G. Dolgolenko, M.A. Kubantsev, hep-ex/0309042.
[8] SVD Collaboration, A. Aleev, et al., hep-ex/0401024.
[9] ZEUS Collaboration, hep-ex/0403051.
[10] COSY-TOF Collaboration, M. Abdel-Bary, et al., hep-ex/0403011.
[11] B. Stech, Phys. Rev. D 36 (1987) 975;
    M. Neubert, B. Stech, Phys. Lett. B 231 (1989) 477;
    M. Anselmino, E. Predazzi, S. Ekelin, S. Fredriksson, D.B. Lichtenberg, Rev. Mod. Phys. 65 (1993) 1199.
[12] M. Karliner, H.J. Lipkin, hep-ph/0307243.
[13] R.L. Jaffe, F. Wilczek, Phys. Rev. Lett. 91 (2003) 232003, hep-ph/0307341.
[14] R. Jaffe, F. Wilczek, hep-ph/0401034.
[15] B.K. Jennings, K. Maltman, Phys. Rev. D 69 (2004) 094020, hep-ph/0308286.
[16] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 385;
    M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 448;
    L.J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rep. 127 (1985) 1;
    S. Narison, World Sci. Lecture Notes Phys. 26 (1989) 1.
[17] B.L. Ioffe, Nucl. Phys. B 188 (1981) 317;
    B.L. Ioffe, Nucl. Phys. B 191 (1981) 591, Erratum;
    B.L. Ioffe, Z. Phys. C 18 (1983) 67;
    Y. Chung, H.G. Dosch, M. Kremer, D. Schall, Phys. Lett. B 102 (1981) 175;
    Y. Chung, H.G. Dosch, M. Kremer, D. Schall, Nucl. Phys. B 197 (1982) 55;
    H.G. Dosch, M. Jamin, S. Narison, Phys. Lett. B 220 (1989) 251.
[18] H.G. Dosch, M. Jamin, B. Stech, Z. Phys. C 42 (1989) 167;
    M. Jamin, M. Neubert, Phys. Lett. B 238 (1990) 387.
[19] J. Sugiyama, T. Doi, M. Oka, Phys. Lett. B 581 (2004) 167, hep-ph/0309271.
[20] S.L. Zhu, Phys. Rev. Lett. 91 (2003) 232002, hep-ph/0307345.
[21] R.D. Matheus, F.S. Navarra, M. Nielsen, R. Rodrigues da Silva, S.H. Lee, Phys. Lett. B 578 (2004) 323, hep-ph/0309001.
[22] P.Z. Huang, W.Z. Deng, X.L. Chen, S.L. Zhu, Phys. Rev. D 69 (2004) 074004, hep-ph/0311108.
[23] K. Cheung, hep-ph/0308176;
    I.M. Narodetskii, Y.A. Simonov, M.A. Trusov, A.I. Veselov, Phys. Lett. B 578 (2004) 318, hep-ph/0310118;
    E. Shuryak, I. Zahed, hep-ph/0310270;
    F.E. Close, hep-ph/0311087;
    J.J. Dudek, hep-ph/0403235;
    N.I. Kochelev, H.J. Lee, V. Vento, hep-ph/0404065.
[24] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Fortschr. Phys. 32 (1985) 585.
[25] K.C. Yang, W.Y.P. Hwang, E.M. Henley, L.S. Kisslinger, Phys. Rev. D 47 (1993) 3001.
[26] D.J. Broadhurst, P.A. Baikov, V.A. Ilyin, J. Fleischer, O.V. Tarasov, V.A. Smirnov, Phys. Lett. B 329 (1994) 103, hep-ph/9403274;
    H.G. Dosch, S. Narison, Phys. Lett. B 417 (1998) 173, hep-ph/9709215;
    M. Jamin, Phys. Lett. B 538 (2002) 71, hep-ph/0201174;
    T.W. Chiu, T.H. Hsieh, Nucl. Phys. B 673 (2003) 217;
    T.W. Chiu, T.H. Hsieh, Nucl. Phys. B (Proc. Suppl.) 129 (2004) 492, hep-lat/0305016;
    A. Di Giacomo, Y.A. Simonov, hep-ph/0404044.
[27] Y. Kondo, O. Morimatsu, T. Nishikawa, hep-ph/0404285.
[28] F. Csikor, Z. Fodor, S.D. Katz, T.G. Kovacs, JHEP 0311 (2003) 070, hep-lat/0309090;
    S. Sasaki, hep-lat/0310014;
    T.W. Chiu, T.H. Hsieh, hep-ph/0403020.