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## Optimal Allocation of Non-Linear Viscous Dampers for Three-Dimensional Building Structures

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### Abstract

A new approach has been proposed by the authors and applied to allocate linear viscous dampers optimally for two- and three-dimensional building structures recently. This approach is extended to search for the optimal allocation of non-linear viscous dampers in this paper. The same initial damper placement is used for conducting the analysis. However, instead of carrying out the linear time-history seismic analysis, the non-linear time-history seismic analysis is performed first to obtain the inter-story drift ratio. Then, check the inter-story drift ratio for the locations where dampers were added and move the damper in the location with the minimal inter-story drift ratio to the location with the maximal inter-story drift ratio. Finally, repeat this process until the prescribed stop criterion is met. The non-linear viscous dampers with two exponents, including 0.3 and 0.5, and two seismic records, including the El Centro earthquake and Chi-Chi earthquake, are used in this paper. Three examples, including two 10-story and one 20-story three-dimensional unsymmetrical building structures, are used to demonstrate the efficiency and accuracy of the proposed approach. The results are compared with those obtained using the simplified sequential search algorithm (SSSA) and are also compared with the optimal placement of linear viscous dampers obtained using the proposed approach. It is found that the proposed approach requires much fewer analyses than the SSSA while their accuracy is comparable. The efficiency of the proposed approach for allocating non-linear viscous dampers is also comparable to or better than for allocating linear viscous dampers.

*Keywords:* Damper, Optimal allocation, SSSA, Non-linear viscous damper, Inter-story drift ratio.

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### 1. INTRODUCTION

Placing dampers properly and effectively on structures has been a research focus for two decades since dampers got started to be applied popularly in earthquake-resistant design of buildings. Many papers related to this subject have been published since early 1980s. Then, more and more researchers got involved in this topic and presented papers in related journals. So far, several methods have been

proposed for allocating linear viscous dampers. Among them, the SSSA is probably the simplest one. Garcia and Soong (2001, 2002) developed and proposed the SSSA on the basis of engineering knowledge and judgment to find the optimal allocation of supplemental dampers. Each time a damper is added to the position with maximal structural response so as to suppress it after carrying out a dynamic analysis; the procedure is repeated until all dampers are added. The more the number of dampers to be added in the building structures, the more the number of dynamic analysis to be performed. To reduce the number of dynamic analysis, a simple approach for relocating dampers with the idea similar to the SSSA is proposed in this paper.

## 2. ESTIMATION OF THE DAMPING CONSTANT OF ADDED DAMPERS

Given a damping ratio, the method for calculating the corresponding damping constant of viscous dampers in common design practice is based on the equivalent energy method. The equivalent damping ratio  $\xi_d$  due to the action of the supplemental nonlinear viscous dampers can be calculated approximately from the fundamental modal energy and given by:

$$\xi_d = \frac{\sum_{j=1}^{n_d} \lambda C_j \phi_{rj}^{1+\alpha} \cos^{1+\alpha} \theta_j}{2\pi A^{1-\alpha} \left(\frac{2\pi}{T}\right)^{2-\alpha} \sum_{i=1}^n m_i \phi_i^2} \quad (1)$$

where  $T$  is the fundamental period of the structure,  $\phi_i$  is the first mode displacement at floor  $i$ ,  $\phi_{rj}$  is the first mode relative displacement between the ends of damper  $j$  in the horizontal direction,  $C_j$  is the damping constant for damper  $j$ ,  $\theta_j$  is the inclined angle of damper  $j$ ,  $m_i$  is the mass of floor  $i$ ,  $A$  is the roof displacement when the modal displacement  $\phi_j$  is normalized to one unit at the roof,  $n$  is the number of floor,  $n_d$  is the number of damper,  $\alpha$  is the damping exponent between 0 and 1, and  $\lambda$  is a parameter which can be calculated by

$$\lambda = 2^{2+\alpha} \frac{\Gamma^2(1+\alpha/2)}{\Gamma(1+\alpha)} \quad (2)$$

in which  $\Gamma$  is the gamma function. The values of  $\lambda$  are tabulated in FEMA 273 based on Eq. (2). Assume all the damping constants of supplemental dampers are the same and their inclined angles are also the same, i.e.,  $C_j = c$ ,  $\theta_j = \theta$ . Re-arrange Eq. (1) and the damping constant of each supplemental damper can be expressed as:

$$c = \frac{2\pi \xi_d A^{1-\alpha} \left(\frac{2\pi}{T}\right)^{2-\alpha} \sum_{i=1}^n m_i \phi_i^2}{\lambda \sum_{j=1}^{n_d} \phi_{rj}^{1+\alpha} \cos^{1+\alpha} \theta_j} \quad (3)$$

Given a damping ratio  $\xi_d$ , Eq. (3) allows one to calculate the damping coefficient of each added damper. Note that only the translational component is considered when calculating the modal kinetic energy in Eq. (1) and Eq. (3), which is the case for symmetrical three-dimensional building structures. However, the torsion effect in unsymmetrical three-dimensional building structures may be significant, which cannot be considered in Eq. (3). To resolve this problem, one needs to consider the associated energies due to the torsional component. If the number of dampers is chosen as twice the number of floors, i.e.,  $n_d = 2n$ . Also, the added dampers are assumed to be placed uniformly along each story of two selected bays. Then, Eq. (3) is modified as:

$$c = \frac{2\pi\xi_d A^{1-\alpha} \left(\frac{2\pi}{T}\right)^{2-\alpha} \sum_{i=1}^n (m_i \phi_{zi}^2 + I_{ci} \phi_{ri}^2)}{\lambda \sum_{j=1}^n (\phi_{rjB1}^{1+\alpha} + \phi_{rjB2}^{1+\alpha}) \cos^{1+\alpha} \theta_j} \quad (4)$$

in which  $\phi_{zi}$  and  $\phi_{ri}$  are the first mode displacement at floor  $i$  in both translational and rotational directions, respectively.  $I_{ci}$  is the mass moment of inertia of floor  $i$  with respect to the principal axis through its center of mass.  $\phi_{rjB1}$  and  $\phi_{rjB2}$  are the first mode relative displacements between the ends of a damper in the horizontal direction on bay 1 and bay 2, respectively.

The total damping coefficient of the supplemental dampers  $C_T$  is equal to  $cn_d$ . After adding the supplemented dampers, the effective damping ratio (or total equivalent damping ratio)  $\xi_T$  of the structure system is given by  $\xi_T = \xi_0 + \xi_d$ , in which  $\xi_0$  is the inherent damping ratio of the structure. The equivalent damping ratio  $\xi_d$  can be obtained if the effective damping ratio and the inherent damping ratio of the structure are assumed to be given.

### 3. PROPOSED APPROACH

In this study, the number of dampers is chosen as twice the number of floors. The damping constant of each damper corresponding to a uniform distribution placement of dampers along two bays can be estimated from Eq. (4) in advance. These two bays are chosen intentionally to place dampers uniformly in the structure model as the initial damper placement. Then, one searches for the optimal damper placement by relocating dampers. The inter-story drift ratio is considered as the performance index herein. Hence, the damper with the minimal inter-story drift ratio is moved to the position with the maximal inter-story drift ratio of the whole structure to suppress the maximal dynamic response. The dynamic response is suppressed effectively and the optimal dampers placement is finally achieved via the above relocation strategy. The procedure of this proposed approach (Leu et al., 2008) is summarized as follows: (1) Model the bare structure without dampers by using commercial program package; perform the modal analysis and extract the modal parameters, including the fundamental period, the first mode displacement and the first mode relative displacement. Choose two bays to place dampers uniformly in the structure model as the initial damper placement. (2) Implement the non-linear time-history seismic analysis to get the inter-story drift ratio. Check the inter-story drift ratio of the locations where dampers were added and move the damper on the location of the minimal inter-story drift ratio to the location of the maximal inter-story drift ratio of the whole structure, moving one damper per step. (3) Repeat the above process until the prescribed stop criterion is met. The maximum drift ratio will decrease and reach a stable value after few steps. According to numerous numerical experiments carried out in this study, the number of steps reaching such a stable value can be set to be one-fifth of the total dampers to be added, which is used as the stop criterion.

### 4. CASE STUDY AND COMPARISONS

The effectiveness of the proposed method for the optimal damper placement of three-dimensional unsymmetrical building structures is investigated using three building structures. Two 10-story and one 20-story structures, are designed intentionally so that their eccentricities between the center of mass and the center of rigidity are larger than 1 meter. The plan layouts of these three structures are shown in Figure 1(a), 1(b) and 1(c). All the floors are assumed to be acted as the rigid diaphragm. Two strong ground motions, including El Centro S00E and Chi-Chi TAP097N (both are scaled to 0.25g), are used and applied to one direction (x direction shown in Figure 1) to conduct the dynamic analysis. Two exponents, including 0.3 and 0.5, for the nonlinear viscous damper are also chosen to evaluate the

damping constant of the supplemental damper. The equivalent damping ratio due to nonlinear viscous dampers is set to 0.18 and the inherent damping ratio is assumed to 0.02. Therefore, the effective damping ratio is equal to 0.20.

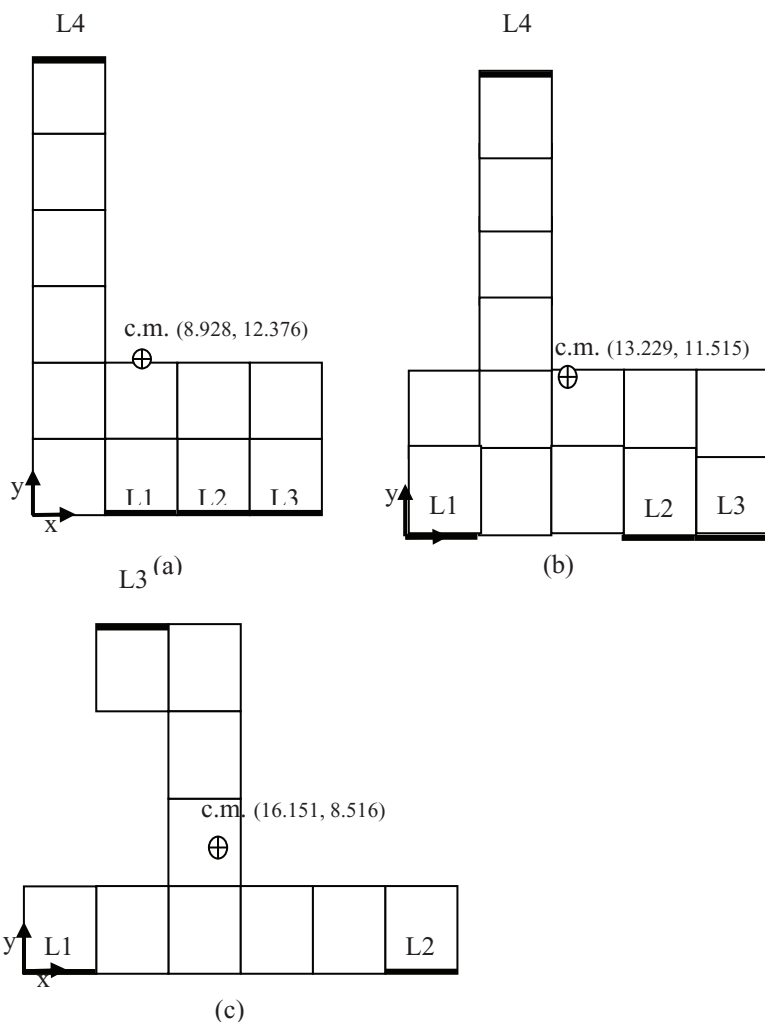


Figure 1: Plan layouts for three different unsymmetrical building structures.

4.1. 10-story Unsymmetrical Building Structure, Case 1 and Case 2

Figure 1(a) and 1(b) show the plan layouts of two 10-story unsymmetrical building structures, which will be indicated as Case 1 and Case 2, respectively. Two earthquakes are applied to these two structures as the external excitation and several initial damper placements are used to conduct the proposed approach to search for the optimal damper placement. The dampers are placed on each story along two selected bays. Three selections are studied here, including (1) bay L1 and bay L4, (2) bay L2 and bay L4,

and (3) bay L3 and bay L4. The maximal inter-story drift ratios obtained from the SSSA and the proposed approach are listed in Table 1 and Table 2 for Case 1 and Case 2, respectively. Compared with the bare structure, the reduction in the maximal inter-story drift ratio for the structure adopting the initial damper placement for these two cases ranges from 71.4% to 75.1% if  $\alpha = 0.3$  and from 69.0% to 74.6% if  $\alpha = 0.5$ . Similarly, compared with the bare structure, the reduction for the structure adopting the optimal damper placement is 78.2%~80.2% if  $\alpha = 0.3$  and 77.7%~80.4% if  $\alpha = 0.5$ . Meanwhile, compared with the structure adopting the uniform damper distribution along two bays (initial placement), the reduction for the structure adopting optimal damper placement is 15.7%~28.8% if  $\alpha = 0.3$  and 14.2%~32.3% if  $\alpha = 0.5$ . Figure 2 shows the optimal damper placement and its inter-story drift ratio evolution of an initial placement, i.e., L1/L4, using the proposed approach.

Table 1: Maximum inter-story drift ratio for the 10-story Case 1 structure

Excitations		El Centro Earthquake, S00E			Chi-Chi Earthquake, TAP097 NS		
Selected bays for initial damper placement		L1/L4	L2/L4	L3/L4	L1/L4	L2/L4	L3/L4
Linear $\alpha = 1.0$	Bare structure	0.009598	0.009598	0.009598	0.018545	0.018545	0.018545
	Initial placement	0.002524	0.002520	0.002613	0.005111	0.005110	0.005129
	Proposed approach	0.002236	0.002232	0.002300	0.004563	0.004559	0.004597
	SSSA	0.002070	0.002070	0.002070	0.004445	0.004445	0.004445
Non-linear $\alpha = 0.3$	Bare structure	0.009598	0.009598	0.009598	0.018545	0.018545	0.018545
	Initial placement	0.002666	0.002662	0.002745	0.004619	0.004618	0.004743
	Proposed approach	<b>0.002093</b>	<b>0.002093</b>	<b>0.002010</b>	0.003826	0.003832	0.003773
	SSSA	<b>0.002116</b>	<b>0.002138</b>	<b>0.002147</b>	0.003175	0.003175	0.003144
Non-linear $\alpha = 0.5$	Bare structure	0.009598	0.009598	0.009598	0.018545	0.018545	0.018545
	Initial placement	0.002512	0.002508	0.002668	0.004779	0.004776	0.004900
	Proposed approach	<b>0.001910</b>	<b>0.001916</b>	<b>0.001899</b>	0.004029	0.004023	0.004009
	SSSA	<b>0.002053</b>	<b>0.002021</b>	<b>0.002047</b>	0.003582	0.003656	0.003641

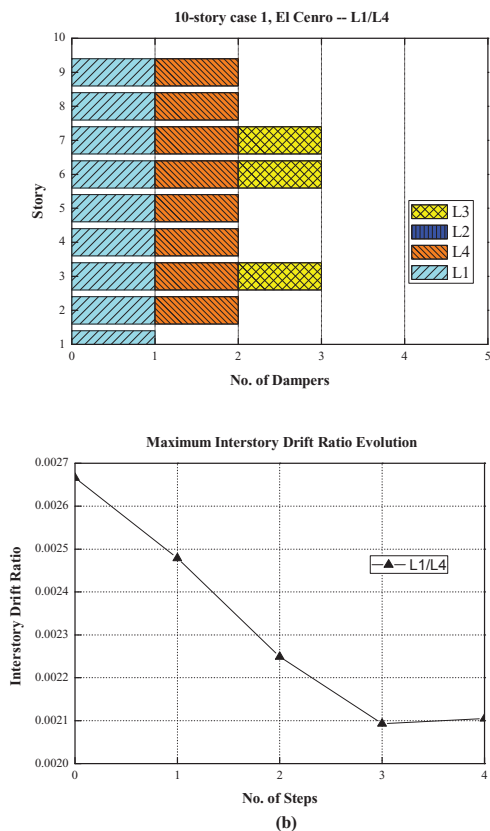


Figure 2: The optimal damper placement and its inter-story drift ratio evolution.

As mentioned before, the number of relocation steps is taken as one-fifth of the number of dampers to be placed, which is equal to four in these two 10-story building cases. The maximal inter-story drift ratio reduces gradually as shown in figure 2(b). The lowest value of the maximal inter-story drift ratio occurs at step 3 and the optimal damper placement is therefore chosen as that corresponding to step 3 as well. Although the optimal damper placement and its inter-story drift ratio evolution for other cases are not reported here, similar observation can be made.

4.2. 20-story Unsymmetrical Building Structure

Figure 1(c) shows the plan layout of a 20-story unsymmetrical building structure. Using the same procedure as mentioned above, 40 dampers are deployed and to be placed uniformly on each story of two selected bays initially. Two selections, including bay L1 and bay L3, bay L2 and bay L3 shown in Figure 1(c), are chosen as the initial damper placement. The maximal inter-story drift ratios obtained from the SSSA and the proposed approach are listed in Table 3. Compared with the bare structure, the reduction in the maximal inter-story drift ratio for the structure adopting either one of the initial damper placements is 63.4%~64.0% if  $\alpha = 0.3$  and 32.2%~36.6% if  $\alpha = 0.5$ . Compared with the bare structure, the reduction for the structure with optimal damper placement is 71.7%~73.2% if  $\alpha = 0.3$  and 45.6%~49.0% if  $\alpha = 0.5$ . Meanwhile, compared with the structure adopting the uniform damper

distribution along two bays (initial placement), the reduction for the structure adopting optimal damper placement is 21.8%~26.4% if  $\alpha = 0.3$  and 18.6%~20.1% if  $\alpha = 0.5$ .

Table 2: Maximum inter-story drift ratio for 10-story Case 2 structure

Excitations		El Centro Earthquake, S00E			Chi-Chi Earthquake, TAP097 NS		
Selected combinations for initial dampers placement		L1/L4	L2/L4	L3/L4	L1/L4	L2/L4	L3/L4
Linear $\alpha = 1.0$	Bare structure	0.009236	0.009236	0.009236	0.018056	0.018056	0.018056
	Initial placement	0.002869	0.002738	0.002893	0.004805	0.004772	0.004813
	Proposed approach	0.002371	0.002304	0.002386	0.004542	0.004503	0.004544
	SSSA	0.002214	0.002214	0.002214	0.004380	0.004380	0.004380
Non-linear $\alpha = 0.3$	Bare structure	0.009236	0.009236	0.009236	0.018056	0.018056	0.018056
	Initial placement	0.002858	0.002764	0.002868	0.004683	0.004585	0.004692
	Proposed approach	<b>0.001936</b>	<b>0.001971</b>	<b>0.002008</b>	0.003585	0.003642	0.003532
	SSSA	<b>0.002180</b>	<b>0.002176</b>	<b>0.002180</b>	0.003198	0.003154	0.003161
Non-linear $\alpha = 0.5$	Bare structure	0.009236	0.009236	0.009236	0.018056	0.018056	0.018056
	Initial placement	0.002738	0.002595	0.002748	0.004794	0.004704	0.004807
	Proposed approach	<b>0.001956</b>	<b>0.001942</b>	<b>0.001982</b>	0.004032	0.004035	0.004004
	SSSA	<b>0.002001</b>	<b>0.002213</b>	<b>0.002001</b>	0.003413	0.003566	0.003430

Table 3: Inter-story drift ratio for 20-story structure

Excitations		El Centro Earthquake, S00E		Chi-Chi Earthquake, TAP097 NS	
Selected combinations for initial dampers placement		L1/L3	L2/L3	L1/L3	L2/L3
$\alpha = 1.0$	Linear				
	Bare structure	0.006519	0.006519	0.011455	0.011455
	Initial placement	0.002428	0.002439	0.006677	0.006674
	Proposed approach	0.002248	0.002252	0.006347	0.006348
	SSSA	0.002063	0.002063	0.005892	0.005892
$\alpha = 0.3$	Non-linear				
	Bare structure	0.006519	0.006519	0.011455	0.011455
	Initial placement	0.002373	0.002387	0.007743	0.007770
	Proposed approach	0.001747	0.001784	0.006187	0.006233
	SSSA	0.001691	0.001692	0.006002	0.005999
$\alpha = 0.5$	Non-linear				
	Bare structure	0.006519	0.006519	0.011455	0.011455
	Initial placement	0.002345	0.002362	0.007267	0.007296
	Proposed approach	0.001833	0.001842	<b>0.005841</b>	0.005938
	SSSA	0.001789	0.001789	<b>0.005895</b>	0.005891

## 5. COMPARISONS

### 5.1. SSSA and the proposed approach

As mentioned before, the number of relocation steps to be performed when using the proposed approach is one-fifth of the number of dampers to be added. For example, there are 20 dampers and 40 dampers, respectively to be added in the 10-story and 20-story buildings, respectively. Therefore, the number of relocation steps is 4 and 8 for the 10-story and 20-story buildings, respectively. However, the number of analysis steps is 20 and 40 for the 10-story and 20-story buildings when the SSSA is employed. Therefore, the proposed approach is very computationally efficient. Even with much fewer analysis steps, the results obtained using the proposed approach are comparable to those obtained using the SSSA as can be seen from Tables 1 to 3.

### 5.2. Linear and nonlinear viscous dampers

To compare the performance of the linear and nonlinear viscous dampers, the linear time-history seismic analysis is also performed to search for the optimal linear viscous dampers placement. Results



are also shown in Tables 1 to 3. Although not reported here, the efficiency of the proposed approach for allocating non-linear viscous dampers is also comparable to or better than for allocating linear viscous dampers. In two 10-story cases (case 1 and case 2) and one 20-story case, compared with the corresponding structure with the initial damper placement, the reduction in the maximal inter-story drift ratio for the structure with the optimal damper placement is 10.6%~12.0%, , 5.5%~17.5%, and 4.9%~7.7%, respectively. It is interesting to find that such reductions are much less than those corresponding to non-linear viscous dampers.

## 6. CONCLUSIONS

Several case studies in this paper shows that the maximum inter-story drift ratio can be reduced quickly and converge to stable values if the initial dampers placement is chosen to place dampers uniformly on two bays of the structure by using the proposed approach. The results are compared with those obtained using the SSSA and are also compared with the optimal placement of linear viscous dampers obtained using the proposed approach. It is found that the proposed approach requires much fewer analyses than the SSSA while their accuracy is comparable. The efficiency of the proposed approach for allocating non-linear viscous dampers is also comparable to or better than for allocating linear viscous dampers.

## 7. ACKNOWLEDGMENTS

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