Large eddy simulation of mixing layer

W.B. Yang\textsuperscript{a}, H.Q. Zhang\textsuperscript{a}, C.K. Chan\textsuperscript{b,\ast}, W.Y. Lin\textsuperscript{a}

\textsuperscript{a}Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China
\textsuperscript{b}Department of Applied Mathematics, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, China

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Abstract

Large eddy simulation (LES) of spatially evolving turbulent mixing layer is carried out in this paper. The predicted evolution of large vortex structures agrees well with the experimental visualization results. The linear growth of momentum thickness along the streamwise direction is obtained, indicating that the pairing and amalgamating of large vortex structures in plane mixing layers occurs randomly in time and space. Self-similarity property of mixing layer is also obtained, which agrees with many experimental discoveries. The mean properties including streamwise velocity, fluctuating velocities and Reynolds shear stress agree well with experimental measurements quantitatively. It shows that LES can achieve accurate quantitative simulation as well as a reasonable vortex structure of mixing layer.

\textsuperscript{\ast}Corresponding author. Tel.: +852-2766-6919; fax: +852-2362-9045.
E-mail address: ck.chan@polyu.edu.hk (C.K. Chan).

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1. Introduction

The mixing layer that forms between two fluid streams moving with different velocities has been the subject of many researches both in experiment and simulation for its technological importance to understand the large coherent vortex structure [1]. There are two main characteristics present in the mixing layer. One is the self-similarity property, which is characterized by linear growth of the layer as well as the mean velocities and turbulent statistics being independent of the down-stream distance nondimensionalized by appropriate length and velocity scales. Another is the presence of large-scale organized structure, whose size is comparable with the transverse length scale of...
the flow. These organized structures play an important role in the momentum and energy transportation, particle dispersion and species diffusion in the mixing layer. Therefore, in order to understand the processes of two fluid streams mixture, particle–vortex interaction as well as vortex–flame interaction, it is required that the numerical methods used to investigate the mixing layer does not only predict self-similarity properties, but also provide the pattern of large vortex structure.

Generally, there are four methods of computing turbulent flow: the correlation-moment closure model based on time-averaged or Favre-averaged properties, LES, direct numerical simulation (DNS) and discrete vortex method. Among these methods, the correlation-moment closure model cannot characterize the large coherent structure in the mixing layer. The DNS requires a large number of grid points and time steps to reach a statistically steady state and are usually limited to relatively low Reynolds numbers. The discrete vortex method is good only for two-dimensional cases and not convenient for considering different profiles of inlet conditions at the end of thin splitter plate. In LES, a low-pass spatial filtering is applied to the Navier–Stokes equations and the filtered equations are solved directly. It is most promising for engineering flows of low-to-moderate Reynolds numbers and is capable of predicting both turbulent statistical properties and large vortex structure. So, the LES is selected for predicting the mixing layer in this paper. Its ability of accurate quantitative simulation of turbulent statistical properties and reasonable prediction of vortex structure is demonstrated in this paper, which paves the way for further investigation on vortex–particle interaction in a gas–particle mixing layer or vortex–flame interaction in a reacting mixing layer.

2. Basic equation

Mixing layer forms when two fluid streams moving with different velocities meet at the end of a splitter plate as shown in Fig. 1. For incompressible flow, the filtered governing equations for the mixing layer can be written as

\[
\begin{align*}
\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} &= 0, \\
\rho \left( \frac{\partial \hat{u}}{\partial t} + u \frac{\partial \hat{u}}{\partial x} + v \frac{\partial \hat{u}}{\partial y} \right) &= -\frac{\partial p}{\partial x} + 2 \frac{\partial \hat{\mu}}{\partial x} \frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{\mu}}{\partial y} \left( \frac{\partial \hat{u}}{\partial y} + \frac{\partial \hat{v}}{\partial x} \right) + \hat{\mu} \left( \frac{\partial^2 \hat{u}}{\partial x^2} + \frac{\partial^2 \hat{u}}{\partial y^2} \right), \\
\rho \left( \frac{\partial \hat{v}}{\partial t} + u \frac{\partial \hat{v}}{\partial x} + v \frac{\partial \hat{v}}{\partial y} \right) &= -\frac{\partial p}{\partial y} + 2 \frac{\partial \hat{\mu}}{\partial y} \frac{\partial \hat{v}}{\partial y} + \frac{\partial \hat{\mu}}{\partial x} \left( \frac{\partial \hat{u}}{\partial y} + \frac{\partial \hat{v}}{\partial x} \right) + \hat{\mu} \left( \frac{\partial^2 \hat{v}}{\partial x^2} + \frac{\partial^2 \hat{v}}{\partial y^2} \right).
\end{align*}
\]
The above equations have been nondimensionalized using momentum thickness of velocity profile at inlet plane \( \theta_0 \) as a length scale, the velocity difference \( \Delta U = U_2 - U_1 \) as a velocity scale and the ratio of \( \theta_0 \) to \( \Delta U \) as a time scale. The effect of the unresolved scale on the resolved scale in the above equations is represented by the sub-grid-scale (SGS) stress, which is modeled by eddy-viscosity hypothesis. The viscosity coefficient in the above equations is composed of two parts as follows:

\[
\tilde{\mu} = \mu + \mu_T, \tag{4}
\]

where \( \mu \) is the molecular viscosity and \( \mu_T \) is turbulent viscosity approximated by the Smagorinsky model in this paper as follows:

\[
\mu_T = \rho(C_s \Delta)^2 |\tilde{S}| = \rho(C_s \Delta)^2 \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}}, \tag{5}
\]

in which \( \tilde{S}_{ij} \) is the rate of strain tensor of the filtered velocity field and \( \Delta \) is the filter width,

\[
\tilde{S}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \tag{6}
\]

\[
\Delta = \sqrt{\Delta x \Delta y}. \tag{7}
\]

The model parameter \( C_s \) has a value of 0.025 in the present work.

3. Boundary condition and numerical procedure

In order to compare with the experimental data of Oster and Wygnanski [8], the calculation parameters should be corresponding with the experiment case. The free-stream velocities \( U_1 \) and \( U_2 \) are taken as 4 and 13 m/s, respectively. The length and width of the calculation domain is taken as 0.6 and 0.2 m, respectively. The molecular viscosity and fluid density are taken as \( 1.798 \times 10^{-5} \) and 1.18 kg/m\(^3\), respectively. The Reynolds number based on the velocity difference \( \Delta U \) and the initial momentum thickness \( \theta_0 \) of the mixing layer is \( 10^4 \).

For the computations in the current study, a uniform grid with 515 nodes in the streamwise direction and 125 nodes in the transverse direction is set up. The second-order 3-point backward Adama–Bashforth scheme is used to carry out the time integration of governing equations. Second-order accurate upwind finite difference scheme is used for the advective terms, while the fourth-order accurate central difference scheme is used for diffusion terms. For convergence, the nondimensional time step is taken as \( 2 \times 10^{-4} \).

The solving processes of velocity and pressure are decoupled using the fractional-step projection method or pressure correction method, which was developed by Chorin [4]. In this method, the momentum equation is solved first without the pressure term for obtaining an intermediate velocity, denoted by \( \tilde{V}^* \). Pressure fields are induced by the divergence of intermediate velocity, being the source term of Poisson equation for pressure,

\[
\nabla^2 p = \frac{\rho}{\Delta t} \nabla \cdot \tilde{V}^*. \tag{8}
\]
The intermediate velocity is modified by the pressure fields obtained from Eq. (8) to obtain a correct velocity $\tilde{V}$, which satisfies momentum equations including the pressure term

$$\tilde{V} = \tilde{V}^* - \frac{\Delta t}{\rho} \nabla P.$$  

(9)

The Poisson equation is discretized using a second-order five-point central difference scheme and solved by cyclic odd–even reduction factorization algorithm [3], which has quite a high efficiency in solving process.

The initial condition of velocity distribution is assumed to be a nonviscous flow field with inlet velocity being step shape. The velocity distribution at the inlet section is assumed as hyperbolic tangent. The so-called traction-free boundary conditions [2] are adopted at the lateral boundary of the computational domain. The improved non-reflective Sommerfeld open boundary conditions [5] are used at the outflow.

4. Results and discussion

Contour distributions of vortices at eight different time instances with an interval of 10 are shown in Fig. 2. A large vortex at one instance can be clearly found in its following instances, so the evolution of large coherent structure can be observed easily in this figure. The large vortex structure and its roll-up, forming and pairing frequently appeared in the mixing layer. Two or more large vortex structures interact by rolling around each other, and then a single larger vortical structure, with approximately twice or more the spacing of former vortices, is formed. The process of vortices arising, growing up, pairing and combining leads to the growth of the mixing layer. The experimental flow visualization result and numerical flow visualization result are compared in Fig. 3. It is shown that the pattern of large vortex structure captured by LES are in satisfactory agreement with experimental results.

Nondimensional time-averaged streamwise velocities, fluctuating velocities and Reynolds shear stresses are shown in Figs. 4–7. These results are obtained by averaging the instantaneous values over time $100 < t < 200$, when the effect of the initial condition on flow fields is neglected. The averaging time period is large enough for obtaining a statistically stationary solution, which has been shown in those figures. The dimensionless coordinate $\eta$ is defined as follows:

$$\eta = \frac{y - y_{0.5}}{\theta}.\quad (10)$$

$y_{0.5}$ denotes the $y$-position at which the time-averaged streamwise velocity, $u$, is equal to half of the sum of two free-stream velocities of $U_1$ and $U_2$. The momentum thickness, $\theta$, at the corresponding streamwise position is defined as

$$\theta = \int_{-\infty}^{\infty} \frac{u - U_1}{U_2 - U_1} \left(1 - \frac{u - U_1}{U_2 - U_1}\right) dy.\quad (11)$$

As shown in Figs. 4–7, the self-similarity of the mixing layer is obtained by simulation, which is one of the two main characteristics demonstrated by many researches. It indicates that the mixing layer is a flow with a self-preserving state. The prediction results of mean streamwise velocity, fluctuating velocity and Reynolds shear stress agree well with the experimental data [4] quantitatively.
The significant difference in transverse fluctuating velocity between predictions and experiments, which is commonly found in two-dimensional simulation of mixing layer [6,7], is induced by the three-dimensionality in actual flows [10].

The growth of the momentum thickness along the x-direction is shown in Fig. 8. It can be found that after an initial transient period, momentum thickness of the mixing layer grows linearly with streamwise distance. The momentum thickness of the mixing layer is mainly dominated by the large vortex structures and their pairing. The vortex pairing process causes a sudden increase of the scale
of the large vortex structures as shown in Fig. 2. The continuous pairing process occurs randomly in space and time, resulting in a linear growth of the mixing layer with streamwise distance.

As described by many researches [1], the self-similar state of mixing layer is characterized by the linear growth of layer thickness and turbulent statistics that, when scaled by the appropriate variables,
are independent of the downstream position. It can be found that the present simulation shows good self-similarity properties both in terms of growth of layer thickness and turbulent statistics.

5. Conclusion

The evolution of a large vortex structure and the nondimensional statistical quantities in the mixing layer are numerically investigated by large eddy simulation. The predicted results including flow visualization, streamwise velocity, fluctuating velocity and Reynolds shear stress agree well with experimental results, and show that LES can achieve accurate quantitative simulation as well as give a reasonable vortex structure of the mixing layer.

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