Available online at www.sciencedirect.com
ScienceDirect
Physics Procedia 2 (2009) 773–779
Physics Procedia
www.elsevier.com/locate/procedia

Proceedings of the JMSM 2008 Conference

The emissivity of conductor Gaussian random rough surfaces: the surface impedance boundary condition method

I. Sassi*, F. Ghmari

Unité de Rayonnement Thermique du département de Physique de la Faculté des Sciences de Tunis. Campus Universitaire El manar I, 2092 Tunis, Tunisia

Received 1 January 2009; received in revised form 31 July 2009; accepted 31 August 2009

Abstract

We are interested in studying the contributions of the physical parameters of materials to the emissivity of random rough surfaces for both transverse electric and transverse magnetic waves. Comparisons between the surface impedance boundary condition (SIBC) and the exact results are presented. The effects of the incident angle, the material surfaces and random roughness on the directional emissivity are quantified. The contributions of the effects to the emissivity are used to investigate the domains of validity of the approximate model.

PACS: Type pacs here, separated by semicolons ;

Keywords: approximate and exact methods, emissivity, random finite conductive surfaces.

1. Introduction

The directional scattering distribution from rough surfaces is of interest in many engineering disciplines. The radiative properties can be quantified by a number of exact and approximate methods. The exact methods provide rigorous solutions through recent advances in numerical methods. The approximate methods, like the geometric optics, Fresnel and the Kirchhoff approximations are computationally less expensive than the exact methods. The range of validity of the approximative models is of interest to many researches [1-5]. The mathematical complexity of many problems in the theory of wave scattering from a metal obstacle is appreciably reduced if the metal is perfectly conducting [5]. For some metals used in optics which are characterized by the finite conductivity, such as Al, W and Au, problems treated using the exact methods usually require large amounts of numerical calculations and the use of sophisticated computer codes. Integral methods based on the electromagnetic theory solve Maxwell's equations at each side of the boundary and the magnetic field H. For the finite conductivity metal the exact methods are very computationally intensive. The scattering problems can also be solved if the relation between the tangential components of the fields $E_{//}$ and $H_{//}$ along the interface is known. The theory is applied to the derivation of a surface impedance condition [6]. The surface impedance Z_0 which is constant along the surface is replaced by a point–

* I. Sassi. Tel.: +0-000-000-0000 ; fax: +0-000-000-0000 .
E-mail address: fss_imedsassi@yahoo.com.

doi:10.1016/j.phpro.2009.11.024

depending surface impedance Z(x). If Z is known the fields outside the scatter can be found without solving Maxwell's equation inside it. The approximation model was used to calculate the efficiency of metallic diffraction grating [7-9].

In this paper, we will consider a one-dimensional random rough surface. The random rough surface is characterized by the correlation length τ and the rms deviation σ. The directional emissivity from a material surface is the radiation property of interest. The influence of roughness, the incident angle and surface materials on surface emission are interested for both transverse electric polarization (TE) and transverse magnetic polarization (TM). The SIBC solutions are compared with the Monte-Carlo solutions for the [10], and the regions of validity of both approximations are discussed.

2. INTEGRAL FORMALISM FOR ROUGH SURFACES

The scattering problem considered here is schematized in Fig. 1. The boundary $\sum$ between medium 1(vacuum) and medium 2 (metal) is denoted by the function $z = h(x)$.

Let $U(x, z)e^{-j\omega t}$ represent the electric field or the magnetic field. The incident field is:

$$U_{inc}(x,z) = \exp jk\left(x\sin\theta_0 - z\cos\theta_0\right), \quad k = 2\pi/\lambda . \tag{1}$$

The total field $U_S(x, z)$ satisfies the Helmholtz wave equations:

$$\left(\nabla^2 + k^2\right)U_S^+(x, z) = 0 \quad z > h(x), \text{ in the air zone,} \tag{2a}$$

$$\left(\nabla^2 + k^2\varepsilon^2\right)U_S^+(x, z) = 0 \quad z < h(x), \text{ in the metal zone.} \tag{2b}$$

It satisfies the radiation conditions at $z \longrightarrow \pm\infty$

$U_S^+(x, z \to \pm\infty) \to$ out going waves,

and the following boundary conditions at Σ :

$$U^+(x,z) = U^-(x,z), \tag{3a}$$

$$C\frac{\partial U^+(x, z)}{\partial n} = \frac{\partial U^-(x, z)}{\partial n}, \tag{3b}$$

where $\partial/\partial n$ Indicates the normal derivative, and C is a constant equal to 1 (TE) or ε (TM).

By using the Green's second theorem and boundary conditions we can derive the integral equations for the scattered electromagnetic field. This field is expressed in terms of the unknown field and its normal derivative (source functions); these functions are defined in vacuum on the boundary (P) as following:

-for TM polarization case

$$H(x) = H^{(1)}(x,z)\Big|_{z=h^+(x)} , \tag{4a}$$

$$L(x) = (-h'(x)\frac{\partial}{\partial x} + \frac{\partial}{\partial z})H^{(1)}(x,z)\Big|_{z=h^+(x)} ; \tag{4b}$$

-for TE polarization case

$$E(x) = E^{(1)}(x,z)\Big|_{z=h^+(x)} , \tag{5a}$$

$$F(x) = (-h'(x)\frac{\partial}{\partial x} + \frac{\partial}{\partial z})E^{(1)}(x,z)\Big|_{z=h^+(x)} . \tag{5b}$$

From integral equations giving a field at any point M(x, z), we can obtain the source functions.

A numerical method used to solve these equations, as described in references [11, 12], consists to convert the infinite systems of integral equations into two a finite systems of linear equations as follows [12].

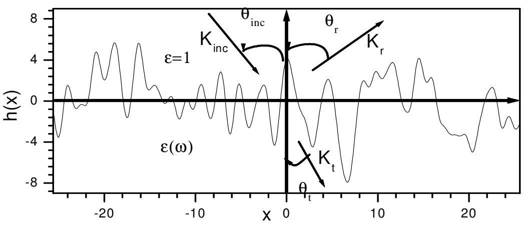


Fig 1: Scattered electromagnetic waves.

3. CURVATURE DEPENDENT SURFACE IMPEDANCE BOUNDARY CONDITION

Some metals used in optics, such as Al, Ag, and Au have a high conductivity. Although when the values of the refraction index of the metals increase, that yield the integration of the equations 4 and 5 very hard because of the appearance of the term $|n^*|$ on the argument of the Hunkel function. This problem can also be solved if the relation between the tangential components of the fields $E_{//}$ and $H_{//}$ along the surface is known. In this second approach Maxwell's boundary conditions are replaced by a boundary condition of the form:

$$E_{//} = Z(\vec{n} \text{ x } H_{//}), \; \mathrm{Re}(Z) > 0. \tag{6}$$

where $\vec{n}$ is a normal unit vector pointing towards the exterior of the scatter and **Z** is a second rank tensor called the surface impedance tensor. If **Z** is known the fields outside the scattered can be found without solving Maxwell's equation inside it.

Using the boundary conditions (equations 3a -3b) and the equation (6), the surface impedance is:

$$Z^{TE} = \frac{k}{i}\frac{E(x)}{\frac{dE(x)}{dn}}. \tag{7a}$$

By using the definition of F given by the equation (5b), we get

$$Z^{TE} = \frac{k}{i}\frac{E(x)}{\gamma F(x)}, \; \gamma = \sqrt{1 + h'^{\,2}(x)}. \tag{7b}$$

In the TM polarization, we get:

$$Z^{TM} = \frac{i}{k}\frac{\frac{dH(x,h(x))}{dn}}{H(x,h(x))}. \tag{8a}$$

By using the definition of F given by the equation (4b), we get

$$Z^{TM} = \frac{i}{k}\frac{\gamma L(x)}{H(x)}. \tag{8b}$$

We define two operators (G and N) associated to the Green function and its normal derivative respectively. The application of such operators to the function ϕ is represented as:

$$G(\phi) = \int_{-\infty}^{+\infty} G(x,x')\,\phi(x')\,dx' \text{ and } N(\phi) = \int_{-\infty}^{+\infty} \frac{dG(x,x')}{dn}\,\phi(x')\,dx'. \tag{9}$$

Using the delta family as $|n^*| \to \infty$ the kernel $G(x,x')$ tends to a delta function $G(\phi)$ can be approximated in the following way:

$$G(\phi) \approx \phi(x) \int_{-\infty}^{+\infty} G(x,x')\,dx'. \tag{10}$$

In the approximation it is found that the operators G and N tend towards multiplicative operators [7-9, 13] so that:

$$G(\phi) = \frac{\phi(x)}{2\,i\,k\,n^{*}\,\gamma(x)} \text{ and } N(\phi) \approx \frac{i}{4kn^{*}} \frac{h''(x)\,\phi(x)}{\gamma^{3}(x)}. \quad (11)$$

When the above relations are introduced into the integral equations [13] we get:

$$\left(1+\frac{jh''}{2kn^{*}\rho^{3}}\right) H(x) = \frac{jn^{*}}{k\rho^{3}} L(x), \qquad \text{TM polarization} \quad (12a)$$

$$\left(1+\frac{jh''}{2kn^{*}\rho^{3}}\right) E(x) = \frac{j}{kn^{*}\rho^{3}} F(x). \qquad \text{TE polarization} \quad (12b)$$

ρ is the radius of curvature of the random rough surface z=h(x),

$$\rho(x) = Z_{0}\frac{\left(1+h'^{2}(x)\right)^{\frac{3}{2}}}{h''(x)}.$$

By using the equations (11a) and (11b) the surface impedance become:

$$Z^{TM}(x) = \left[1+\frac{j}{2kn^{*}\rho}\right], \quad (13a)$$

$$Z^{TE}(x) = \left[1+\frac{j}{2kn^{*}\rho}\right]^{-1}. \quad (13b)$$

Where $Z_0 = 1/n^*$ is the constant value obtained for flat boundary [14].

The surface radiative properties are defined in reference [5].

4. RESULTS AND DISCUSSION

Figs. 2 show curves displaying the emissivity $\varepsilon'^{\cap}$ versus the ratio τ/λ at normal incidence $(\theta = 0°)$. It is seen that the curves obtained by the SIBC method has the same horizontal asymptotic behavior for both materials. The horizontal asymptote is defined by the value of the emissivity of a flat surface given by using the Fresnel formula [5]. Figs. 2(a) and 2(b) show the emissivity for an rms deviation equal to λ of a gold and tungsten RRS. For the roughness the behavior of the TE-polarized emissivity is the same for the two surface materials (Au and W). But for random rough surfaces the behavior of the TM-polarized emissivity is not the same. Fig. 1 (a) shows the resonance region for gold RRS in TM polarization, defined by the ratio τ/λ varying from 2 to 6 approximately, and a peak of emissivity due to the emission mediated by surface Plasmon [16]. The phenomenon treated in Ref. 17, showed that two different peaks appeared on the emissivity.

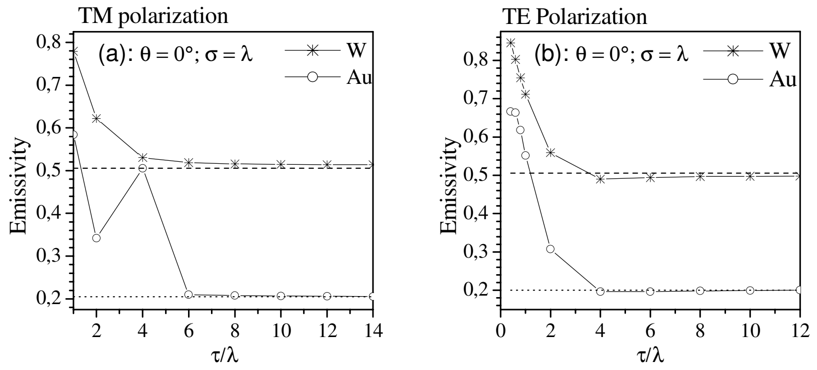


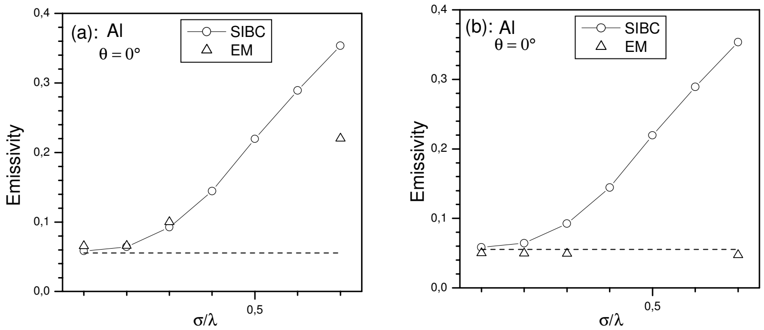
Fig 2: Effect of materials to the emissivity of random rough surfaces, at normal incidence ($\theta_0 = 0°$).

Parameters: $\lambda = 0.55\ \mu m$, optical constants of gold: $n = 0.4$ and $\chi = 2.45$, optical constants of tungsten: $n = 3.5$ and $\chi = 2.7$.

For figs. 3 and 4, the random rough surfaces are characterized by the same correlation length $\tau = 1\ \mu m$, the range of rms roughness to wavelength ratios is from $\sigma/\lambda = 0$ to 0.7. A comparison of the approximate model (SIBC) with the exact model of the directional emissivity at two incident angles, $\theta_0 = 0°$ and $\theta_0 = 30°$, for a two conductor materials, tungsten (W) and aluminium (Al) are presented. The wavelength for the tungsten is $\lambda = 0.55\ \mu m$ and for the aluminium is $\lambda = 1\ \mu m$, with the optical constants equal to $n = 3.5$ and $\chi = 2.7$, and $n = 1.35$ and $\chi = 9.58$ respectively.

For the conductor material aluminium, figs. 3 shows that for the incidence angle 0°, the approximate method (SIBC) and the exact method are in good agreement for the TM and TE polarization cases if the ratio σ/λ is less than 0.3 and 0.2 respectively. For the incident angle 30°, for all values of the rms deviation, it's noticed that the approximate method SIBC is valid for the TM polarization case, but is not valid for the TE polarization case.

For the tungsten conductor at incident angle 0° (figs 4e and f) the approximate method and the exact method are in good agreement if the ratio σ/λ is less than 0.3 with an error less than 7.29 % for the TM polarization case. The SIBC is considered valid only for σ/λ less than 0.2 with an error not beyond 9.72 %. At incident angle 30°, for the TM polarization the SIBC is in good agreement for the rms deviation less than $0.3\,\lambda$ with an error limited by the value equal to 6.1 %. At the same incident angle for the TE polarization case it is necessary to use an exact method because the approximate method is not in good agreement for all values of rms deviation (fig. 4h).



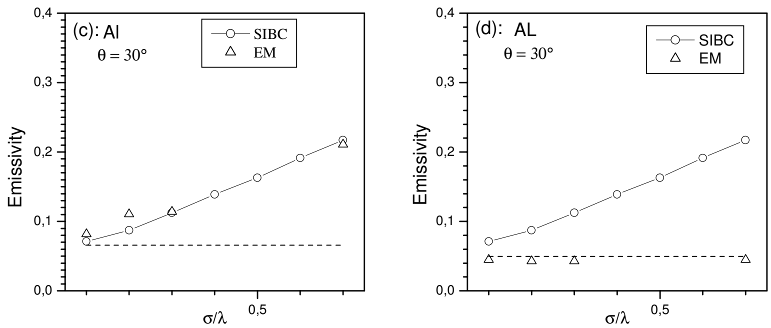


Fig 3: Comparison of directional emissivities solutions for aluminium random rough surfaces, at $\theta_0 = 0°$ versus the ratio σ/λ, for the exact method and the surface impedance boundary condition approximation. (a) and (c): TM polarization, (b) and (d): TE polarization.

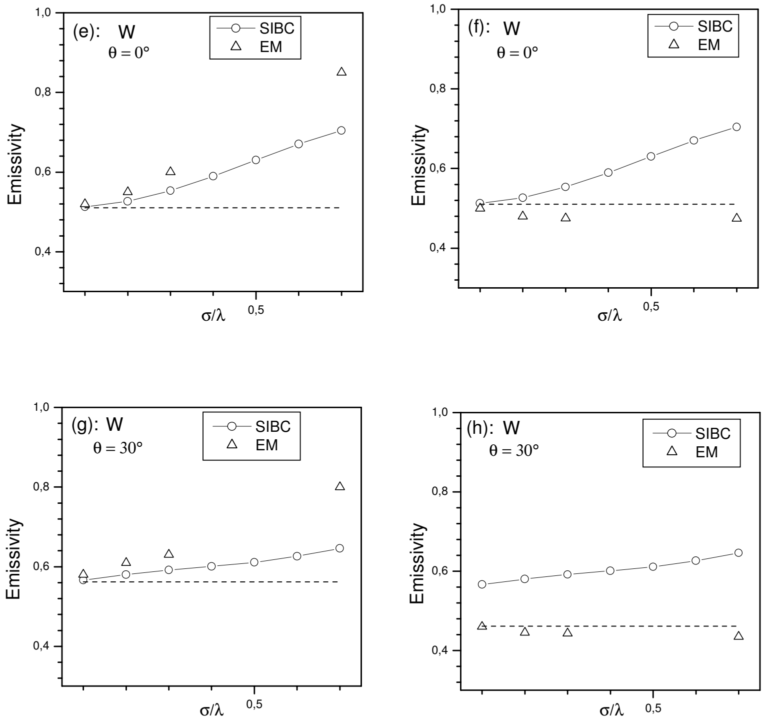


Fig 4: Comparison of directional emissivities solutions for aluminium random rough surfaces, at $\theta_0 = 0°$ versus the ratio σ/λ, for the exact method and the surface impedance boundary condition approximation. (e) and (g): TM polarization, (f) and (h): TE polarization.

CONCLUSION

A comparison between the exact method and the approximate method for the emissivity of the finite conductivity random rough surfaces has been presented. The effects of the incident angle, the rms deviation versus the wavelength and the physical parameters to the validity of the method called the surface impedance boundary condition are discussed. The regions of validity depend on the polarization, the incident angle and the index of refraction of the conductor. The domains of validity are more extended for TM than the TE polarization and for the aluminium than the tungsten. The approximate method is valid when the rms deviation is less than $0.2\ \lambda$ at incident angle 0° for both TE and TM, but at incident angle 30° for all values of the rms deviation, it's remarked that the approximate method SIBC is valid for the TM polarization case, but is not for the TE polarization case.

REFERENCES

[1] E.I. Thorsos, J. Acoust. Soc. Am. 83 (1988) 78-92.
[2] A. Ralph Dimenna, O. Richard Buckius, J. of Thermo Physics and Heat Transfer 8 (1994) 393-399.
[3] K. Tang, A. Ralph Dimenna, O. Richard Buckius, Int. J. Heat Mass Transfer 40 (1997) 49-59.
[4] F. Ghmari, I. Sassi, M.S. Sifaoui, Waves in Random and Complex Media 15 (2005) 469-486.
[5] I. Sassi, M.S. Sifaoui, J. Opt. Soc. Am. A24 (2007) 451-462.
[6] M.A. Leontovich, V.A. Fork, JETP (USSR) 16 (1946) 557.
[7] R.A. Depine, J.M. Simon, Optica-Acta 29 (1982) 1459-1473.
[8] R.A Depine, J. M. Simon, Optica-Acta 30 (1983) 313.
[9] R.A Depine, Appl. Opt. 26 (1987) 2348.
[10] R. Carminati, J.J Greffet, Heat transfer 1998, proceeding of 11th IHTS. 7 August Kyongyu, Koria 23-28 (1998).
[11] A.A. Maradudin, T. Michel, A.R. McGum, E.R. Mendez, Annals of Physics 203 (1999) 255-307.
[12] J.A. Sanchez-Gil, M. Nieto-Vesperinas, J. Opt. Soc. Am. A 8 (1991) 1270-1286.
[13] A. Depine Richardo, Optik 79 (1988) 75-80.
[14] L.M. Brekhovskikh, Waves in layered media (Academic Press, New York, 1960) p. 14.
[15] M.Q. Brewster, Thermal Radiative Transfer and Properties, 1st cdn. John Wiley, New York, 1992.
[16] F. Ghmari, T. Ghbara, M. Laroche, R. Carminati, J.J. Greffet, App. Phys. 96 (2004) 2656-2664.
[17] H. Sai, Y. Kanamori, H. Yugami, J. Micromech. Microeng. 15 (2005) 243-249.