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A heterotic standard model

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Abstract

Within the context of the $E_8 \times E_8$ heterotic superstring compactified on a smooth Calabi–Yau threefold with an $SU(4)$ gauge instanton, we show the existence of simple, realistic $N = 1$ supersymmetric vacua that are compatible with low-energy particle physics. The observable sector of these vacua has gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, three families of quarks and leptons, each with an additional *right-handed neutrino*, *two* Higgs–Higgs conjugate pairs, a small number of uncharged moduli and *no exotic matter*. The hidden sector contains non-Abelian gauge fields and moduli. In the strong coupling case there is *no exotic matter*, whereas for weak coupling there are a *small number* of additional matter multiplets in the hidden sector. The construction exploits a mechanism for "splitting" multiplets. The minimal nature and rarity of these vacua suggest the possible theoretical and experimental relevance of spontaneously broken $U(1)_{B-L}$ gauge symmetry and two Higgs–Higgs conjugate pairs. The $U(1)_{B-L}$ symmetry helps to naturally suppress the rate of nucleon decay.

The discovery of non-vanishing neutrino masses indicates that, in supersymmetric theories without exotic multiplets, a right-handed neutrino must be added to each family of quarks and leptons [1]. It is well known that this augmented family fits exactly into the **16** spin representation of *Spin*(10), making this group very compelling from the point of view of grand unification and string theory. Within the context of $N = 1$ supersymmetric $E_8 \times E_8$ heterotic string vacua, a *Spin*(10) group can arise from the spontaneous breaking of the observable sector E_8 group by an $SU(4)$ gauge instanton on an internal Calabi–Yau threefold [2]. The *Spin*(10) group is then broken by a Wilson line to a gauge group containing $SU(3)_C \times SU(2)_L \times U(1)_Y$ as a factor [3]. To achieve this, the Calabi–Yau manifold must have, minimally, a fundamental group $\mathbb{Z}_3 \times \mathbb{Z}_3$.

Until now, such vacua could not be constructed since (a) Calabi–Yau threefolds with fundamental group $\mathbb{Z}_3 \times \mathbb{Z}_3$ were not known and (b) it was unknown how to find $SU(4)$ gauge instantons on such manifolds. Recently, the first problem was rectified

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in [4]. We have now solved the second problem, exhibiting a large class of $SU(4)$ gauge instantons on the Calabi–Yau manifolds presented in [4]. Generalizing the results in [5,6], these instantons are obtained as connections on stable, holomorphic vector bundles with structure group $SU(4)$. The technical details will be given elsewhere [7]. In addition to these considerations, we also use a natural method for "splitting" multiplets that was introduced for general bundles in [6]. In this Letter, we present the results of our search for realistic vacua in this context.

The results are very encouraging. We find $N = 1$ supersymmetric vacua whose minimal observable sector, for both the weakly and strongly coupled heterotic string, has the following properties.

- Observable sector. Weak and strong coupling.
  1. Gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$.
  2. *Three families* of quarks and leptons, each with a *right-handed neutrino*.
  3. *Two* Higgs–Higgs conjugate pairs.
  4. *Six* geometric moduli and a *small number* of vector bundle moduli.
  5. *No exotic matter fields*.

These are, to our knowledge, the first vacua in any string theory context whose observable sector contains no exotic matter. We emphasize that, although very similar to the supersymmetric standard model, our observable sector differs in three significant ways. First, there is an extra right-handed neutrino in each family. Closely related to this is the appearance of an additional gauged B–L symmetry. Finally, we find, not one, but two Higgs–Higgs conjugate pairs.

The structure of the hidden sector depends on whether one is in the weakly or strongly coupled regime of the heterotic string. In the strongly coupled context, we find the following minimal hidden sector.

- Hidden sector. Strong coupling.
  1. Gauge group $E_7 \times U(6)$.
  2. A *small number* of vector bundle moduli.
  3. *No matter fields*.

Again, note that this hidden sector has no exotic matter. Combining this with the above, we have demonstrated, within the context of the strongly coupled heterotic string, the existence of realistic vacua containing no exotic matter fields. We emphasize that the hidden sector gauge group $E_7 \times U(6)$ is sufficiently large to allow acceptable supersymmetry breaking via condensation of its gauginos.

In the weakly coupled context, we find the following minimal hidden sector (this is also a valid vacuum in the strongly coupled case).

- Hidden sector. Weak coupling.
  1. Gauge group $Spin(12)$.
  2. A *small number* of vector bundle moduli.
  3. *Two* matter field multiplets in the **12** of $Spin(12)$.

Note that, in this case, there are a small number of exotic matter multiplets in the hidden sector. Again, the hidden sector gauge group $Spin(12)$ is sufficiently large to allow acceptable supersymmetry breaking via gaugino condensation.

The vacua presented above are the result of an extensive search within the wide context made precise in [7]. They appear to be the minimal vacua, all others containing exotic matter fields, either in the observable sector, the hidden sector, or both, usually with a large number of Higgs–Higgs conjugate pairs. We have been unable to find any vacuum in this context with only a single pair of Higgs–Higgs conjugate fields. Furthermore, to our knowledge, phenomenological vacua in all other string contexts [6,8,10–12] have substantial amounts of exotic matter, both in the observable and hidden sectors. For all these reasons, we refer to the class of vacua presented in this Letter as a *heterotic standard model* and speculate that it may be of phenomenological significance. In particular, it would seem to motivate renewed interest, both theoretical and experimental, in its characteristic properties; namely, (1) the physics of a $U(1)_{B-L}$ gauge symmetry spontaneously broken at, or above, the electroweak scale and (2) the physics of two pairs of Higgs–Higgs conjugate fields, particularly their experimental implications for flavor changing neutral currents. It is immediately clear that the B–L symmetry will help to naturally suppress the rate of nucleon decay. This potentially resolves a long-standing problem in phenomenological string vacua. At the least, our results go a long way toward demonstrating that realistic particle physics can be the low-energy manifestation of the $E_8 \times E_8$ heterotic superstring, as originally envisaged in [8,9].

We now specify, in more detail, the properties of the these minimal vacua and indicate how they are determined. Following [8], the requisite Calabi–Yau threefold, X, is constructed as follows. We begin by considering a simply connected Calabi–Yau threefold, $\tilde{X}$, which is an elliptic fibration over a rational elliptic surface, $d\mathbb{P}_9$. In a six-dimensional region of moduli space, such manifolds can be shown to admit a $\mathbb{Z}_3 \times \mathbb{Z}_3$ group action which is fixed point free. It follows that

$$X = \frac{\tilde{X}}{\mathbb{Z}_3 \times \mathbb{Z}_3} \tag{1}$$

is a smooth Calabi–Yau threefold that is torus-fibered over a singular $d\mathbb{P}_9$ and has non-trivial fundamental group

$$\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3, \tag{2}$$

as desired. It was shown in [4] that X has

$$h^{1,1}(X) = 3, \qquad h^{2,1}(X) = 3 \tag{3}$$

Kähler and complex structure moduli, respectively. To our knowledge, this is the only Calabi–Yau threefold with $\mathbb{Z}_3 \times \mathbb{Z}_3$ fundamental group that has been constructed. We note [13] that the transpose of the configuration matrix [14] associated with $\tilde{X}$ defines another simply connected Calabi–Yau threefold. Interestingly, this is precisely the manifold introduced by Tian and Yau [15] which, when quotiented by $\mathbb{Z}_3$, was used to construct three generation heterotic string vacua within the context of the standard gauge embedding.

We now construct a stable, holomorphic vector bundle, V, on X with structure group

$$G = SU(4) \tag{4}$$

contained in the E_8 of the observable sector. This bundle admits a gauge connection satisfying the Hermitian Yang–Mills equations. The connection spontaneously breaks the observable sector E_8 gauge symmetry to

$$E_8 \to Spin(10), \tag{5}$$

as desired. We produce V by building stable, holomorphic vector bundles $\tilde{V}$ with structure group $SU(4)$ over $\tilde{X}$ that are equivariant under the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$. This is accomplished by generalizing the method of "bundle extensions" introduced in [5]. The bundle V is then given as

$$V = \frac{\tilde{V}}{\mathbb{Z}_3 \times \mathbb{Z}_3}. \tag{6}$$

Realistic particle physics phenomenology imposes additional constraints on $\tilde{V}$. To ensure that there are three generations of quarks and leptons after quotienting out $\mathbb{Z}_3 \times \mathbb{Z}_3$ one must require that

$$c_3(\tilde{V}) = \pm 54, \tag{7}$$

where $c_3(\tilde{V})$ is the third Chern class of $\tilde{V}$. Recall that with respect to $SU(4) \times Spin(10)$ the adjoint representation of E_8 decomposes as

$$\begin{aligned} \mathbf{248} = {} & (\mathbf{1}, \mathbf{45}) \oplus (\mathbf{15}, \mathbf{1}) \oplus (\mathbf{4}, \mathbf{16}) \\ & \oplus (\bar{\mathbf{4}}, \overline{\mathbf{16}}) \oplus (\mathbf{6}, \mathbf{10}). \end{aligned} \tag{8}$$

The number of $\overline{\mathbf{16}}$ zero modes is given by $h^1(\tilde{X}, \tilde{V}^*)$ [6]. Therefore, if we demand that there be no exotic matter fields arising from vector-like $\overline{\mathbf{16}}$–$\mathbf{16}$ pairs, $\tilde{V}$ must be constrained so that

$$h^1\big(\tilde{X}, \tilde{V}^*\big) = 0. \tag{9}$$

Similarly, the number of $\mathbf{10}$ zero modes is $h^1(\tilde{X}, \wedge^2 \tilde{V})$. However, since the Higgs fields arise from the decomposition of the $\mathbf{10}$, we must not set the associated cohomology to zero. Rather, we restrict $\tilde{V}$ so that $h^1(\tilde{X}, \wedge^2 \tilde{V})$ is minimal, but non-vanishing. Subject to all the constraints that $\tilde{V}$ must satisfy, we find

$$h^1\big(\tilde{X}, \wedge^2 \tilde{V}\big) = 14. \tag{10}$$

Finally, for the gauge connection to satisfy the Hermitian Yang–Mills equations the holomorphic bundle $\tilde{V}$ must be stable. A complete proof of the stability of $\tilde{V}$ is technically very involved and has not been carried out. However, there are a number of non-trivial checks of stability that can be made. Specifically, stability constrains the cohomology of $\tilde{V}$ to satisfy

$$\begin{aligned} & h^0(\tilde{X}, \tilde{V}) = 0, \qquad h^0\big(\tilde{X}, \tilde{V}^*\big) = 0, \\ & h^0\big(\tilde{X}, \tilde{V} \otimes \tilde{V}^*\big) = 1. \end{aligned} \tag{11}$$

We have shown [7] that vector bundles $\tilde{V}$ satisfying constraints (7), (9), (10) and (11) indeed exist. Henceforth, we will restrict our discussion to such bundles.

We now extend the observable sector bundle V by adding a Wilson line, W, with holonomy

$$\mathrm{Hol}(W) = \mathbb{Z}_3 \times \mathbb{Z}_3 \subset Spin(10). \tag{12}$$

The associated gauge connection spontaneously breaks *Spin*(10) as

$$Spin(10) \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}, \tag{13}$$

where $SU(3)_C \times SU(2)_L \times U(1)_Y$ is the standard model gauge group. Since $\mathbb{Z}_3 \times \mathbb{Z}_3$ is Abelian and rank$(Spin(10)) = 5$, an additional rank one factor must appear. For the chosen embedding of $\mathbb{Z}_3 \times \mathbb{Z}_3$, this is precisely the gauged B–L symmetry.

As discussed in [6], the zero mode spectrum of $V \oplus W$ on X is determined as follows. Let R be a representation of $Spin(10)$, and denote the associated $\tilde{V}$ bundle by $U_R(\tilde{V})$. Find the representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on $H^1(\tilde{X}, U_R(\tilde{V}))$ and tensor this with the representation of the Wilson line on R. The zero mode spectrum is then the invariant subspace under this joint group action. Let us apply this to the case at hand. First consider the $\overline{\mathbf{16}}$ representation. It follows from Eq. (9) that no such representations occur. Hence, no exotic $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ fields arising from vector-like $\overline{\mathbf{16}}$–$\mathbf{16}$ pairs appear in the spectrum, as desired. Now examine the $\mathbf{16}$ representation. The Atiyah–Singer index theorem, Eqs. (7) and (9) imply that

$$h^1(\tilde{X}, \tilde{V}) = 27. \tag{14}$$

We can calculate the $\mathbb{Z}_3 \times \mathbb{Z}_3$ representation on $H^1(\tilde{X}, \tilde{V})$ as well as the Wilson line action on $\mathbf{16}$. Tensoring these together, we find that the invariant subspace consists of three families of quarks and leptons, each family transforming as

$$(\mathbf{3}, \mathbf{2}, 1, 1), \quad (\bar{\mathbf{3}}, \mathbf{1}, -4, -1), \quad (\bar{\mathbf{3}}, \mathbf{1}, 2, -1) \tag{15}$$

and

$$(\mathbf{1}, \mathbf{2}, -3, -3), \quad (\mathbf{1}, \mathbf{1}, 6, 3), \quad (\mathbf{1}, \mathbf{1}, 0, 3) \tag{16}$$

under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. We have displayed the quantum numbers $3Y$ and $3(B$–$L)$ for convenience. Note from Eq. (16) that each family contains a right-handed neutrino, as desired.

Finally, consider the $\mathbf{10}$ representation. Recall from Eq. (10) that $h^1(\tilde{X}, \wedge^2 \tilde{V}) = 14$. We find that the representations of the two generators of $\mathbb{Z}_3 \times \mathbb{Z}_3$ in $H^1(\tilde{X}, \wedge^2 \tilde{V})$ are given by the 14×14 matrices

$$\mathrm{diag}\big(1, 1, 1, \omega_1, \omega_1^2, \omega_1, \omega_1^2, 1, 1, 1, \omega_1, \omega_1^2, \omega_1, \omega_1^2\big) \tag{17}$$

and

$$\mathrm{diag}\big(1, \omega_2, \omega_2^2, 1, 1, \omega_2^2, \omega_2, 1, \omega_2, \omega_2^2, 1, 1, \omega_2^2, \omega_2\big) \tag{18}$$

respectively, where ω_1 and ω_2 are third roots of unity. Furthermore, the Wilson line W can be chosen so that

$$\mathbf{10} = \big(\omega_1^2\big)\mathbf{5} \oplus (\omega_1)\bar{\mathbf{5}} \tag{19}$$

and

$$\mathbf{10} = \big(\mathbf{2} \oplus \big(\omega_2^2\big)\mathbf{3}\big) \oplus \big(\bar{\mathbf{2}} \oplus (\omega_2)\bar{\mathbf{3}}\big) \tag{20}$$

are the representations on $\mathbf{10}$ of the first and second generators. Tensoring these actions together, one finds that the invariant subspace consists of *two* copies of the vector-like pair

$$(\mathbf{1}, \mathbf{2}, 3, 0), \quad (\mathbf{1}, \bar{\mathbf{2}}, -3, 0). \tag{21}$$

That is, there are two Higgs–Higgs conjugate pairs occurring as zero modes of our vacuum.

Putting these results together, we conclude that the zero mode spectrum of the observable sector (1) has gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, (2) contains *three families* of quarks and leptons each with a *right-handed neutrino*, (3) has *two* Higgs–Higgs conjugate pairs and (4) *contains no exotic fields of any kind*. Additionally, there are (5) a *small number* of uncharged vector bundle moduli. These arise from the invariant subspace of $H^1(\tilde{X}, \tilde{V} \otimes \tilde{V}^*)$ under the action of $\mathbb{Z}_3 \times \mathbb{Z}_3$.

Thus far, we have discussed the vector bundle of the observable sector. However, the vacuum can contain a stable, holomorphic vector bundle, V', on X whose structure group is in the E_8' of the hidden sector. As above, one can construct V' by building stable, holomorphic vector bundles $\tilde{V}'$ over $\tilde{X}$ which are equivariant under $\mathbb{Z}_3 \times \mathbb{Z}_3$ using the method of “bundle extensions”. V' is then obtained by taking the quotient of $\tilde{V}'$ with $\mathbb{Z}_3 \times \mathbb{Z}_3$. The requirement of anomaly cancellation relates the observable and hidden sector bundles, imposing the constraint that

$$[\mathcal{W}] = c_2(T\tilde{X}) - c_2(\tilde{V}) - c_2(\tilde{V}') \tag{22}$$

must be an effective class. Here c_2 is the second Chern class. In the strongly coupled heterotic string, $[\mathcal{W}]$ is the class of the holomorphic curve around which a bulk space five-brane is wrapped. In the weakly coupled case $[\mathcal{W}]$ must vanish. We have previously constructed $\tilde{X}$ and $\tilde{V}$ and, hence, can compute $c_2(T\tilde{X})$

and $c_2(\tilde{V})$. Then Eq. (22) becomes a constraint on the hidden sector bundle $\tilde{V}'$. The easiest possibility is that $\tilde{V}'$ is the trivial bundle. However, in this case, we find that $[\mathcal{W}]$ is not effective.

The next simplest choice is to take $\tilde{V}'$ to have structure group

$$G' = SU(2) \tag{23}$$

in E_8'. This spontaneously breaks the hidden sector E_8' symmetry to

$$E_8' \to E_7. \tag{24}$$

Recall that with respect to $SU(2) \times E_7$ the adjoint representation of E_8' decomposes as

$$\mathbf{248}' = (\mathbf{1}, \mathbf{133}) \oplus (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{2}, \mathbf{56}). \tag{25}$$

We now require that there be no exotic matter fields in the hidden sector. This imposes the additional constraint that

$$h^1(\tilde{X}, \tilde{V}') = 0. \tag{26}$$

Finally, the requirement that $\tilde{V}'$ be stable implies the conditions

$$h^0(\tilde{X}, \tilde{V}') = 0, \qquad h^0(\tilde{X}, \tilde{V}'^*) = 0,$$
$$h^0(\tilde{X}, \tilde{V}' \otimes \tilde{V}'^*) = 1. \tag{27}$$

It can be shown [7] that vector bundles $\tilde{V}'$ satisfying Eqs. (22), (23), (26) and (27) can be constructed. For these bundles $[\mathcal{W}]$ is non-vanishing and, hence, this is a vacuum of the strongly coupled heterotic string. The five-brane wrapped on a holomorphic curve associated with $[\mathcal{W}]$ contributes non-Abelian gauge fields, but no matter fields, to the hidden sector. Following the results in [16], we find that the five-brane gauge group is

$$G_5' = U(6). \tag{28}$$

Moving in the moduli space of the holomorphic curve, this group can be maximally broken to $U(1)^6$. We conclude that, within the context of the strongly coupled heterotic string, our observable sector is consistent with a hidden sector with gauge group $E_7 \times U(6)$ and *no exotic matter*. There are, additionally, a *small number* of uncharged vector bundle moduli that arise from the invariant subspace of $H^1(\tilde{X}, \tilde{V}' \otimes \tilde{V}'^*)$ under $\mathbb{Z}_3 \times \mathbb{Z}_3$, as well as some five-brane moduli.

We now exhibit a hidden sector, compatible with our observable sector, that has no five-branes; that is, for which

$$[\mathcal{W}] = 0. \tag{29}$$

This does not occur for structure group $G' = SU(2)$. From the results in [17], we expect that the appropriate group may be the product of two non-Abelian groups. The simplest choice is

$$G' = SU(2) \times SU(2). \tag{30}$$

This bundle, which is the sum of two $SU(2)$ factors $\tilde{V}' = \tilde{V}_1' \oplus \tilde{V}_2'$, spontaneously breaks the hidden sector E_8' gauge group to

$$E_8' \to Spin(12). \tag{31}$$

With respect to $SU(2) \times SU(2) \times Spin(12)$ the adjoint representation of E_8' decomposes as

$$\mathbf{248}' = (\mathbf{3}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{66})$$
$$\oplus (\mathbf{1}, \mathbf{2}, \mathbf{32}) \oplus (\mathbf{2}, \mathbf{1}, \mathbf{32}) \oplus (\mathbf{2}, \mathbf{2}, \mathbf{12}). \tag{32}$$

The hidden sector will have no exotic matter fields if

$$h^1(\tilde{X}, \tilde{V}_1') = 0, \qquad h^1(\tilde{X}, \tilde{V}_2') = 0, \tag{33}$$

and

$$h^1(\tilde{X}, \tilde{V}_1' \otimes \tilde{V}_2') = 0. \tag{34}$$

Finally, note that the stability of each bundle $\tilde{V}_i'$, $i = 1, 2$ implies the conditions

$$h^0(\tilde{X}, \tilde{V}_i') = 0, \qquad h^0(\tilde{X}, \tilde{V}_i'^*) = 0,$$
$$h^0(\tilde{X}, \tilde{V}_i' \otimes \tilde{V}_i'^*) = 1, \quad i = 1, 2. \tag{35}$$

Subject to Eq. (22) and the condition Eq. (29) that there be no five-brane, we are unable to simultaneously satisfy all of the constraints in Eqs. (33), (34) and (35). Demanding that the stability conditions Eq. (35) hold, it is possible to choose $\tilde{V}_i'$, $i = 1, 2$ so that only the first condition in Eq. (33) is fulfilled. One finds that, minimally,

$$h^1(\tilde{X}, \tilde{V}_2') = 4 \tag{36}$$

and

$$h^1(\tilde{X}, \tilde{V}_1' \otimes \tilde{V}_2') = 18. \tag{37}$$

However, we can show that the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on $H^1(\tilde{X}, \tilde{V}_2')$ has no invariant subspace. It follows that the associated matter fields will be projected out under the quotient by $\mathbb{Z}_3 \times \mathbb{Z}_3$. Unfortunately, this is not the case for $H^1(\tilde{X}, \tilde{V}_1' \otimes \tilde{V}_2')$. Here, we find that the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action is two copies of its regular representation, which leaves a two-dimensional subspace of $H^1(\tilde{X}, \tilde{V}_1' \otimes \tilde{V}_2')$ invariant. Hence, after quotienting by $\mathbb{Z}_3 \times \mathbb{Z}_3$, one finds two **12** multiplets of *Spin*(12). We conclude that, for vacua with no five-branes, our observable sector is consistent with a hidden sector with gauge group *Spin*(12) and *two* **12** *multiplets of exotic matter*. There are also vector bundle moduli arising from the $\mathbb{Z}_3 \times \mathbb{Z}_3$ invariant subspace of $H^1(\tilde{X}, \tilde{V}' \otimes \tilde{V}'^*)$. These vacua can occur in the context of both the weakly and strongly coupled heterotic string.

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