# Understanding the penguin amplitude in $B \rightarrow \phi K$ decays 

S. Mishima<br>Department of Physics, Nagoya University, Nagoya 464-8602, Japan

Received 19 July 2001; received in revised form 12 September 2001; accepted 24 September 2001 Editor: T. Yanagida


#### Abstract

We calculate branching ratios for pure penguin decay modes, $B \rightarrow \phi K$ decays using perturbative QCD approach. Our results of branching ratios are consistent with the experimental data and larger than those obtained from the naive factorization assumption and the QCD-improved factorization approach. This is due to a dynamical penguin enhancement in perturbative QCD approach.


© 2001 Elsevier Science B.V. Open access under CC BY license
PACS: 13.25.Hw; 11.10.Hi; 12.38.Bx

## 1. Introduction

Recently the branching ratios of $B \rightarrow \phi K$ decays have been measured by the BaBar [1], BELLE [2] and CLEO [3] Collaborations. There is an interesting problem related a penguin contribution to decay amplitudes [4]. A naive estimate of the loop diagram leads to $P / T \sim \alpha_{s} /(12 \pi) \log \left(m_{t}^{2} / m_{c}^{2}\right) \sim \mathrm{O}(0.01)$ where $P$ is a penguin amplitude and $T$ is a tree amplitude. But experimental data for $\operatorname{Br}(B \rightarrow K \pi)$ and $\operatorname{Br}(B \rightarrow \pi \pi)$ leads to $P / T \sim \mathrm{O}(0.1)$. Therefore, there must be a dynamical enhancement of the penguin amplitude. This problem is studied by Keum et al. using perturbative QCD (PQCD) approach [5]. $B \rightarrow \phi K$ modes are important understanding penguin dynamics, because these modes are dominated by penguins. Here we report our study of $B \rightarrow \phi K$ decays using PQCD.

PQCD method for inclusive decays was developed by many authors over many years, and this formalism has been successful. Recently, PQCD has been applied to exclusive $B$ meson decays, $B \rightarrow K \pi$ [5], $\pi \pi$ [6], $\pi \rho, \pi \omega$ [7], $K K$ [8] and $K \eta^{(1)}$ [9]. PQCD approach is based on the three scale factorization theorem [10,11]. For example, $B \rightarrow K$ transition form factor can be written as

$$
\begin{equation*}
F^{B K} \sim \int[d x][d b] C_{i}(t) \Phi_{K}\left(x_{2}, b_{2}\right) H(t) \Phi_{B}\left(x_{1}, b_{1}\right) \exp \left[-\sum_{j=1,2} \int_{1 / b_{j}}^{t} \frac{d \bar{\mu}}{\bar{\mu}} \gamma_{\phi}\left(\alpha_{s}(\bar{\mu})\right)\right], \tag{1}
\end{equation*}
$$

where $x_{1}$ and $x_{2}$ are momentum fractions of partons, $b_{1}$ and $b_{2}$ are conjugate variables of parton transverse momenta $k_{1 T}$ and $k_{2 T}$, and $\gamma_{\phi}$ is the anomalous dimension of mesons. The hard part $H(t)$ can be calculated

[^0]

Fig. 1. Feynman diagrams contributing to factorizable amplitudes for $B \rightarrow \phi K$.

(a)

(b)

(c)

(d)

Fig. 2. Feynman diagrams contributing to nonfactorizable amplitudes for $B \rightarrow \phi K$.
perturbatively. $C_{i}(t)$ is the Wilson coefficient corresponding to the four-quark operator causing $B \rightarrow K$ transition. The scale $t$ is given explicitly in terms of $x_{1}, x_{2}, b_{1}, b_{2}$ and $M_{B}$, and it is of $\mathrm{O}\left(\sqrt{\bar{\Lambda} M_{B}}\right)$. Here $\bar{\Lambda}=M_{B}-m_{b}$, where $M_{B}$ and $m_{b}$ are $B$ meson mass and $b$ quark mass, respectively. It is important to note that in PQCD, the scale of the Wilson coefficient $t$ can reach below $M_{B} / 2$. In the factorization assumption [12], this scale is fixed at $M_{B} / 2$. The Wilson coefficient for a penguin operator increases as the scale evolves down. This explains the enhancement of the penguin amplitude in PQCD compared to the amplitude obtained by the factorization assumption.

In this method, we can calculate not only factorizable amplitudes but also nonfactorizable and annihilation amplitudes. In case of $B \rightarrow \phi K$ decays, the factorizable amplitudes which can be written in terms of form factors $F^{B K}$ and $F^{\phi K}$ are shown in Fig. 1(a)-(d). The nonfactorizable amplitudes are shown in Fig. 2(a)-(d). Ellipses denote meson wave functions in these figures. For illustration purposes, we show the hard part of the nonfactorizable diagram as the dashed box in Fig. 2(a). The parameters in meson wave functions are calculated from the light-cone QCD sum rules, and the theoretical uncertainty of the parameters is about $30 \%$. The hard part depends on the particular processes, but it is calculable. The wave functions contain nonperturbative dynamics and are not calculable, but once it is known, it can be used for other decay processes.

In this Letter, we calculate branching ratios for $B \rightarrow \phi K$ modes using PQCD approach. The detail is discussed in Ref. [13]. We predict the branching ratios for $B \rightarrow \phi K$ decays, and our predictions agree with the current experimental data and are larger than the values obtained from the naive factorization assumption (FA) and the QCD-improved factorization (QCDF) [14,15].

## 2. $B \rightarrow \phi K$ amplitudes

We consider $B$ meson to be at rest. In the light-cone coordinate, the $B$ meson momentum $P_{1}$, the $K$ meson momentum $P_{2}$ and the $\phi$ meson momentum $P_{3}$ are taken to be

$$
\begin{equation*}
P_{1}=\frac{M_{B}}{\sqrt{2}}\left(1,1, \mathbf{0}_{T}\right), \quad P_{2}=\frac{M_{B}}{\sqrt{2}}\left(1-r_{\phi}^{2}, 0, \mathbf{0}_{T}\right), \quad P_{3}=\frac{M_{B}}{\sqrt{2}}\left(r_{\phi}^{2}, 1, \mathbf{0}_{T}\right), \tag{2}
\end{equation*}
$$

where $r_{\phi}=M_{\phi} / M_{B}$, and the $K$ meson mass is neglected. The momentum of the spectator quark in the $B$ meson is written as $k_{1}$. Since the hard part is independent of $k_{1}^{+}$, the $\delta\left(k_{1}^{+}\right)$function appears after integrating over its conjugate spacial variable. Therefore, $k_{1}$ has only the minus component $k_{1}^{-}$and small transverse components $\mathbf{k}_{1 T}$. $k_{1}^{-}$is given as $k_{1}^{-}=x_{1} P_{1}^{-}$, where $x_{1}$ is a momentum fraction. The quarks in the $K$ meson have plus components $x_{2} P_{2}^{+}$and $\left(1-x_{2}\right) P_{2}^{+}$, and the small transverse components $\mathbf{k}_{2 T}$ and $-\mathbf{k}_{2 T}$, respectively. The quarks in the $\phi$ meson have the minus components $x_{3} P_{3}^{-}$and $\left(1-x_{3}\right) P_{3}^{-}$, and the small transverse components $\mathbf{k}_{3 T}$ and $-\mathbf{k}_{3 T}$, respectively. The $\phi$ meson longitudinal polarization vector $\epsilon_{\phi}$ and two transverse polarization vector $\epsilon_{\phi T}$ are given by $\epsilon_{\phi}=\left(1 / \sqrt{2} r_{\phi}\right)\left(-r_{\phi}^{2}, 1, \mathbf{0}_{T}\right)$ and $\epsilon_{\phi T}=\left(0,0, \mathbf{1}_{T}\right)$.

The $B$ meson wave function for incoming state and the $K$ and $\phi$ meson wave functions for outgoing state with up to twist- 3 terms are written as

$$
\begin{align*}
& \Phi_{B, \alpha \beta, i j}^{(\text {(in })}=\frac{i \delta_{i j}}{\sqrt{2 N_{c}}} \int d x_{1} d^{2} \mathbf{k}_{1 T} e^{-i\left(x_{1} P_{1}^{-} z_{1}^{+}-\mathbf{k}_{1 T} \mathbf{z}_{1 T}\right)}\left[\left(\not p_{1}+M_{B}\right) \gamma_{5} \phi_{B}\left(x_{1}, \mathbf{k}_{1 T}\right)\right]_{\alpha \beta},  \tag{3}\\
& \Phi_{K, \alpha \beta, i j}^{\text {(out) }}=\frac{-i \delta_{i j}}{\sqrt{2 N_{c}}} \int_{0}^{1} d x_{2} e^{i x_{2} P_{2} \cdot z_{2}} \gamma_{5}\left[\not \phi_{2} \phi_{K}^{A}\left(x_{2}\right)+m_{0 K} \phi_{K}^{P}\left(x_{2}\right)+m_{0 K}(\nmid \nmid h-1) \phi_{K}^{T}\left(x_{2}\right)\right]_{\alpha \beta},  \tag{4}\\
& \Phi_{\phi, \alpha \beta, i j}^{\text {(out) }}=\frac{\delta_{i j}}{\sqrt{2 N_{c}}} \int_{0}^{1} d x_{3} e^{i x_{3} P_{3} \cdot z_{3}}\left[M_{\phi} \not \phi_{\phi} \phi_{\phi}\left(x_{3}\right)+\not \not_{\phi} \not \phi_{3} \phi_{\phi}^{t}\left(x_{3}\right)+M_{\phi} \phi_{\phi}^{s}\left(x_{3}\right)\right]_{\alpha \beta}, \tag{5}
\end{align*}
$$

where $i$ and $j$ is color indices, and $\alpha$ and $\beta$ are Dirac indices. $m_{0 K}$ is related to the chiral symmetry breaking scale, $m_{0 K}=M_{K}^{2} /\left(m_{d}+m_{s}\right) . v$ and $n$ are defined as $v^{\mu}=P_{2}^{\mu} / P_{2}^{+}$and $n^{\mu}=z_{2}^{\mu} / z_{2}^{-}=\left(0,1, \mathbf{0}_{T}\right)$. We neglect the terms which are proportional to the transverse polarization vector $\epsilon_{\phi}^{T}$, because these terms drop out from our calculation kinematically. The explicit form of these wave functions will be shown in Section 3.

Widths of $B \rightarrow \phi K$ decays can be expressed as

$$
\begin{equation*}
\Gamma=\frac{G_{F}^{2}}{32 \pi M_{B}}|\mathcal{A}|^{2} . \tag{6}
\end{equation*}
$$

The decay amplitudes, $\mathcal{A}$, and $\overline{\mathcal{A}}$, corresponding to $B^{0} \rightarrow \phi K^{0}$, and $\bar{B}^{0} \rightarrow \phi \bar{K}^{0}$, respectively, are written as

$$
\begin{align*}
& \mathcal{A}=f_{\phi} V_{t s} V_{t b}^{*} F_{e}^{P}+V_{t s} V_{t b}^{*} \mathcal{M}_{e}^{P}+f_{B} V_{t s} V_{t b}^{*} F_{a}^{P}+V_{t s} V_{t b}^{*} \mathcal{M}_{a}^{P},  \tag{7}\\
& \overline{\mathcal{A}}=f_{\phi} V_{t s}^{*} V_{t b} F_{e}^{P}+V_{t s}^{*} V_{t b} \mathcal{M}_{e}^{P}+f_{B} V_{t s}^{*} V_{t b} F_{a}^{P}+V_{t s}^{*} V_{t b} \mathcal{M}_{a}^{P}, \tag{8}
\end{align*}
$$

where $F_{e}$ is the amplitude for factorizable diagrams which are considered in FA. $F_{a}$ and $\mathcal{M}$ are the annihilation factorizable and the nonfactorizable diagrams which are neglected in FA. The indices $e$ and $a$, denote the tree topology, and annihilation topology, respectively. The index $P$ denotes the contribution from diagrams with a penguin operator. The decay amplitudes $\mathcal{A}^{+}$and $\mathcal{A}^{-}$, corresponding to $B^{+} \rightarrow \phi K^{+}$, and $B^{-} \rightarrow \phi K^{-}$, respectively, are written as

$$
\begin{align*}
& \mathcal{A}^{+}=f_{\phi} V_{t s} V_{t b}^{*} F_{e}^{P}+V_{t s} V_{t b}^{*} \mathcal{M}_{e}^{P}+f_{B} V_{t s} V_{t b}^{*} F_{a}^{P}+V_{t s} V_{t b}^{*} \mathcal{M}_{a}^{P}-f_{B} V_{u s} V_{u b}^{*} F_{a}^{T}-V_{u s} V_{u b}^{*} \mathcal{M}_{a}^{T},  \tag{9}\\
& \mathcal{A}^{-}=f_{\phi} V_{t s}^{*} V_{t b} F_{e}^{P}+V_{t s}^{*} V_{t b} \mathcal{M}_{e}^{P}+f_{B} V_{t s}^{*} V_{t b} F_{a}^{P}+V_{t s}^{*} V_{t b} \mathcal{M}_{a}^{P}-f_{B} V_{u s}^{*} V_{u b} F_{a}^{T}-V_{u s}^{*} V_{u b} \mathcal{M}_{a}^{T}, \tag{10}
\end{align*}
$$

where the index $T$ denotes tree contributions. Since the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements for the tree amplitudes are much smaller than for the penguin amplitudes, the tree contributions are very small.

The factorizable diagrams are given as Fig. 1. The factorizable penguin amplitude, $F_{e}^{P}$, which comes from Fig. 1(a) and Fig. 1(b) is written as

$$
\begin{align*}
F_{e}^{P}= & 8 \pi C_{F} M_{B}^{4} \int_{0}^{1} d x_{1} d x_{2} \int_{0}^{\infty} b_{1} d b_{1} b_{2} d b_{2} \phi_{B}\left(x_{1}, b_{1}\right) \\
& \times\left\{\left[\left(1+x_{2}\right) \phi_{K}^{A}\left(x_{2}\right)+r_{K}\left(1-2 x_{2}\right)\left(\phi_{K}^{P}\left(x_{2}\right)+\phi_{K}^{T}\left(x_{2}\right)\right)\right] E_{e}\left(t_{e}^{(1)}\right) N_{t}\left\{x_{2}\left(1-x_{2}\right)\right\}^{c} h_{e}\left(x_{1}, x_{2}, b_{1}, b_{2}\right)\right. \\
& \left.\quad+2 r_{K} \phi_{K}^{P}\left(x_{2}\right) E_{e}\left(t_{e}^{(2)}\right) N_{t}\left\{x_{1}\left(1-x_{1}\right)\right\}^{c} h_{e}\left(x_{2}, x_{1}, b_{2}, b_{1}\right)\right\} \tag{11}
\end{align*}
$$

where $N_{t}\{x(1-x)\}^{c}$ is the factor for the threshold resummation [16]. We use $N_{t}=1.775$ and $c=0.3$ [17]. The evolution factors are defined by $E_{e}(t)=\alpha_{S}(t) a_{e}(t) \exp \left[-S_{B}(t)-S_{K}(t)\right]$ where $\exp \left[-S_{i}(t)\right]$ is the factor for the $k_{T}$ resummation [18,19]. The explicit forms of the factor $S_{i}(t)$ are given, for example, in Ref. [5]. The hard scales $t_{e}^{(1)}$ and $t_{e}^{(2)}$, which are the scales in hard process, are given by $t_{e}^{(1)}=\max \left(\sqrt{x_{2}} M_{B}, 1 / b_{1}, 1 / b_{2}\right)$ and $t_{e}^{(2)}=\max \left(\sqrt{x_{1}} M_{B}, 1 / b_{1}, 1 / b_{2}\right)$. The Wilson coefficient is given by

$$
\begin{equation*}
a_{e}(t)=C_{3}+\frac{C_{4}}{N_{c}}+C_{4}+\frac{C_{3}}{N_{c}}+C_{5}+\frac{C_{6}}{N_{c}}-\frac{1}{2}\left(C_{7}+\frac{C_{8}}{N_{c}}+C_{9}+\frac{C_{10}}{N_{c}}+C_{10}+\frac{C_{9}}{N_{c}}\right) \tag{12}
\end{equation*}
$$

The hard function, which is the Fourier transformation of the virtual quark propagator and the hard gluon propagator, is given by

$$
\begin{equation*}
h_{e}\left(x_{1}, x_{2}, b_{1}, b_{2}\right)=K_{0}\left(\sqrt{x_{1} x_{2}} M_{B} b_{1}\right)\left[\theta\left(b_{1}-b_{2}\right) K_{0}\left(\sqrt{x_{2}} M_{B} b_{1}\right) I_{0}\left(\sqrt{x_{2}} M_{B} b_{2}\right)+\left(b_{1} \leftrightarrow b_{2}\right)\right] \tag{13}
\end{equation*}
$$

The factorizable annihilation diagrams shown in Fig. 1(c) and Fig. 1(d), and the nonfactorizable diagrams shown in Fig. 2(a)-(d) can be also calculated in the same way as $F_{e}^{P}$ [13].

## 3. Numerical results

We use the model of the $B$ meson wave function written as

$$
\begin{equation*}
\phi_{B}(x, b)=N_{B} x^{2}(1-x)^{2} \exp \left[-\frac{1}{2}\left(\frac{x M_{B}}{\omega_{B}}\right)^{2}-\frac{\omega_{B}^{2} b^{2}}{2}\right] \tag{14}
\end{equation*}
$$

where $\omega_{B}=0.40 \mathrm{GeV}$ [20]. $N_{B}$ is determined by normalization condition given by

$$
\begin{equation*}
\int_{0}^{1} d x \phi_{B}(x, b=0)=\frac{f_{B}}{2 \sqrt{2 N_{c}}} \tag{15}
\end{equation*}
$$

The $K$ meson wave functions are given as

$$
\begin{align*}
& \phi_{K}^{A}(x)= \frac{f_{K}}{2 \sqrt{2 N_{c}}} 6 x(1-x)\left[1+3 a_{1}(1-2 x)+\frac{3}{2} a_{2}\left\{5(1-2 x)^{2}-1\right\}\right]  \tag{16}\\
& \begin{aligned}
\phi_{K}^{P}(x)= & \frac{f_{K}}{2 \sqrt{2 N_{c}}}\left[1+\frac{1}{2}\left(30 \eta_{3}-\frac{5}{2} \rho_{K}^{2}\right)\left\{3(1-2 x)^{2}-1\right\}\right. \\
& \left.\quad-\frac{1}{8}\left(3 \eta_{3} \omega_{3}+\frac{27}{20} \rho_{K}^{2}+\frac{81}{10} \rho_{K}^{2} a_{2}\right)\left\{3-30(1-2 x)^{2}+35(1-2 x)^{4}\right\}\right]
\end{aligned}
\end{align*}
$$

$$
\begin{equation*}
\phi_{K}^{T}(x)=\frac{f_{K}}{2 \sqrt{2 N_{c}}}(1-2 x)\left[1+6\left(5 \eta_{3}-\frac{1}{2} \eta_{3} \omega_{3}-\frac{7}{20} \rho_{K}^{2}-\frac{3}{5} \rho_{K}^{2} a_{2}\right)\left(1-10 x+10 x^{2}\right)\right], \tag{18}
\end{equation*}
$$

where $\rho_{K}=\left(m_{d}+m_{s}\right) / M_{K}$ [21,22]. The parameters of these wave functions are given as $a_{1}=0.17, a_{2}=$ $0.20, \eta_{3}=0.015$ and $\omega_{3}=-3.0$ where the renormalization scale is 1 GeV .

The $\phi$ meson wave functions are given as

$$
\begin{align*}
& \phi_{\phi}(x)=\frac{f_{\phi}}{2 \sqrt{2 N_{c}}} 6 x(1-x),  \tag{19}\\
& \phi_{\phi}^{t}(x)=\frac{f_{\phi}^{T}}{2 \sqrt{2 N_{c}}}\left[3(1-2 x)^{2}+\frac{35}{4} \zeta_{3}^{T}\left\{3-30(1-2 x)^{2}+35(1-2 x)^{4}\right\}\right. \\
&\left.\quad+\frac{3}{2} \delta_{+}\left\{1-(1-2 x) \log \frac{1-x}{x}\right\}\right],  \tag{20}\\
& \phi_{\phi}^{s}(x)=\frac{f_{\phi}^{T}}{4 \sqrt{2 N_{c}}}[ \left.(1-2 x)\left(6+9 \delta_{+}+140 \zeta_{3}^{T}-1400 \zeta_{3}^{T} x+1400 \zeta_{3}^{T} x^{2}\right)+3 \delta_{+} \log \frac{x}{1-x}\right], \tag{21}
\end{align*}
$$

where $\zeta_{3}^{T}=0.024$ and $\delta_{+}=0.46$ [23]. We have found that the final results are insensitive to the values chosen for $\zeta_{3}^{T}$ and $\delta_{+}$.
We use the Wolfenstein parameters for the CKM matrix elements $A=0.819, \lambda=0.2196, R_{b} \equiv \sqrt{\rho^{2}+\eta^{2}}=$ 0.38 [24], and choose the angle $\phi_{3}=\pi / 2$ [5]. We have found that the final results are quite insensitive to the values of $\phi_{3}$. For the values of meson masses, we use $M_{B}=5.28 \mathrm{GeV}, M_{K}=0.49 \mathrm{GeV}$ and $M_{\phi}=1.02 \mathrm{GeV}$. In addition, for the values of meson decay constants, we use $f_{B}=190 \mathrm{MeV}, f_{K}=160 \mathrm{MeV}, f_{\phi}=237 \mathrm{MeV}$ and $f_{\phi}^{T}=215 \mathrm{MeV}$. The $B$ meson life times are given as $\tau_{B^{0}}=1.55 \times 10^{-12} \mathrm{~s}$ and $\tau_{B^{ \pm}}=1.65 \times 10^{-12} \mathrm{~s}$. And we use $\Lambda_{\mathrm{QCD}}^{(4)}=0.250 \mathrm{GeV}$ and $m_{0 K}=1.70 \mathrm{GeV}$ [5].

We show the numerical results of each amplitude for $B^{0} \rightarrow \phi K^{0}$ and $B^{ \pm} \rightarrow \phi K^{ \pm}$decays in Table 1. The factorizable penguin amplitude $F_{e}^{P}$ gives a dominant contribution to $B \rightarrow \phi K$ decays. The factorizable annihilation penguin amplitude $F_{a}^{P}$ generates a large strong phase. In $B^{ \pm} \rightarrow \phi K^{ \pm}$modes, there are contributions from $f_{B} F_{a}^{T}$ and $M_{a}^{T}$. These tree amplitudes contribute only a few percent to the whole amplitude, since the CKM matrix elements related to the tree amplitudes are very small. In order to isolate the trivial uncertainty from $f_{B}, f_{K}$ and $f_{\phi}$, we express our prediction for $B \rightarrow \phi K$ as

$$
\begin{align*}
& \operatorname{Br}\left(B^{0} \rightarrow \phi K^{0}\right)=\left\lvert\, \frac{f_{B} f_{K} f_{\phi}}{\left.190 \mathrm{MeV} \mathrm{160MeV237MeV}\right|^{2} \times\left(9.43 \times 10^{-6}\right)}\right.  \tag{22}\\
& \operatorname{Br}\left(B^{ \pm} \rightarrow \phi K^{ \pm}\right)=\left|\frac{f_{B} f_{K} f_{\phi}}{190 \mathrm{MeV} 160 \mathrm{MeV} 237 \mathrm{MeV}}\right|^{2} \times\left(10.1 \times 10^{-6}\right) \tag{23}
\end{align*}
$$

We found that our result is insensitive to $f_{\phi}^{T} / f_{\phi}$. For example, $10 \%$ variation of $f_{\phi}^{T} / f_{\phi}$ leads to less than $1 \%$ variation in our final result. The current experimental values are summarized in Table 2. The values which are predicted in PQCD are consistent with the current experimental data. However, our branching ratios have the theoretical error from the $\mathrm{O}\left(\alpha_{s}^{2}\right)$ corrections, the higher twist corrections, and the error of input parameters. Large uncertainties come from the meson decay constants, the shape parameter $\omega_{B}$, and $m_{0 K}$. These parameters are fixed from the other modes ( $B \rightarrow K \pi, D \pi$, etc.). We try to vary $\omega_{B}$ from 0.36 to 0.44 GeV , then we obtain $\operatorname{Br}\left(B^{ \pm} \rightarrow \phi K^{ \pm}\right)=(7.54 \sim 13.9) \times 10^{-6}$. Next, we set $\omega_{B}=0.40$ and try to vary $m_{0 K}$ from 1.40 to 1.80 GeV , then we obtain $\operatorname{Br}\left(B^{ \pm} \rightarrow \phi K^{ \pm}\right)=(6.65 \sim 11.4) \times 10^{-6}$.

Now, we consider the ratio of the branching ratio for the $B^{0} \rightarrow \phi K^{0}$ decay to the one for the $B^{+} \rightarrow \phi K^{+}$ decay. The theoretical uncertainties from various parameters are small, since the parameters in the numerator cancel out those in the denominator. The difference between the two branching ratios come in principle from $B$

Table 1
Contribution to $B^{0} \rightarrow \phi K^{0}$ and $B^{ \pm} \rightarrow \phi K^{ \pm}$decays from each amplitude

|  | $B^{0} \rightarrow \phi K^{0}$ | $B^{ \pm} \rightarrow \phi K^{ \pm}$ |
| :--- | ---: | ---: |
| $f_{\phi} F_{e}^{P}$ | $-1.03 \times 10^{-1}$ | $-1.03 \times 10^{-1}$ |
| $f_{B} F_{a}^{P}$ | $6.45 \times 10^{-3}+i 4.28 \times 10^{-2}$ | $6.17 \times 10^{-3}+i 4.20 \times 10^{-2}$ |
| $M_{e}^{P}$ | $5.24 \times 10^{-3}-i 3.61 \times 10^{-3}$ | $5.24 \times 10^{-3}-i 3.61 \times 10^{-3}$ |
| $M_{a}^{P}$ | $-8.03 \times 10^{-4}-i 1.73 \times 10^{-3}$ | $-6.56 \times 10^{-4}-i 7.22 \times 10^{-4}$ |
| $f_{B} F_{a}^{T}$ |  | $-1.11 \times 10^{-1}-i 3.75 \times 10^{-2}$ |
| $M_{a}^{T}$ |  | $1.60 \times 10^{-2}+i 2.77 \times 10^{-2}$ |

Table 2
The experimental data of $B \rightarrow \phi K$ branching ratios from BaBar [1], BELLE [2] and CLEO [3]

|  | $\operatorname{Br}\left(B^{0} \rightarrow \phi K^{0}\right)$ | $\operatorname{Br}\left(B^{ \pm} \rightarrow \phi K^{ \pm}\right)$ |
| :--- | ---: | ---: |
| BaBar | $\left(8.1_{-2.5}^{+3.1} \pm 0.8\right) \times 10^{-6}$ | $\left(7.7_{-1.4}^{+1.6} \pm 0.8\right) \times 10^{-6}$ |
| BELLE | $\left(8.7_{-3.0}^{+3.8} \pm 1.5\right) \times 10^{-6}$ | $\left(10.6_{-1.9}^{+2.1} \pm 2.2\right) \times 10^{-6}$ |
| CLEO | $<12.3 \times 10^{-6}$ | $\left(5.5_{-1.8}^{+2.1} \pm 0.6\right) \times 10^{-6}$ |

meson life times, tree and electroweak penguin contributions in annihilation amplitudes. We found that the tree and electroweak penguin amplitudes in the annihilation diagrams are negligible. Tree amplitudes are suppressed by two factors. First, they are annihilation processes which are suppressed by helicity. Second, they are multiplied by small CKM matrix elements. We predict that this ratio is

$$
\begin{equation*}
\frac{\operatorname{Br}\left(B^{0} \rightarrow \phi K^{0}\right)}{\operatorname{Br}\left(B^{+} \rightarrow \phi K^{+}\right)}=0.95 \tag{24}
\end{equation*}
$$

where the theoretical uncertainties from $m_{0 K}$ and $\omega_{B}$ are less than $1 \%$. The ratio is essentially given by the life time difference. The experimental value of this ratio from BELLE [2] is $\operatorname{Br}\left(B^{0} \rightarrow \phi K^{0}\right) / \operatorname{Br}\left(B^{+} \rightarrow \phi K^{+}\right)=$ $0.82_{-0.32}^{+0.39} \pm 0.10$.

In FA, the branching ratio is very sensitive to the effective number of colors $N_{c}^{\text {eff }}$. If we set $N_{c}^{\text {eff }}=3$, then the branching ratio is about $4.5 \times 10^{-6}$ where the scale of the Wilson coefficient is taken to $M_{B} / 2$ and $F^{B K}$ is 0.38 from the BSW model. In QCDF, branching ratios for $B \rightarrow \phi K$ decays are predicted as $\operatorname{Br}\left(B^{0} \rightarrow \phi K^{0}\right)=$ $\left(4.0_{-1.4}^{+2.9}\right) \times 10^{-6}$ and $\operatorname{Br}\left(B^{-} \rightarrow \phi K^{-}\right)=\left(4.3_{-1.4}^{+3.0}\right) \times 10^{-6}$ including the annihilation diagram [15]. Our predicted values are larger than these results. This is due to the enhancement of the Wilson coefficient for the penguin amplitude as explained in Section 1. In PQCD approach, the scale of the Wilson coefficients, which is equal to the hard scale $t$, can reach lower values than $M_{B} / 2$.

## 4. Summary

In this Letter, we calculate $B^{0} \rightarrow \phi K^{0}$ and $B^{ \pm} \rightarrow \phi K^{ \pm}$decays in PQCD approach. Our predicted branching ratios agree with the current experimental data and are larger than the values obtained by FA and QCDF. Because the Wilson coefficients for penguin operators are enhanced dynamically in PQCD.

## Note added

After this work has been completed, we become aware of a similar calculation by Chen et al. [25]. Our results are in agreement.

## Acknowledgements

The topic of this research was suggested by Professor A.I. Sanda. The author thanks his PQCD group members: Y.Y. Keum, E. Kou, T. Kurimoto, H.-N. Li, T. Morozumi, R. Sinha, K. Ukai for useful discussions. The author thanks JSPS for partial support. The work was supported in part by Grant-in Aid for Special Project Research (Physics of CP violation); Grant-in Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan.

## References

[1] G. Cavoto (for BaBar Collaboration), hep-ex/0105018.
[2] A. Bozek (for BELLE Collaboration), Talk presented at the International Europhysics Conference on High Energy Physics, Budapest, Hungary, July 12-18, 2001.
[3] CLEO Collaboration, Phys. Rev. Lett. 86 (2001) 3718.
[4] For example, see, I.I. Bigi, A.I. Sanda, CP Violation, Cambridge, 2000.
[5] Y.Y. Keum, H.-N. Li, A.I. Sanda, Phys. Lett. B 504 (2001) 6; Y.Y. Keum, H.-N. Li, A.I. Sanda, Phys. Rev. D 63 (2001) 054008.
[6] C.D. Lü, K. Ukai, M.Z. Yang, Phys. Rev. D 63 (2001) 074009.
[7] C.D. Lü, M.Z. Yang, hep-ph/0011238.
[8] C.H. Chen, H.-N. Li, Phys. Rev. D 63 (2001) 014003.
[9] E. Kou, A.I. Sanda, hep-ph/0106159.
[10] C.H. Chang, H.-N. Li, Phys. Rev. D 55 (1997) 5577.
[11] T.W. Yeh, H.-N. Li, Phys. Rev. D 56 (1997) 1615.
[12] M. Bauer, B. Stech, M. Wirbel, Z. Phys. C 34 (1987) 103; M. Bauer, B. Stech, M. Wirbel, Z. Phys. C 29 (1985) 637.
[13] S. Mishima, hep-ph/0107163.
[14] X.G. He, J.P. Ma, C.Y. Wu, Phys. Rev. D 63 (2001) 094004.
[15] H.Y. Cheng, K.C. Yang, Phys. Rev. D 64 (2001) 074004.
[16] H.-N. Li, hep-ph/0102013.
[17] T. Kurimoto, H.-N. Li, A.I. Sanda, hep-ph/0105003.
[18] J.C. Collins, D.E. Soper, Nucl. Phys. B 193 (1981) 381.
[19] J. Botts, G. Sterman, Nucl. Phys. B 325 (1989) 62.
[20] M. Bauer, M. Wirbel, Z. Phys. C 42 (1989) 671.
[21] P. Ball, JHEP 09 (1998) 005.
[22] P. Ball, JHEP 01 (1999) 010.
[23] P. Ball, V.M. Braun, Y. Koike, K. Tanaka, Nucl. Phys. B 529 (1998) 323.
[24] Particle Data Group, Eur. Phys. J. C 15 (2000) 1.
[25] C.H. Chen, Y.Y. Keum, H.-N. Li, hep-ph/0107165.


[^0]:    E-mail address: mishima@eken.phys.nagoya-u.ac.jp (S. Mishima).

