Available online at www.sciencedirect.com
SciVerse ScienceDirect
Procedia Engineering 48 (2012) 636 – 642
Procedia Engineering
www.elsevier.com/locate/procedia

MMaMS 2012

Evaluation of fatigue tests by means of mathematical statistics

Peter Sivák[a]*, Eva Ostertagová[b]

[a]Technical University of Košice, Faculty of Mechanical Engineering, Letná 9, 042 00 Košice, Slovak republic
[b]Technical University of Košice, Faculty of Electrical Engineering and Informatics, Letná 9, 042 00 Košice, Slovak republic

Abstract

Wöhler curve is the most frequently used S-N curve in life calculations and in determination of fatigue damage accumulation in components of supporting structures. The curve can solely be derived from long-term fatigue tests with constant stress amplitude by means of Experimental Stress Analysis (EAN). Fatigue tests are performed using special testing devices while the whole testing process follows relevant regulations. The set of data gained by measurements undergoes further processing and evaluation by means of mathematical statistics using relevant application software. This paper presents an example of a statistical evaluation of a data set obtained from fatigue tests of construction steel KONOX 345T. The paper further includes theoretical foundations for calculation and application of a simple linear regression model. In the applications the model is fitted to relevant data using the method of least squares. For quality measurements of the regression model we used two error measures – root mean squared error and coefficient of determination.

Keywords: fatigue life; Wöhler curve; simple linear regression model; method of least squares; coefficient of determination.

Nomenclature

a, b	parameters: intercept (-), slope (MPa^{-1})
M_k	torsion moment (Nm)
N	number of cycles (-)
MSE	mean squared error (-)
$RMSE$	root mean squared error (-)
R^2	coefficient of determination (-)
SSE	error sum of squares (-)
Greek symbols	
σ	normal stress (MPa)
τ	shear stress (MPa)
σ_a, τ_a	stress amplitude (MPa)
σ_m, τ_m	mean stress (MPa)
σ_c, τ_c	fatigue limit (MPa)

* Corresponding author. Tel.: +421556022466.
E-mail address: peter.sivak@tuke.sk.

doi:10.1016/j.proeng.2012.09.564

1. Introduction

In life analysis and in determination of fatigue damage accumulation in supporting structures and their components it is crucial to determine life curves of both used material and structure detail. These curves, together with their parameters, represent relations which are present in the complex fatigue process during mechanical cyclic loading. Wöhler curve is an often used S-N curve which can be obtained from fatigue tests with constant stress amplitude. In this case, we deal with the relationship of harmonic stress amplitude (in some cases maximum stress) under controlled force and the number of cycles to fracture [1].

2. Fatigue tests

Wöhler curves as representations of a relation between stress amplitude and the number of cycles to fracture are obtained as a result of data evaluation. In this place we refer to data that were gained by experimental measurements carried out on material samples. The samples share the same material, production technology, prescribed number, shape, size and coating and are subjected to endurance tests performed while using special testing devices as to whether they withstand fatigue failure. Loading for tensile, compression, bending or torsional stress should be quick, smooth and without sudden impacts. The aim of experimental investigation of fatigue characteristics possessed by some metal material is to determine the slope (the oblique line) of the S-N curve in the range of time-related fatigue strength and the horizontal line in the range of permanent fatigue strength or to directly determine fatigue limit.

2.1. Assumptions for fatigue tests

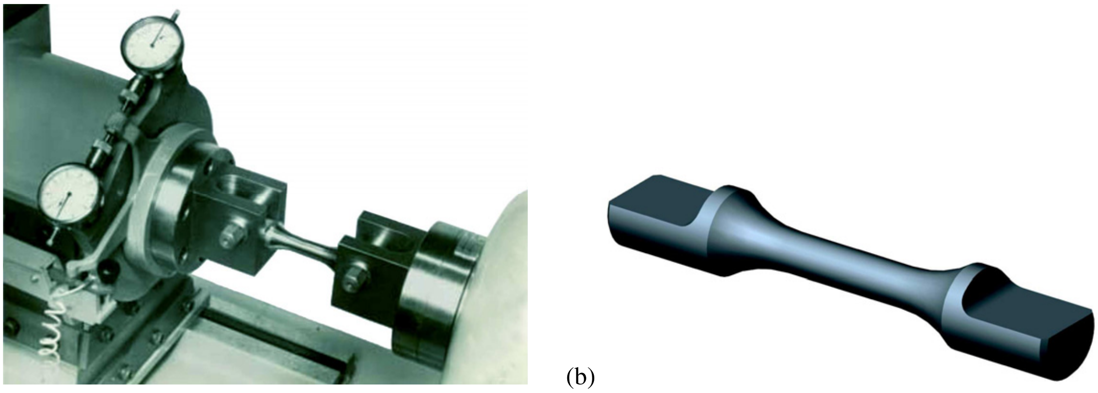
For the S-N curve fitting the measured values in the range of time-related fatigue strength represent the slopes of S-N curves provided that there is a linear relationship in coordinates log N-σ or log N-log σ [2]. The slopes normally represent regression lines with 50 % survival probability. It is further possible to evaluate the variance and in that sense the standard deviation as well as confidence and tolerance intervals for reliability or other statistical characteristics.

The results gained on the basis of the following relations cannot be considered valid unless some principal conditions are met. First, it is assumed that the distribution of the number of cycles log N is normal. It is further required that all tests which are represented by coordinate pairs $[\sigma_i, N_i]$, where the number of the pairs is n (the range of statistical set), are finished, i.e. that the tests are carried out until complete failure (fracture). Some methods imply that the occurrence of fatigue crack of a prescribed size may suffice. Tests are to be conducted at least at 4 or 5 stress levels above assumed fatigue limit and it is appropriate to conduct 6 to 15 tests at each stress level. Some standards for fatigue strength calculations and for detail category determination prescribe the minimum of ten tests at least at three different levels of fatigue loading and at least three tests at each particular level. It is further important to ensure that the difference between the lowest stress magnitude at which the sample failure would still occur (fatigue fracture) and the stress magnitude at which the failure no longer occurs or at which the failure does not occur even after given basic number of cycles N_c is not higher than the relevant stress σ. The above-mentioned tests can be conducted for both symmetrical reversed cycle - i.e. for zero mean stress σ_m - and for asymmetrical reversed cycle - i.e. non-zero mean stress.

2.2. Performing fatigue tests

In fatigue tests for determination of fatigue characteristics of structural materials through bending and torsional loading it is possible to use the testing device depicted in Fig. 1 (a). The sample can either be subjected to the load until fracture (the range of low- or high-cycle fatigue) or the test is finished even if no failure occurs provided that a high number of cycles, that is 10^7 or 10^8 cycles, was reached (the range of permanent fatigue strength). Testing samples can be flat, though round-section samples or some other simple structural components can be used as well.

We were conducting the fatigue tests in line with the procedure prescribed by the standard STN 42 0363. We were performing torsion-fatigue tests in symmetrical reversed cycle, i.e. for cycle asymmetry coefficient $\tau_{\min} / \tau_{\max} = -1$, i.e. for mean stress $\tau_m = 0$. The tests were carried out on smooth round-section specimens with diameter d = 6 mm (Fig. 1 (b)) at frequency 35 Hz. The steel specimens were made from metal sheet with thickness 12.5 mm. Differentiated were samples gained through linear sampling (L) from samples gained through transverse (cross) sampling (T). Stated sampling directions were related to the rolling direction. The specimens were made in line with the relevant standard. As prescribed by the relevant standard, the limiting number of cycles was $N_c = 10^7$ cycles while in some cases the tests were interrupted even after reaching higher numbers.



(a) (b)

Fig. 1. (a) sample bound to the testing device ready for torsion-fatigue test, (b) sample of the testing material for torsion-fatigue testing.

Shear stresses were expressed through relationship (1). Torsion moment M_k was obtained from known calibration relationship M_k of the testing device which was found by means of centesimal deviation meters.

$$\tau = \frac{16 M_k}{\pi d^3} \tag{1}$$

Values for the number of cycles to fracture N, which were obtained by the experiment at given shear stress τ and represented by coordinate pairs [τ_i, N_i] under torsion loading in symmetrical reversed cycle, were statistically processed as regression relation. This is the case of the slope (oblique line) in Wöhler curve which stands for the curve with 50 % survival probability. In line with the procedure as described in the relevant STN standard, fatigue limit τ_c was set from the horizontal line of the Wöhler curve.

Initiation and spreading of the fatigue fracture were similar in all samples. The main crack was spreading from the surface in transition region approximately at angle 45°, in the central part at angle 0° or 90°. Further spreading of the crack was then affected by the inner structure.

2.3. Determination of fatigue limit

When gradually decreasing the loading, the Wöhler curve for carbon steel bends and continues in horizontal intercept known as the range of permanent fatigue strength or the range of safe cyclic loading. This intercept does not imply any further fatigue failure. Relevant stress magnitude is referred to as fatigue limit σ_c of the material and is defined as the maximum stress at which no failure occurs even after theoretically unlimited number of cycles is applied. Fatigue limit is generally expressed as cyclic stress relevant to limiting stress amplitude specified by fatigue test at given mean stress σ_C from

$$\sigma_C = \sigma_m \pm \sigma_a \, . \tag{2}$$

At symmetrical reversed cycle ($\sigma_m = 0$) relevant fatigue limit will be expressed as follows

$$\sigma_C = \pm \sigma_a \, . \tag{3}$$

The region in which the Wöhler curve bends and so the relevant number of cycles N_c, are dependant from steel type. Wöhler curve for some alloyed steels and light and non-ferrous metals does not necessarily need to bend, which means that such materials do not exhibit fatigue limit. In such cases, when determining the number of cycles relevant to the bending region of other Wöhler curves the so called contractual (agreed) value is used. This value is prescribed by relevant standard for materials such as light metals and their alloys, steel, cast-irons, copper and its alloys. Similarly, as for determination of the slope (the oblique line) of the S-N curve, when specifying its horizontal line, it is necessary to state conditions and circumstances under which it occurred. This especially includes mean stress values, loading frequency f_z, basic number of cycles N_c, coating of the test samples etc. It is further possible to statistically evaluate gained data, define confidence and tolerance reliability intervals etc [3]. All given relations are applicable for normal stresses σ as well as shear stresses τ.

3. Evaluation of fatigue tests

For the purposes of experimental investigation of fatigue endurance while torsion we examined construction steel KONOX 345T [4]. Steel type KONOX 345T ranks among low-alloyed construction steel types which are determined for middle-loaded structural components and structures with higher resistance to atmospheric corrosion. This material exhibits higher strength together with good ductility, appropriate compressibility and good weldability.

As a result we gained fatigue characteristics and S-N curves under torsion loading. These characteristics are, for instance in comparison with the characteristics gained during tensile-compressive or bending loading, less common subject of experimental testing. This fact results in the number of relevant information and fatigue characteristics available for instance for the needs of engineering practice. The obvious importance of fatigue tests for the above-mentioned materials derives from practical application of gained results.

3.1. Using of the simple linear regression model

We wish to determine the relationship between an independent (predictor) variable τ and a dependent (response) variable log *N*. If we assume that the true relationship between these variables is a straight line and that the observed value of log *N* at any particular value of τ is a random variable, then we may write

$$\log N = a + b\,\tau + \varepsilon \qquad (4)$$

where log *N* is the dependent variable, τ is the independent variable, *a* and *b* are the parameters (i.e., intercept and slope) to be estimated and ε is a random error.

The simple linear model assumptions [5-7]:

1. The relationship between τ and log *N* is a straight line relationship.
2. The values of the independent variable τ are assumed fixed (not random); the only randomness in the values of log *N* comes from the error term ε.
3. The errors ε are normally distributed with mean $E(\varepsilon \mid \tau) = E(\varepsilon) = 0$ and a constant variance $V(\varepsilon) = S^2$. The errors are uncorrelated (not related) with each other in successive observations, i.e. covariance $K(\varepsilon_i, \varepsilon_j) = 0$, for $i \neq j$, $i, j = 1, 2, \ldots, n$ where *n* is the total number of the paired observations [τ_i, log N_i].

Taking the expected value of (4) conditional on τ and using $E(\varepsilon \mid \tau) = 0$ gives

$$E(\log N \mid \tau) = a + b\,\tau\,. \qquad (5)$$

There are many techniques that could be used to estimate the unknown parameters *a*, *b* in equation (4). A method used quite frequently is least squares, in which estimates of *a*, *b* are chosen to minimize the error or residual sum of squares *SSE*. The error sum of squares or least squares function is [5-7]

$$SSE(\hat{a},\hat{b}) = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \left(\log N_i - \log\hat{N}_i\right)^2 = \sum_{i=1}^{n} (\log N_i - \hat{a} - \hat{b}\,\tau_i)^2\,. \qquad (6)$$

The random error terms ε_i are estimated by the residuals e_i – the difference between observed values $\log N_i$ and the fitted (forecast) values $\log\hat{N}_i$.

To estimate parameters *a* and *b* by the method of least squares, we must choose $\hat{a}$ and $\hat{b}$ to minimize equation (4). We can write the results in the following normal equations

$$\hat{a}\,n + \hat{b}\sum_{i=1}^{n}\tau_i = \sum_{i=1}^{n}\log N_i\,,\;\; \hat{a}\sum_{i=1}^{n}\tau_i + \hat{b}\sum_{i=1}^{n}\tau_i^2 = \sum_{i=1}^{n}\tau_i\,\log N_i\,. \qquad (7)$$

The solution to the normal equations (7) are the least squares estimators for *a* and *b*

$$\hat{b} = \frac{n\sum_{i=1}^{n}\tau_i \log N_i - \sum_{i=1}^{n}\tau_i \sum_{i=1}^{n}\log N_i}{n\sum_{i=1}^{n}\tau_i^2 - \left(\sum_{i=1}^{n}\tau_i\right)^2}, \quad \hat{a} = \frac{\sum_{i=1}^{n}\log N_i - \hat{b}\sum_{i=1}^{n}\tau_i}{n} = \overline{\log N} - \hat{b}\,\overline{\tau} \tag{8}$$

where $\overline{\tau}$ and $\overline{\log N}$ are the arithmetic means of the τ and $\log N$ variables.

The variance S^2 of the random error term is estimated by the mean squared error (*MSE*) and the standard deviation S is estimated by the root mean squared error (*RMSE*)

$$MSE = \frac{SSE}{n-2}, \quad RMSE = \sqrt{MSE} \tag{9}$$

where $n - 2$ is the degree of freedom associated with *SSE*.

The *R*-squared of the regression, sometimes called the coefficient of determination R^2, is defined as

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{\sum_{i=1}^{n}\left(\log N_i - \log\hat{N}_i\right)^2}{\sum_{i=1}^{n}\left(\log N_i - \overline{\log N}\right)^2} \tag{10}$$

where *SST* is the total sum of squares. The value of R^2 is always between zero and one, $0 \le R^2 \le 1$. When interpreting R^2, we usually multiply it by 100 to change it into a percent. Thus, $100R^2$ is the percentage of the variability in $\log N$ that is explained by τ. An R^2 value of 0.9 or above is very good, a value above 0.8 is good, and a value of 0.6 or above may be satisfactory in some applications, although we must be aware of the fact that, in such cases, errors in prediction may be relatively high. When the R^2 value is 0.5 or below, the regression explains only 50 % or less of the variation in the data; therefore, prediction may be poor. If we are interested only in understanding the relationship between the variables, lower values of R^2 may be acceptable, as long as we realize that the model does not explain much [6].

3.2. Mathematical interpretation of experiment results

For experiment were collected $n = 12$ of the paired observations [τ_i, $\log N_i$] for steel KONOX 345T under shear stress in selected direction L and $n = 13$ of the paired observations[τ_i, $\log N_i$] for this same steel under shear stress in selected direction T. We want to set-up a simple linear regression model to predict $\log N$ with τ.

A graphical scatter plot is here helpful. Fig. 2 (a) shows a scatter plot of the data for direction L and Fig. 2 (b) for direction T. We can see that in both of selected directions L and T is strong correlation between τ and $\log N$.

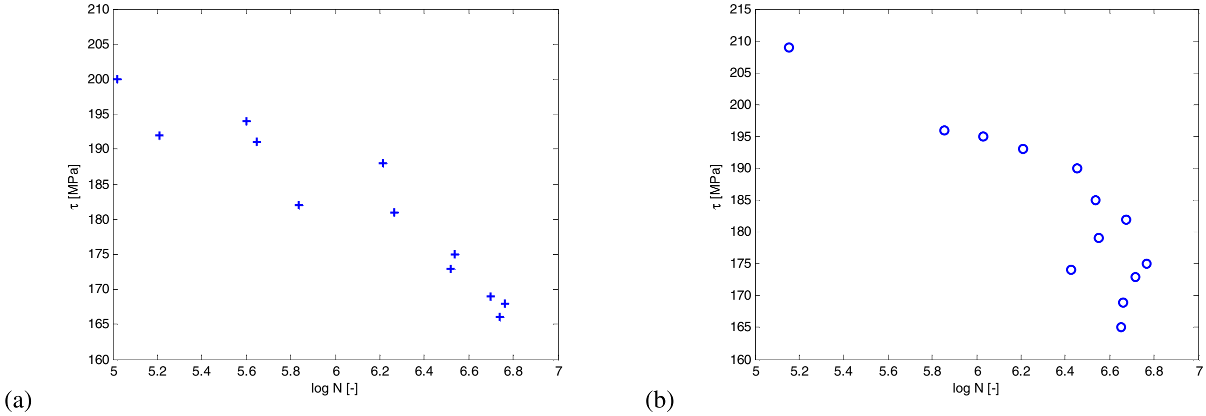


Fig. 2. (a) scatter plot of log N versus τ for direction L, (b) scatter plot of log N versus τ for direction T.

We perform the model of the simple linear regression using MATLAB with the Curve Fitting Toolbox too [7].

- Results for direction L:

Fig. 3 (a) represents the least squares straight line in a scatter plot of the data for direction L. We can see that this line fits the data very well.

The least squares estimates of the regression parameters are given by $\hat{a} = 15.2048$, $\hat{b} = -0.0502$. So, the fitted least squares regression straight line has equation $\log\hat{g}\, N_i = \hat{a} + \hat{b}\,\tau = 15.2048 - 0.0502\,\tau$.

We can find anymore confidence bounds for the regression parameters. The 95 % confidence intervals for *a* and *b* are, respectively, $a \in \langle 12.8690\,, 17.5407 \rangle$, $b \in \langle -0.0630\,, -0.0374 \rangle$. Thus, there is 95 % chance that the true values of parameters *a*, *b* are in this intervals.

The standard deviation *S* of the variation of observations around the regression line is estimated by $\hat{S} = RMSE = 0.2183$. This value is little (near zero) and represents very good result.

Coefficient of determination in this case is high. Result $R^2 = 0.8836$ implies that 88.36 % of the variability in log *N* is explained by *τ*. This result is good.

- Results for direction T:

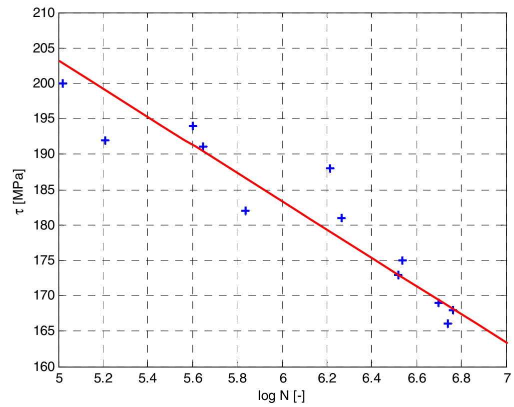
Fig. 3 (b) shows the least squares straight line in a scatter plot of the data for direction T. This regression line fits the data good.

For the least squares estimates of the intercept and slope parameters we get $\hat{a} = 12.1293$, $\hat{b} = -0.0314$. The fitted least squares regression straight line has equation $\log\hat{g}\, N_i = 12.1293 - 0.0314\,\tau$.

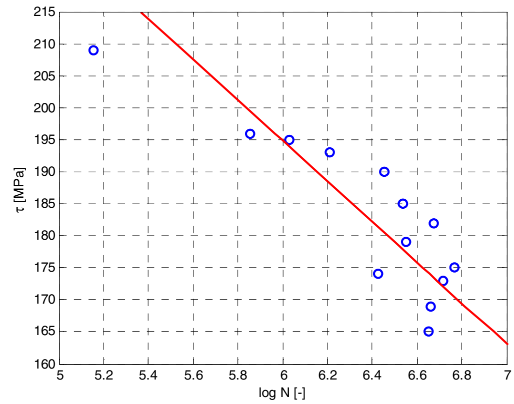
The 95 % confidence intervals for *a* and *b* are, respectively, $a \in \langle 9.9867\,, 14.2720 \rangle$, $b \in \langle -0.0431\,, -0.0198 \rangle$.

The estimate of the standard deviation of the regression errors is $\hat{S} = RMSE = 0.2321$. Between results of *RMSE* for both directions is only minor difference.

The *R*-squared is $R^2 = 0.7623$, so 76.23 % of the variability in log *N* is explained by *τ*. This result is a little inferior to result for direction T, but it is still good. Larger values of the coefficient of determination indicate a better fitting model.



(a)



(b)

Fig. 3. (a) least squares straight line in a scatter plot of log *N* versus *τ* for direction L, (b) least squares straight line in a scatter plot of log *N* versus *τ* for direction T.

Table 1 shows the summary results of the statistical study.

Table 1. Table of summary results

Material	KONOX 345T (direction L)	KONOX 345T (direction T)
Least squares estimate of the intercept	15.2048	12.1293
95 % confidence interval for the intercept	$\langle 12.8690\,, 17.5407 \rangle$	$\langle 9.9867\,, 14.2720 \rangle$
Least squares estimate of the slope	– 0.0502	– 0.0314
95 % confidence interval for the slope	$\langle -0.0630\,, -0.0374 \rangle$	$\langle -0.0431\,, -0.0198 \rangle$
Estimate of the standard deviation – *RMSE*	0.2183	0.2321
Coefficient of determination R^2	0.8836	0.7623

4. Conclusion

On the basis of performed fatigue tests under torsional symmetrical reversed cycle and static tests, test results and their analysis it is possible to formulate the following conclusion.

Measured values of fatigue limit in case of steel type KONOX 345T point out that the effect of sampling direction against rolling direction is very slight. From the course of fatigue relations that were processed in coordinate system log N-τ further arises the conclusion that the slope (the oblique line) of the S-N curve for tested material is relatively flat. This proves that initiation of fatigue cracks is being hindered as a result of a very good plastic property of the steel. Eventually, it can be stated that the tested strip steel exhibits very good fatigue failure endurance.

Acknowledgements

This contribution is the result of the project implementation: Center for research of control of technical, environmental and human risks for permanent development of production and products in mechanical engineering (ITMS:26220120060) supported by the Research & Development Operational Programme funded by the ERDF and project VEGA 1/0289/11.

References

[1] Ostertag, O., Sivák, P., 2010. Degradation processes and fatigue life prediction (in Slovak). 1st edition. TU Košice, Slovakia, ISBN 978-80-553-0486-1.
[2] Trebuňa, F., Buršák, M., 2002. Limit states, fractures (in Slovak). 1st edition. ManaCon Prešov, p. 348. ISBN 80-7165-362-4.
[3] Veles, P., 1989. Mechanical properties and testing of metals. 2nd edition. ALFA, SNTL Bratislava. p. 401.
[4] Sivák, P., Kachman, J., 2011. "Determination of fatigue characteristics in selected construction materials," Proceedings of the 4th International conference Modelling of Mechanical and Mechatronic Systems, Faculty of Mechanical engineering, Technical university of Košice, pp 452-459.
[5] Aczel, A. D., 1989. Complete Business Statistics. Irwin, p. 1056. ISBN 0-256-05710-8.
[6] Hanke, J. E., Reitsch, A. G., 1991. Understanding Business Statistics. Irwin, p. 878. ISBN 0-256-06627-2.
[7] Ostertagová, E., 2012. Applied Statistic with MATLAB (in Slovak). Equilibria Košice, Slovakia, p. 193. ISBN 978-80-8143-006-0.