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SEVENTEENTH CONFERENCE ON STOCHASTIC PROCESSES AND THEIR APPLICATIONS

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Introduction

The Seventeenth Conference on Stochastic Processes and their Applications was held at the Centre Linceo Interdisciplinare Benjamino Segre, Accademia Nazionale dei Lincei, Roma, Italy, over the period 27 June - 2 July 1988. The Conference was arranged under the auspices of the Committee for Conferences on Stochastic Processes of the ISI's Bernoulli Society for Mathematical Statistics and Probability. It was attended by some 210 participants from 25 countries, and the scientific program consisted of 22 invited and 121 contributed papers together with a special session in commemoration of A.N. Kolmogorov organized in collaboration with the Academy of Sciences of the USSR.

Abstracts[1]

1. Invited papers

Some new developments in Markov random fields and quantum fields

Sergio Albeverio, Ruhr-Universität Bochum, FR Germany

Dedicated to the memory of Raphael Høegh-Krohn (10.02.1938 - 24.01.1988).

We give a survey of some new developments in the study of Markov random fields and their relations with quantum fields. In particular we discuss the following four topics.

1. *The theory of Dirichlet forms on infinite dimensional spaces and the associated diffusion processes with infinite dimensional state space.* We report on recent work of M. Röckner and ourselves yielding necessary and sufficient conditions for the existence of the forms (closability problem) in the general setting of spaces for which a disintegration theory for measures holds (like locally convex Souslin spaces). Even in finite dimensions these results are new and give an optimal 'reduction' to the one-dimensional case. We also report on recent work by S. Kusuoka and ourselves characterizing and proving the maximality of the natural Sobolev Dirichlet forms.

2. *The theory of scalar generalized random fields: the associated Gibbs states, Dirichlet forms, and the global Markov property.* We discuss briefly the application of the results of Section 1 to the infinite dimensional Dirichlet forms associated with the Euclidean resp. 'time zero' probability

[1] An asterisk is attached to the name of the speaker in the case of a joint paper.

measures of the scalar generalized homogeneous Markovian random fields with polynomial, exponential on trigonometric interactions. In particular we get a characterization of the processes or of 'stochastic quantization' by Dirichlet forms.

In the case of exponential interaction over $\mathbb{R}^2$ the time zero Sobolev Dirichlet form coincides with the one associated with the infinite dimensional diffusion provided by the global Markov Euclidean diffusion field.

We also report on a recent breakthrough by R. Høegh-Krohn, B. Zegarliński and ourselves in the solution of the long standing open problem of the global Markov property for the φ_2^4-model, as well as in the study of the structure of Gibbs states for this and more general models.

3. *Vector-valued and group-valued Markovian random fields.* We report shortly on some recent results by R. Høegh-Krohn, K. Iwata and ourselves on the construction of random fields which are covariant with respect to proper Euclidean transformations in $\mathbb{R}^4$ and have Markovian properties. The fields are obtained by solving stochastic partial differential equations, best formulated using the isomorphism of $\mathbb{R}^4$ with the quaternions (as a normed vector space). This can be looked upon the construction of a quaternionic valued stochastic integral or alternatively as a $u(2)$-valued stochastic integral, over $\mathbb{R}^4$, which can also be lifted over to a $U(2)$-valued stochastic integral.

We thus get the connection with a general theory of stochastic multiplicative integrals, noises, measures and semigroups developed recently by R. Høegh-Krohn, H. Holden, T. Kolsrud and ourselves.

4. *Fields associated with Riemannian surfaces, probability measures on infinite dimensional manifolds and bosonic strings.* We close by reporting briefly on recent work (particularly by R. Høegh-Krohn, S. Paycha, S. Scarlatti and ourselves) on the connection between scalar fields with exponential interactions on a compact Riemann surface M, probability measures on some infinite dimensional manifolds and Polyakov models of bosonic strings. The restriction on the exponential rate of the interaction is related with the restriction in space-time dimensions for the existence of the string (and also with irreducibility resp. possible reducibility of the energy representation of the group of mappings from M into a compact semisimple Lie group).

Applications of random walks on graphs

David Aldous, University of California, Berkeley, CA, USA

Consider a graph G and a particle stepping randomly around the vertices in the natural way: from a vertex v it chooses uniformly at random an edge e incident at v, and steps to the vertex v' at the other end of e. Such *random walks on graphs* arise in many settings, and many elementary (and not-so-elementary) results have been repeatedly rediscovered. The purpose of this talk is to publicize the fact that a lot is known about these random walks, even though the subject has not been well-organized. Our thesis is that 'random walks on graphs' deserves to be regarded as a subject with a distinctive flavor of its own, analogous to (say) 'branching processes'. In this abstract, let me merely list some contexts where random walks arise.

Analogy with electric networks [7]. Leveling networks [4]. Universal traversal sequences on graphs [2]. Minimization algorithms [1]. Approximate counting, or simulating uniform distributions on combinatorial sets [8]. Voter Models [6]. Diffusions on regular fractals [11, 3, 10]. Random fractals [9]. Planar graphs [5].

A more extensive bibliography is available from the author.

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Ergodic theory of stochastic flows with applications

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We first present the multiplicative ergodic theorem for stochastic flows. This theorem provides the stochastic and nonlinear analogues of the deterministic eigenspaces and real parts of eigenvalues of a matrix. We then present the concept of rotation number which is the stochastic and nonlinear analogue of the deterministic imaginary part of eigenvalues of a matrix. We outline how this information can be used to attack the basic problems for nonlinear stochastic systems: stochastic stability, stochastic bifurcation, stochastic chaos.

Applications in physics and engineering are discussed.

Limit theorems for the distributions of functionals of random walks

A.N. Borodin, Yu.A. Davydov and I.A. Ibragimov, University of Leningrad, USSR*

The report contains the survey of recent results concerning the limit behaviour of distributions of functionals of random walks. All results under consideration are devoted to the cases when the application of the classical invariance principle is impossible. There are two reasons which make such an application unavailable. The first one is the strong discontinuity of the functionals and the second one is the replacement of weak convergence by convergence in variation.

Let ν_k be a recurrent random walk, i.e. $\nu_k = \sum_{l=1}^{k} \xi_l$ where $\{\xi_l\}_{l=1}^{\infty}$ are i.i.d. random variables with $\mathbf{E}\xi_1 = 0$. The general class of additive functionals of a random walk ν_k has the form $\eta_n = \sum_{k=1}^{n-l} f_n(\nu_k, \ldots, \nu_{k+l})$. A.V. Skorokhod and N.P. Slobodenyuk were the first to consider the convergence of distribution of η_n under wide conditions. However some of the assumptions, especially on the distribution of ν_1, were rather restrictive. The new methods which will be discussed in the report allow one to solve the problem of convergence of η_n in considerably more general situations. The next class of results is connected with the functionals $F(\nu_n(\cdot))$, where $\nu_n(\cdot)$ are the random broken lines constructed in the usual way by ν_k, $k = 1, 2, \ldots, n$. Assume that the distribution of ν_1 is absolutely continuous and has the finite variation. Then for the wide class of functionals, which involves such functionals as integral, ones, convex and many others, it is proved that the distributions of $F(\nu_n(\cdot))$ converge in variation to the distribution of $F(w(\cdot))$, where $w(\cdot)$ is Brownian motion.

Ergodicity and stability of Markov chains and of their generalizations

A.A. Borovkov, Mathematical Institute, Academy of Sciences, Novosibirsk, USSR

Let $\{X_n, n \geqslant 0\}$ be a Markov chain (MC) in the space $\mathscr{X}$. The starting point of the talk is some version of the ergodicity theorem of MC which is based on the results of Doeblin, Doob, Harrris, Athreya, Ney, Nummelin and others. The advantage of this version is that it allows one to make advances in several directions, in particular to obtain rate of convergence and stability results. This version is very helpful in investigations of the minimal ergodicity conditions for multidimensional MC when $\mathscr{X}=\mathbb{R}^d$, $d\geq 1$.

The other section of the talk is devoted to the generalization of above mentioned ergodicity results on the stochastically recurrent sequence (SRS),

$$X_{n+1}=f(X_n,\xi_n)\in\mathscr{X}, \tag{1}$$

where $\xi_n\in\mathscr{X}'$ are strictly stationary r.v., $f:\mathscr{X}\times\mathscr{X}'\to\mathscr{X}$. SRS is a more general subject than MC: if ξ_n are i.i.d. r.v. then SRS is MC. Vice versa: if X_n is MC then there exists $\mathscr{X}'$, i.i.d. r.v. $\xi_n\in\mathscr{X}'$, the function f, such that (1) holds.

These results are a very helpful tool in the investigations, connected with queueing theory, random access communication nets and in other cases; they were obtained jointly with S. Foss.

Diffusions on manifolds

K.D. Elworthy, University of Warwick, Coventry, UK

In order to illustrate how the diffusion theory can be used in differential geometry a specific application was discussed in detail. It was taken from K.D. Elworthy and S. Rosenberg, Generalized Bochner Theorems and the Spectrum of Complete Manifolds, to appear in Acta Applic. Math. The main result described was that, for $\rho(x)$ the lower bound of the Ricci curvature at x on a compact Riemannian manifold M, positivity of $-\Delta+\rho$ on $L^2(M)$ implies that every covering space of M is connected at infinity. The techniques involved were the probabilistic solution to the heat equation for 1-forms, the existence of continuous flows for solutions of stochastic differential equations, and the h-transform/Girsanov theorem.

On large deviations and surface entropy

H. Föllmer, ETH-Zentrum, Zürich, Switzerland

For a random field given by an ergodic Gibbs measure P on a product space $\Omega=S^T$ with $T=Z^d$, large deviations of the empirical field

$$R_n(\omega)=|V_n|^{-1}\sum_{t\in V}\delta_{\theta(\omega)}$$

along d-dimensional boxes V_n are of the form

$$|V_n|^{-1}\log P[R_n\in A]\sim -\inf h(Q;P), \tag{1}$$

where the infimum is taken over stationary probability measures in A, and where $h(Q;P)$ is the specific relative entropy of Q with respect to P. We sketch different approaches due to Comets, Olla and F.-Orey, with special emphasis on the role of the d-dimensional Shannon-McMillan theorem, and discuss some applications to an infinite collection of Brownian motions.

For a Markov random field which exhibits a phase transition, the right side of (1) may be 0 even though P is not in the closure of A. This leads to a refinement of (1) in terms of surface entropy where volumes $|V|$ are replaced by surface areas. In particular, we discuss corresponding versions of the Shannon-McMillan theorem which are based on joint work with M. Ort.

Statistical models and problems in image analysis

Alan F. Karr, The Johns Hopkins University, Baltimore, MD, USA

Digital image analysis is a field of dramatically increasing activity and importance. Applications range from medicine to geography to robotics. We survey several probabilistic models for images and the processes that produce them, as well as statistical approaches to problems such as noise removal; edge detection and feature extraction; image segmentation; and reconstruction.

Two problems—a Poisson process model of positron emission tomography and models based on Markov random fields—will receive particular emphasis.

Asymptotic behavior of Brownian flows

Hiroshi Kunita, Kyushu University, Fukuoka, Japan

Consider the stochastic differential equation on $\mathbb{R}^d$:

$$\varphi_t(x) = x + \int_0^t F(\varphi_r(x), \mathrm{d}r),$$

where $F(x, t)$ is an $\mathbb{R}^d$-valued Brownian motion with parameter x such that its mean vector is $b(x)t$ and the covariance matrix is $(a^{ij}(x, y))t$. Suppose that $a(x, y)$ and $b(x)$ are smooth functions and their derivatives are bounded. It is known that φ_t defines a Brownian flow of diffeomorphisms: It is continuous in (t, x) and for each t the map $\varphi_t : \mathbb{R}^d \to \mathbb{R}^d$ is a C^∞-diffeomorphism a.s. Further it is time homogeneous and has independent increments.

Let π be a σ-finite measure on $\mathbb{R}^d$. We define the image measures by the map φ_t and φ_t^{-1} by $\varphi_t(\pi)$ and $\varphi_t^{-1}(\pi)$, respectively. Namely, $\varphi_t(\pi)(A) \equiv \pi(\varphi_t^{-1}(A))$ and $\varphi_t^{-1}(\pi)(A) \equiv \pi(\varphi_t(A))$. In this lecture we discuss the asymptotic behavior of these image measures as t tends to infinity. The three cases will be discussed separately. We first discuss the case that φ_t is π-preserving, i.e., $\varphi_t(\pi) = \pi$ holds. We characterize the π-preserving property by $F(x, t)$ or its mean and covariance. Secondly it will be shown that the time average of $\varphi_t(\pi)$ converges to the invariant measure of the diffusion process $\varphi_t(x)$. Finally we show that $\varphi_t^{-1}(\pi)$ converges to a random measure π_∞ which is singular to π if π is an invariant measure of the inverse flow $\varphi_t^{-1}(x)$.

Singular perturbations in stochastic control and filtering with wide bandwidth noise

Harold J. Kushner, Brown University, Providence, RI, USA

We consider stochastic control problems where the system dynamics have fast (scaled by ε) and slow states, and the driving noise is not necessarily 'white' (bandwidth scaled by λ). It is shown that the optimal value function converges as $(\varepsilon, \lambda \to 0)$ to the optimal value functions for the 'averaged' limit system. Results are available for both control over a finite time interval and for the average cost per unit time over an infinite time interval. The methods are based on the theory of weak convergence and the results seem to include currently available results—which have been restricted largely to the Itô equation model. Analogous results are available for linear and non-linear filtering problems. Such results are of considerable interest in applications, since the models characterize many common systems, and the model simplifications provided by the averaging are of practical use.

A new example of chaotic representation

P.A. Meyer, Université L. Pasteur, Strasbourg, France

Let (X_t) be a martingale with $X_0 = 0$, such that $X_t^2 - t$ is also a martingale. Examples of this situation are standard Brownian motion and compensated Poisson processes. These conditions allow the definition of multiple integrals in the sense of Wiener, and one is naturally led to the question whether these multiple integrals generate the whole space L^2 (as they do in the Wiener and Poisson cases). This property is called the *chaotic representation property* (CRP) in reference to Wiener's 'polynomial chaos'; it is stronger than the *predictable representation property* (PRP), which means that every element of L^2 has an Itô stochastic integral representation with respect to X.

The talk reports on results by M. Emery, who proved recently that a martingale introduced by Azéma in the theory of Markov random sets

$$X_t = \sqrt{t - g_t}\,\mathrm{sgn}(B_t),$$

where B_t is Brownian motion and g_t is the last zero before time t, has the CRP. Emery's method uses the fact that X satisfies a 'structure equation',

$$[X, X]_t = t + \int_0^t \Phi(s)\,\mathrm{d}X_s,$$

where $[X, X]$ is the usual 'square bracket' of martingale theory, and $\Phi(s)$ is a predictable functional of X, here $\Phi(s) = -X_{s-}$. Indications on the interest of structure equations and other examples of such equations and of 'Azéma-like' martingales satisfying the CRP are given. In discrete time, structure equations are equivalent both to the CRP and the PRP, but the situation is much more complicated in continuous time. Structure equations are also related with special quantum stochastic differential equations driven by the number operator.

Stochastic calculus for anticipating processes

David Nualart, University of Barcelona, Spain

Suppose that $W = \{W(t), 0 \leq t \leq 1\}$ is a standard one-dimensional Brownian motion. The problem of defining the stochastic integral $\int_0^1 u(t)\,\mathrm{d}W(t)$ for a nonadapted square integrable process u has been investigated using different methods (approximation by Riemann sums, expansion of u along an orthonormal basis of $L^2([0, 1])$, theory of 'grossissements', ...).

The purpose of this talk is to present some of the main properties and applications of the noncausal stochastic integral introduced by Skorohod. Most of the results we will describe have been obtained in two joint papers of the author with M. Zakai and E. Pardoux.

The Skorohod integral can be defined using the Wiener-Chaos expansion of the process u and it turns out to coincide with the adjoint of the derivative operator on the Wiener space. Roughly speaking, the role of the adaptability condition is replaced by some differentiability property, in a weak sense. On the other hand, the Skorohod integral generalizes the stochastic integral of adapted processes, and it possesses the basic properties (local character, quadratic variation and path continuity) which allow to develop a stochastic calculus like in Itô's classical theory.

The relation of this type of integral to other nonadapted stochastic integrals like the Stratonovich integral, and some recent applications to noncausal stochastic differential equations will also be discussed.

Cohomology of finite power sets, Clifford–Wiener algebras and boson–fermion bridges

K.R. Parthasarathy, Indian Statistical Institute, Delhi Centre, New Delhi, India

This is a brief account of recent work done jointly with J.M. Lindsay at the Delhi Centre of the Indian Statistical Institute.

Let $(\Omega, \mathcal{F}, P)$ be the Wiener probability space of the standard Brownian motion process $\{B(t), t \geq 0\}$ and let $\mathcal{A}(\Omega)$ denote the algebra of bounded random variables on $(\Omega, \mathcal{F}, P)$. Any $f \in \mathcal{A}(\Omega)$ can be expanded in terms of Wiener Chaos: $\tilde{f} = \tilde{f}_0 + \tilde{f}_1 + \cdots + \tilde{f}_n + \cdots$, where $\tilde{f}_0 = \mathbb{E}f$,

$$\tilde{f}_n = \int_{0 < t_1 < \cdots < t_n < \infty} f_n(t_1, t_2, \ldots, t_n)\,\mathrm{d}B(t_1)\,\mathrm{d}B(t_2) \cdots \mathrm{d}B(t_n)$$

where $\mathrm{d}B(t)$ indicates stochastic integration with respect to $B(t)$. The whole sequence $\{f_n(t_1, t_2, \ldots, t_n), n = 0, 1, 2, \ldots\}$ where f_0 is the constant $\tilde{f}_0$, can be visualized as a function F

it became possible to construct canonical versions of stochastic processes on spaces Ω of the form $E^{\mathbb{R}}$, where E, the state space, was assumed to be a Polish space or, more generally, a Lusinian space.

The second type of canonical spaces are metric spaces of functions as C and D. Prokhorov and Skorokhod's results in the fifties gave the foundations of the theory of probability in that context.

Since then, in stochastic analysis it has been of common practice to work with *functions* as canonical objects to develop the theory. Certainly, these functions become more and more complex from the point of view of their values: vector-valued functions, measure-valued functions, tempered distribution-valued functions, ... But the essential fact is we are trying to describe results of experiences as functions and physical models by means of probability distributions over a function space.

The actual proposal is founded on the essential assumption that 'measures' are better than 'points' and 'functions' as a model for physical states. This point of view is not new, but we develop it systematically as a framework to analyze unstable systems.

Thus, our basic space is constructed as follows. Take E to be a locally compact space, and define Ω to be the set of all Radon measures over the product space $E \times \mathbb{R}_+$ whose projection over the real line is Lebesgue measure. Elements $\omega \in \Omega$ will be called *states*. States can be written, using the Desintegration Theorem, in the following manner:

$$\omega(\mathrm{d}x, \mathrm{d}t) = \int_0^\infty \alpha_s(\mathrm{d}x) \otimes \boldsymbol{\delta}_s(\mathrm{d}t)\, \mathrm{d}s,$$

where $\boldsymbol{\delta}_s$ is Dirac measure supported by $s \geq 0$.

In this context, *classical mechanics states* are all elements $\omega \in \Omega$ for which there exist a borelian function $\chi : \mathbb{R}_+ \to E$ such that for all $s \geq 0$, except a Lebesgue null set, it holds $\alpha_s(\mathrm{d}x) = \boldsymbol{\delta}_{\chi(s)}(\mathrm{d}x)$.

Ω is convex and classical mechanics states coincides with the set of its extremal points.

Ω is endowed with the weak $*$-topology for measures. This gives nice criteria on tightness for probability measures defined over Ω. Some particular subsets of the set of classical mechanics states have been characterized (namely, the image of the space of right-continuous functions with left-hand limits was derived in a previous work by Meyer and Zheng).

Furthermore, using convex analysis it is possible to introduce a thermodynamical formalism: concepts as *free energy*, *pressure*, and *entropy functionals* are available.

With these tools one gets a theoretical framework for the analysis of *unstable* stochastic dynamical systems. In particular, to study local equilibrium, and metastable phenomena.

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on the space $\Gamma(\mathbb{R}_+)$ of all finite subsets of the halfline $\mathbb{R}_+$ by putting $F(\phi)=f_0$, $F(\{t_1, t_2, \ldots, t_n\}) = f_n(t_1, t_2, \ldots, t_n)$, $n = 1, 2, \ldots$, where $t_1 < t_2 < \cdots < t_n$. The correspondence $f \to F$ yields a commutative and associative algebra isomorphic with $\mathscr{A}(\Omega)$ which is denoted by $\mathscr{K}(\mathbb{R}_+)$. If $fg \to F * G$ under this correspondence then

$$(F * G)(\sigma) = \int \sum_{\alpha+\beta=\sigma} F(\alpha \cup \omega) G(\beta \cup \omega)\, \mathrm{d}\omega$$

where $\mathrm{d}\omega$ denotes the symmetric measure on $\Gamma(\mathbb{R}_+)$ generated by the Lebesgue measure on $\mathbb{R}_+$ and summation is over all ordered pairs (α, β), $\alpha \cap \beta = \phi$, $\alpha, \beta \subset \sigma$, $\sigma \in \Gamma(\mathbb{R}_+)$. The operation $*$ was used by Maassen (1985) to find explicit solutions of quantum stochastic differential equations investigated by Hudson and Parthasarathy (1984). The algebraic features of $\mathscr{K}(\mathbb{R}_+)$ and their ramifications were studied by Meyer (1986, 1986/87) with inspiration derived from the commutation rules of quantum theory. In the present account, motivated by several remarks made by Meyer (1986) and innumerable personal communications from him as well as the recent work of Lindsay and Maassen at the University of Nijmegen, we present a unified approach to the subject by introducing an operation $*$ by

$$(F * G)(\sigma) = \int \sum_{\alpha+\beta=\sigma} p(\omega, \alpha, \beta) F(\alpha \cup \tilde{\omega}) G(\beta \cup \omega)\, \mathrm{d}\omega$$

where F, G are functions on the symmetric measure space $(\Gamma(S), \mu)$ over a nonatomic measure space (S, m), p is a 'weight function' on $\Gamma(S)^3$ and $\omega \to \tilde{\omega}$ is a 'twist' map. Restrictions on $(p, \sim)$ are sought to ensure the associativity of the operation $*$ and the different possible weights are classified upto equivalence using principles of cohomology.

With the help of ideas arising naturally from our investigations we construct a whole spectrum of operator fields which can be interpreted as deformations of the classical free and quasifree boson or fermion fields. We are tempted to call them a boson-fermion bridge since they obey commutation or anticommutation relations depending on the configuration of the supports of the pair of test functions under consideration and the classical boson and fermion fields appear as special cases. Finally a counterexample to the conjecture made in Accardi and Parthasarathy (1988) is constructed by showing the existence of a Levy field for which the past and future operators do not commute or anticommute.

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General theory of processes and stability problems for stochastic dynamical systems

Rolando Rebolledo, Universidad Católica de Chile, Santiago, Chile

This lecture deals with a new type—say a *third type*—of canonical space to construct models for stochastic dynamical systems.

Now, which is the meaning of this kind of canonical space and why do we need it? One of the main results on the so called first canonical space is due to Kolmogorov: he proved the existence of probability measures on an infinite product of measurable spaces. With this important result

Applications of Dirichlet forms in probability theory
Daniel W. Stroock, M.I.T., Cambridge, MA, USA

This talk will present a selection of applications of Dirichlet form techniques to problems in probability theory.

Emphasis will be placed on those applications for which there appears to be no other technique available.

Semilinear stochastic equations: Regularity and asymptotics
J. Zabczyk, Polish Academy of Sciences, Warsaw, Poland

Semilinear stochastic equations are of the form:

$$\mathrm{d}X = [AX + F(X)]\,\mathrm{d}t + \mathrm{d}W, \quad X(0) = x \in E, \tag{1}$$

where A and F are respectively linear and nonlinear operators defined on a Banach space E and W is a Wiener process on a Hilbert space H. Equations of this type have numerous applications. They model motion of an elastic string in a noisy environment, some chemical reactions, behaviour of neurons, the Euclidean free field and the anharmonic oscillator of the quantum mechanics.

We assume that E is embedded into H and that the operator A together with its extension to H generate semigroups of operators $S(t)$, $S_0(t)$, $t \geq 0$.

Existence of solutions to (1) is closely related to regularity in time and space of the Ornstein-Uhlenbeck (OU) process $Z(t)$, $t \geq 0$, which solves the linear equation ($F = 0$),

$$Z(t) = S(t)x + \int_0^t S_0(t-s)\,\mathrm{d}W(s), \quad t \geq 0.$$

We first describe regularity results for OU-processes and an existence theorem for general (1) obtained in [1] and [2]. Then we concentrate on symmetric solutions to (1). We follow [5] and give necessary and sufficient conditions for an OU-process to be symmetrizable and show also that a large class of nonlinear gradient systems gives rise to symmetric solutions. Finally we present an infinite dimensional version of the exit theorems of Freidlin and Wentzell following [4] and using large deviation results for semilinear equations obtained in [1] and [3].

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2. Contributed papers

2.1. Asymptotics and convergence

On extremal theory for stationary processes

J.M.P. Albin, University of Lund, Sweden

Let $\{\xi(t)\}_{t\geq 0}$ be a stationary stochastic process, with one-dimensional distribution function G. We develop a general method to determine an asymptotic expression for $\Pr\{\sup_{0\leq t\leq h} \xi(t) > u\}$, when $u\to\infty$, of special interest in the case when G belongs to a domain of attraction of extremes, and show that if G belongs to such a domain of attraction, then so does $\sup_{0\leq t\leq h} \xi(t)$. Applications are given to hitting probabilities for small sets for multidimensional Gaussian processes, and to high level extrema of χ^2-processes. Further, we prove the double exponential law and the Fréchet law, for maxima over increasing intervals when G belongs to the Type I- and the Type II-domain of attraction of extremes, respectively, and establish the asymptotic Poisson character of ε-upcrossings and local ε-maxima. We also prove the complete asymptotic Poisson character in time and space for the visits of certain multidimensional differentiable stationary processes in rare sets.

On normal approximations of distributions in terms of dependency graphs

Pierre Baldi, University of California, San Diego, La Jolla, CA, USA
Yosef Rinott, Hebrew University, Jerusalem, Israel*

Bounds on the error of the normal approximation of sums of dependent random variables introduced by Stein (1986) are interpreted in terms of dependency graphs. This leads to improvements on a limit theorem of Petrovskaya and Leontovich (1982) and recent applications by Baldi and Rinott (1988). In particular, bounds on rates of convergence are obtained. As an application we study the normal approximation to the number of local maxima of a random function on a graph.

An approximate martingale functional central limit theorem on a group

M.S. Bingham, University of Hull, UK

Let G be a locally compact second countable abelian group, $\hat{G}$ be the dual group of G and g be a local inner product on $G\times\hat{G}$. Let $\{S_{nj}, \mathcal{F}_{nj}: 0\leq j\leq k_n, n\geq 1\}$ be an adapted triangular array of G-valued random variables with differences $X_{nj}=S_{nj}-S_{n,j-1}$ where S_{n0} is the identity of G. Define the random element S_n in the Skorokhod space D of right-continuous G-valued functions with left-hand limits on $[0, 1]$ by $S_n(t)=S_{n,[k_n t]}$, $0\leq t\leq 1$, where $[\cdot]$ denotes the integer part.

Suppose that for each $t\in[0,1]$ there is a random continuous nonnegative quadratic form Φ_t on $\hat{G}$ such that $\Phi_0(y)=0$ and $t\to\Phi_t(y)$ is continuous for each $y\in\hat{G}$, $\omega\in\Omega$. Assume that for each $y\in\hat{G}$ and $t\in[0,1]$,

$$\sum_{j=1}^{[k_n t]} g(X_{nj}, y)^2 \xrightarrow{\mathrm{P}} \Phi_t(y) \quad \text{as } n\to\infty$$

and

$$\sum_{j=1}^{k_n} |E[g(X_{nj}, y)\,|\,\mathcal{F}_{n,j-1}]| \xrightarrow{\mathrm{P}} 0 \quad \text{as } n\to\infty.$$

Assuming also a uniform infinitesimality condition and that $\mathcal{F}_{nj}\subseteq\mathcal{F}_{n+1,j}$ for all n, j, it is shown that S_n converges stably in law on D to a continuous process which is a mixture of continuous, independent increment, Gaussian processes. In particular, if there exists a non-trivial Gaussian measure on G then there exist G-valued Brownian motions.

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On stochastic center manifold theory

Petra Boxler, Universität Bremen, FR Germany

The solution of a stochastic differential equation, which models the influence of white noise on a nonlinear dynamical system, generates a cocycle of diffeomorphisms. The question of stability may be tackled by means of the stochastic counterpart to the real parts of the eigenvalues, the Lyapunov exponents attached to the linearized cocycle. If at least one Lyapunov exponent vanishes then the existence of a stochastic analogue to the deterministic center manifold can be proved. It is invariant and tangent to the Oseledec space that corresponds to the zero Lyapunov exponent and which replaces the deterministic eigenspace. Furthermore it is shown that the asymptotic behaviour on the stochastic center manifold is crucial for the stability of the entire system.

Collecting all the points with a certain growth behaviour in *both* directions of time the stochastic center manifold may be characterized dynamically.

Finally an approximation theorem is shown and the polynomial approximation up to second order is given explicitly.

Asymptotic normality and convergence of eigenvalues

Louis H.Y. Chen and J.H. Lou, National University of Singapore, Singapore*

For any random variable X with finite variance $\sigma^2 > 0$, let

$$U(X) = \sup_g \frac{\mathrm{Var}[g(X)]}{\sigma^2 E[g'(X)^2]},$$

where the supremum is taken over the class of absolutely continuous functions g such that $0 < \mathrm{Var}[g(X)] < \infty$. Borovkov and Utev (1984) proved that if $X_1, X_2, \ldots,$ is a sequence of random variables such that $U(X_n) \to 1$, then the moment generating function of $(X_n - EX_n)/[\mathrm{Var}(X_n)]^{1/2}$ converges to that of a standard normal random variable in a neighborhood of the origin.

Suppose $X_n = (Y_1 + \cdots + Y_n)/n^{1/2}$ where $Y_1, Y_2, \ldots,$ are independent and identically distributed random variables with mean 0 and variance 1. By the central limit theorem, X_n converges in distribution to a standard normal random variable. If $U(Y_1) < \infty$, then the convergence of X_n in distribution to a standard normal random variable can be strengthened to the convergence of the corresponding moment generating functions. Furthermore, it can be proved that $U(X_n)$ is n^{-1} times the nth term of a subadditive sequence of real numbers and therefore has a finite limit as $n \to \infty$. The result of Borovkov and Utev raises the following interesting question: Is the limit 1?

In this paper we attempt to answer this question by finding a set of conditions on an arbitrary sequence of random variables $X_1, X_2, \ldots,$ such that the asymptotic normality of X_n in the sense of the convergence of moment generating functions is equivalent to the convergence of $U(X_n)$ to 1.

Confiners: a stochastic approach of the attractors

Jacques Demongeot, Université Médicale de Grenoble, La Tronche, France
Christine Jacob INRA, Jouy-en-Josas, France*

The stochastic analogue of the notion of attractor for a deterministic dynamical system, is defined. Similarly to the deterministic concept, the basic tool is the set of accumulation points of a trajectory and the definition is based on the algorithm of research of the confiner from the properties of invariance (research first of the confining basin of a set and then of the limit set of this confining basin), maximality for the stochastic c-connexity and then minimality for the inclusion.

Some examples of confiners are given. In particular, this notion is reduced to an attractor in invariant probability when the process is recurrent and possesses such a probability (more exactly, in this case, the confiners correspond to the sets defined by level curves of the invariant probability).

Finally, the notion of stochastic isochrones which allow one to localize the starting points of trajectories with identical phasing, are defined as well as the notion of geometric or temporal bifurcations which define any change in the topology or in the temporal description of the confiners induced by a change of a parameter of the dynamical system.

Tail triviality for sums of stationary random variables

W.Th.F. den Hollander, Delft University of Technology, The Netherlands*
H.C.P. Berbee, CWI, Amsterdam, The Netherlands

Let $(X_n)_{n\in\mathbb{Z}}$ be a stationary sequence of integer-valued random variables, and let $(S_n)_{n\in\mathbb{Z}}$ be the sequence of sums given by

$$S_0=0, \qquad S_n-S_{n-1}=X_n \quad (n\in\mathbb{Z}).$$

We give general conditions for triviality of the following tail σ-fields:

$$\mathcal{G}_\infty^+=\bigcap_{N\geq 0}\sigma(S_n:n\geq N),$$

$$\mathcal{G}_\infty=\bigcap_{M,N\geq 0}\sigma((S_n,S_{-m}):n\geq N, m\geq M),$$

$$\mathcal{G}_\infty^{\text{inv}}=\bigcap_{M,N\geq 0}\sigma(S_n-S_{-m}:n\geq N, m\geq M).$$

The results are applied to 0–1 sequences to prove convergence in distribution of interarrival times.

Asymptotic properties of stochastic difference equations

G. Kersting, J.W. Goethe-Universität, Frankfurt, FR Germany

We consider random variables $X_0, X_1, \ldots,$ such that

(i) $X_n\geq 0$,

(ii) there are functions $g(x)>0$ and $\sigma^2(x)>0$ such that a.s. $X_{n+1}=X_n+g(X_n)+\xi_{n+1}$, $E(\xi_{n+1}|X_0,\ldots,X_n)=0$, $E(\xi_{n+1}^2|X_0,\ldots,X_n)=\sigma^2(X_n)$,

(iii) $g(x)=O(x)$, as $x\to\infty$.

We present two results concerning the behaviour of X_n on the event $\{X_n\to\infty\}$. The first is a law of large numbers, which relates the rate of divergence of X_n to that of the deterministic sequence

$$a_{n+1}=a_n+g(a_n), \qquad a_0=1.$$

Our result is close to the best possible, as can be demonstrated by counterexamples.

The second result deals with the asymptotic distribution of X_n. Usually one expects that a law of large numbers involves a central limit theorem. In our model this is not always true. There is a certain situation where X_n, conditioned on the event $\{X_n\to\infty\}$ and properly rescaled, has an asymptotic Γ-distribution. This theorem contains (among others) known results on the square Euclidean norm of a zero-mean random walk and on critical branching processes with immigration.

Lyapunov exponents in stochastic dynamics

W. Kliemann, Iowa State University, Ames, IA, USA

Multiplicative ergodic theory provides the basis for the analysis of nonlinear stochastic systems via linearization: stable, center, and unstable manifold theorems describe the local behavior around stationary solutions. These manifolds can be described through the 'eigenspaces' of the linearized system, corresponding to the negative, zero, and positive Lyapunov exponents. If the stochastic system is not uniquely ergodic, the regions of attraction for different stationary solutions can be obtained via the analysis of associated (deterministic) control systems. Consequences for a stochastic bifurcation theory will be indicated.

On a class of Lévy processes with partial scaling property

Jiann-Hua Lou, National University of Singapore, Singapore

First of all, we study a sequence of random variables $\{X_n\}$ constructed as follows: X_0 is an arbitrary non-degenerate random variable, and X_n's are defined inductively by $X_{n+1} = (X_n - X'_n)/2^\gamma$ for $n = 0, 1, 2, \ldots$, where γ is some positive constant and X'_n denotes an independent copy of X_n. The limit law of this sequence, if there is, is characterized to be an infinitely divisible distribution whose Lévy canonical representation can be described. It is also noted that γ must be $\geq \frac{1}{2}$ in order to have a limit law, and in particular, when $\gamma = \frac{1}{2}$, we obtain a (well-known) characterization of normal laws.

It leads to the study of a Lévy process $X(\cdot)$ satisfying a partial scaling property in the sense that $X(ct) \overset{\mathscr{L}}{=} c^\gamma X(t)$ for some particular value of c, e.g. $c = 2$. Properties of this process are investigated in this paper.

On the rate of convergence in Strassen's theorem

Joop Mijnheer, University of Leiden, The Netherlands

Let $\{W(t): 0 \leq t < \infty\}$ be a Wiener process on $(\Omega, \mathscr{F}, \mathscr{P})$. We define $f_n : [0, 1] \times \Omega \to \mathbb{R}$ by

$$f_n(t, \omega) = W(nt, \omega)(2n \log \log n)^{-1/2}.$$

Let

$$K = \left\{ x: x(0) = 0,\ x \text{ absolutely continuous},\ \int_0^1 \{\dot{x}(t)\}^2 \, \mathrm{d}t \leq 1 \right\}.$$

Theorem (Strassen, 1964). *For almost all ω, the set $\{f_n(\cdot, \omega): n \geq 3\}$ is relatively compact, with limitset K.*

Results about the rate of convergence are obtained by Bolthausen (1978) and Grill (1987).

Theorem (Grill, 1987).

$$P(d_\alpha(f_n, K) \geq (\log \log n)^{-\alpha} \text{ i.o.}) = \begin{cases} 0 & \text{as } \alpha < \frac{2}{3}, \\ 1 & \text{as } \alpha > \frac{2}{3}. \end{cases}$$

We give a new method to obtain the rate of convergence in the case $\alpha = \frac{2}{3}$.

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Large deviations for functionals of stationary processes

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Let $(X_n;\ n \geq 1)$ be a stationary sequence of random variables and let $(S_n;\ n \geq 1)$ be a sequence of functionals, i.e. a sequence of random variables adapted to the internal history $(\sigma(X_1, \ldots, X_n);\ n \geq 1)$. Define the 'deviations from additivity' $S_{n,m}$, $n, m \geq 1$, through the formulas

$$S_{n+m} = S_n + S_m \circ \theta_n + S_{n,m}.$$

We show that under a certain lower mixing condition on the probability law of the stationary sequence and under a stochastic dominance condition on the deviations $S_{n,m}$ the large deviation principle holds true for the sequence (S_n).

Martingale limit theorems

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If $S_{n,k}=\sum_{1\leq i_1<\cdots<i_R\leq m_n} X_{n_{i_1}}\cdots X_{n_{i_R}}$ where: $\{X_{n_j},\mathscr{F}_{n_j}, 1\leq j\leq m_n, n\geq 1\}$, $m_n\uparrow+\infty$, is a martingale difference array, conditions are given for the distribution and moment convergence of $S_{n,k}$ the distribution and moments of $(1/k!)H_k(z)$ where H_k is the Hermite polynomial of degree k and z is standard normal random variable. This is intimately related to an identity for multiple Wiener integrals. Under alternative conditions, similar results hold for $S_{n,k}/V_n^k$ and $S_{n,k}/V_n^k$ where $V_n^z=\sum_{j=1}^{m_n} X_{n_j}^2$ and V_n^z is the conditional variance.

Future independent times and Markov chains

Hermann Thorisson, Chalmers University of Technology, Göteborg, Sweden

Let $X=(X_k)_0^\infty$ be an irreducible aperiodic recurrent Markov chain with state space E and transition matrix P. Let T be a nonnegative integer valued a.s. finite random time.

Call T *future independent* if T is independent of the future, $(X_{T+k})_0^\infty$. Call T *regular* if $(X_{T+k})_0^\infty$ is a Markov chain with transition matrix P. Say that T *achieves* μ if X_T has the distribution μ.

We show that a regular future independent time T achieving μ *exists* if $\mu(B)=1$ for some finite subset B of E, while T need not exist in general. This result is used to give a new and short proof of the *basic limit theorem of Markov chains* improving somewhat the result in the null recurrent case. Of basic importance is the existence of a *stationary measure* ν (i.e. $\nu=\nu P$) satisfying $\nu(E)=\infty$ if and only if X is null recurrent. Instead of using the stationary distribution $\pi=\nu/\nu(E)$ (as in the classical coupling proof of this theorem for positive recurrent chains), we use the *conditional* stationary measure, i.e. the distribution π^B defined for a finite subset B of E by

$$\pi^B(A)=\nu(A\cap B)/\nu(B)\quad \text{for all subsets } A \text{ of } E.$$

There are connections between this method and the coupling method: If T is regular, future independent and achieves a stationary distribution π, then T can be regarded as a *coupling epoch*. The classical coupling argument can be extended to *null recurrent* chains using π^B instead of π (see Thorisson, 1988b,c). An improved limit result for null recurrent *Harris chains* can also be obtained using π^B (see Thorisson, 1988c).

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Lyapunov exponents of nilpotent systems

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We consider the Lyapunov exponent of the stochastic dynamical system

$$\mathrm{d}x=A_0x\,\mathrm{d}t+\sigma\sum_{i=1}^{r}A_ix\circ\mathrm{d}W_t^i$$

($A_0, A_1,\ldots,A_r$ constant $d\times d$-matrices, $(W_t^1,\ldots,W_t^r)$ standard r-dimensional Wiener process) and investigate its dependence on the noise intensity σ in the highly singular case of a nilpotent drift matrix A_0.

This covers the critical case of the random frequency oscillator $\ddot{y}+(\sigma V_t - E)y=0$, V_t noise, where the energy level E is 0.

Simple and exact formulas for the Lyapunov exponent as a function of σ are given rather than asymptotic expansions.

2.2. Branching and renewal processes

Branching processes, Polyá schemes and Ferguson processes

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The classical Polyá urn scheme involving two colors and adding just one ball each time can be generalized to many colors and addition of a random number of balls. The proof of convergence results in the classical case using exchangeability and martingales breaks down for the general case. However a branching process argument saves the situation. This and the extension of the Blackwell and McQueen approach to Ferguson processes will be explored. A central limit theorem for the comparison at time n will be proved, and then applied of the Polyá posterior Bayesian approach to finite population sampling.

Interparticle correlation in death processes

Frank Ball, University of Nottingham, UK

We consider a pure death process, starting with N individuals, with death rates μ_n, $n=1,2,\ldots,N$. We show that the fates of distinct individuals are positively correlated if μ_n/n decreases with n, and negatively correlated if μ_n/n increases with n. We describe the application of this result to the problem of variability in stochastic compartmental models.

A discrete non-linear Markov model for a population of interacting cells

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The goal of this paper is a construction of a discrete mathematical model for the evolution of a population of cells, classified by their DNA content. In order to take into account 'crowding effects' the growth is modeled as a pure jump non-linear Markov process. Existence and uniqueness of solutions for the corresponding martingale problem is proved by means of fixed point techniques.

Approximations to the lifetime distribution of k-out-of-n systems with cold standby

D.P. Kroese and W.C.M. Kallenberg, University of Twente, Enschede, The Netherlands*

Consider the following k-out-of-n system with cold standby. Let S be a system that consists of n identical components and k component positions. In order that the system works, each position has to be occupied by a working component. The components are however subject to failure and they cannot be repaired. Let k be fixed. We are now interested in the lifetime distribution of the entire system for large n.

For fixed k let M_n be the lifetime of the k-out-of-n system and $N_i(t)$ the number of failures during $[0,t]$ at position i. Let $N(t)=N_1(t)+\cdots+N_k(t)$, then obviously $\{M_n\leq t\}\Leftrightarrow\{N(t)\geq n-k+1\}$. For each renewal process $(N_i(t), t\geq 0)$, Edgeworth's expansions for $P(N_i(t)\leq x)$ are found. These expansions are then used to construct associated expansions for $P(N(t), t\geq 0)$ and $P(M_n\leq x)$. From these expansions several approximations to the lifetime distribution are found. By using these approximations we can improve considerably on the known normal approximation of $P(M_n\leq x)$.

The case in which the components are repaired at a slower rate than the total failure rate is also considered. Finding expansions for $P(M_n \leqslant x)$ in this case is closely related to finding boundary crossing expansions in sequential analysis. Several results in sequential analyses can be generalised to sums of independent renewal processes.

Theory of semiregenerative phenomena

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We develop a theory of semiregenerative phenomena. These may be viewed as a family of linked regenerative phenomena, for which J.F.C. Kingman developed a theory within the framework of quasi-Markov chains. We use a different approach and explore the correspondence between semiregenerative sets and the range of a Markov subordinator with a unit drift (or a Markov renewal process in the discrete time case). We use techniques based on results from Markov renewal theory.

When is the superposition of independent renewal processes regenerative?

Karl Sigman, Columbia University, New York, USA

When is the superposition of two independent renewal processes a regenerative process? We show that if one of the cycle length distributions is spread-out then a one dependent regenerative (od-R) structure exists. The corresponding regeneration points can be chosen as a subsequence of the event times of one of the renewal processes. This result is motivated by the study of queues in continuous time where the renewal processes can be viewed as the arrival streams of different classes of customers. We generalize our result to the case of any finite number of renewal processes as well as to other kinds of point processes.

Poisson processes, Bessel functions and electrons

F.W. Steutel, Eindhoven University of Technology, The Netherlands

The motion of electrons through a gas of slow, heavy particles is modelled by competing Poisson processes with random splitting.

The average number of moving electrons, i.e. the current, as a function of time, obeys a Volterra integral equation, which can be solved in terms of Bessel-function integrals, by probabilistic means.

Asymptotic results are obtained by means of the central limit theorem, which yields much better results than standard asymptotic expansions of Bessel functions.

Uniformly strong ergodicity for non-homogeneous semi-Markov processes

W. Wajda, Adam Mickiewicz University, Poznań, Poland

Let $N(t)$ be a non-homogeneous renewal process. Consider a non-homogeneous semi-Markov process $X(t)=X_{N(t)}$ generated by a sequence of the semi-Markov matrices $(Q^n(t))_{n\in\mathbb{N}}$ and an initial distribution over a state space $B\subset\mathbb{N}$. For $i,j\in B$, $n\in\mathbb{N}$, t, $h\geqslant 0$, let us denote

$$F_{ij}(t,h)=P\{X(t+h)=j\,|\,X(t)=i\},$$

$$e^n_{ij}(t)=E\{T_{N(t)+n}\,|\,X_{N(t)+n-1}=i,\ X_{N(t)+n}=j\}.$$

The processes considered here have at least one positive persistent state so, without loss of generality, we can assume that the state 0 is positive persistent. For the embedded Markov chain the mean visit time from the state i at step s to the state 0 will be denoted by ${}_sm_i$. For the process $X(t)$ the mean visit time from state i at the moment t to state 0 will be denoted by $\mu_i(t)$. Let us introduce the following definition.

Definition. A non-homogeneous semi-Markov process is uniformly strongly ergodic if there exists a row-constant stochastic matrix Π such that

$$\lim_{h\to\infty} \|F(t,h)-\Pi\|=0 \quad \text{uniformly in } t.$$

Theorem 1. *A non-homogeneous semi-Markov process is uniformly strongly ergodic if and only if* $\sup_{t,i} \mu_i(t)<\infty$.

Theorem 2. *Let $X(t)$ be a non-homogeneous semi-Markov process. Then*

(a) *if* $\sup_{t,i}\mu_i(t)<\infty$ *and* $\inf_{t,i,j,n} e_{ij}^n(t)\geq c>0$ *then* $\sup_{s,i}{}_s m_i<\infty$,

(b) *if* $\sup_{s,i}{}_s m_i<\infty$ *and* $\sup_{t,i,j,n} e_{ij}^n(t)\leq C<\infty$ *then* $\sup_{t,i}\mu_i(t)<\infty$.

Corollary. *If for a non-homogeneous semi-Markov process $X(t)$,*

$$0<c\leq \inf_{t,i,j,n} e_{ij}^n(t) \quad \text{and} \quad \sup_{t,i,j,n} e_{ij}^n(t)\leq C<\infty$$

then the following statements are equivalent:

(a) *$X(t)$ is uniformly strongly ergodic.*

(b) $\sup_{t,i}\mu_i(t)<\infty$.

(c) $\sup_{s,i}{}_s m_i<\infty$.

(d) *The embedded Markov chain is uniformly strongly ergodic.*

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2.3. Infinite particle systems

Boltzmann processes with multiple particle interactions: An exponential rate of convergence

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The Boltzmann equations with cut-off are considered. The solution, of such an equation, is expressed as the temporal law of a process build with the help of an exponential age-dependent binary branching process. (A.L. Sznitman, Equations de type de Boltzmann, spatialement homogène, Z. Wahrsch. Verw. Gebiete 70 (1985) 117–129.)

When the interaction mechanism is scaling invariant, it is shown that this temporal law converges geometrically fast to the equilibrium law. Our techniques are applicable to other mechanisms: 'binomial', Kac's caricature (already shown by H.P. McKean, Speed of approach to equilibrium for Kac's caricature of a Maxwellian gas, Arch. Rational Mech. Anal. 21 (1966) 347–367). They even apply for a general branching process since using the underlying tree structure of the process we avoid some of the hard computations in McKean's approach.

Cyclic cellular automata

David Griffeath, University of Wisconsin, Madison, WI, USA

Start by randomly populating each site of the d-dimensional integer lattice with any one of N types, labeled $0,1,\ldots,N-1$. The type $\xi(x)$ can *eat* the type $\xi(y)$ at neighboring site y (i.e., replace the type at y with $\xi(x)$) *provided that* $\xi(x)-\xi(y)=1$ mod N. We will describe the dynamics of various one- and two-dimensional systems with local transitions of this sort. Computer graphics, with types represented by *colors*, will illustrate our findings.

(i) $d=1$, *continuous time, random eating.* At exponential rate one, each type chooses a random neighbor and eats its type if it can. Bramson and Griffeath (1988) have proved that there is a qualitative change in behavior between systems with $N=3,4$, and those with $N\geqslant 5$. Specifically, if $N\geqslant 5$ the process *fixates*. That is, each site is occupied by a final type with probability one. For $N=3,4$, on the other hand, every site changes type at arbitrarily large times with probability one.

(ii) $d=1$, *discrete time, deterministic eating.* During any unit of time, each type eats *every* neighboring type that it can. Fisch (1988) has proved the result analogous to that of Bramson and Griffeath above for this deterministic cellular automaton. Moreover, he has shown for the three-type model that *clustering* occurs at rate $\sqrt{t}$ and he has computed the precise asymptotic constant. Recent experimental data concerning the four-type model will be discussed briefly.

(iii) $d=2$, *discrete time, random eating.* During any unit of time, each type chooses a random neighbor and eats its type if it can. Fisch and Griffeath (1988) have recently implemented high-performance simulations of these random cellular automata, with up to 16 types and $\approx$65 000 sites, by customizing the Cellular Automaton Machine (CAM) (see Toffoli and Margolus, 1987) developed at M.I.T. The CAM exhibits spectacular complex dynamics, strongly suggesting that these models *never fixate*, no matter how large the number N of types. The *metastable* evolution is characterized by three stages:

(1) an early stage in which most of the lattice fixates, but a few 'critical droplets' of wave activity are formed;

(2) an intermediate stage in which the wave droplets expand at a linear rate until they free up all of the fixated sites;

(3) a final stage in which an oscillating equilibrium forms, consisting of stable cyclic spiral formations with very large but finite size.

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Time reversal and stationarity of infinite-dimensional Markov birth-and-death processes

Ingemar Kaj, University of Uppsala, Sweden

By using stochastic calculus for pure jump martingales, we study a class of infinite-dimensional birth-and-death processes. A technique based on a relative entropy condition, adopted from diffusion process theory, enables us to also handle the corresponding processes obtained by reversing the direction of time. The duality between the processes of forward and backward time is considered for Markov processes, defined by a prescribed family of upward and downward jump rates.

A new characterization is obtained of probability measures, which are invariant with respect to the stochastic evolution associated with a specific set of jump rates. It leads to the conclusion that, if phase transition occurs, then all measures, with a given set of local conditional distributions in common, are invariant provided one of them is.

The hydrodynamical limit of a one-dimensional nearest neighbour gradient system

Michael Mürmann, Universität Heidelberg, FR Germany

We derive the hydrodynamical behaviour of the following one-dimensional nearest neighbour gradient system.

Let $\Phi:\mathbb{R}\backslash\{0\}\to\mathbb{R}$ be an even positive convex potential with a singularity of finite order at the origin, which vanishes at infinity.

The time evolution of the respective gradient system $\{x_i(t)\}$ $(1\leq i\leq N, t\geq 0)$ is given by the system of equations:

$$\mathrm{d}x_i/\mathrm{d}t = -F(x_{i+1}-x_i)+F(x_i-x_{i-1}) \quad (1\leq i\leq N),$$

with the force $F=-\Phi'$ and $x_i<x_{i+1}$ for $1\leq i<N$.

We study the behaviour of this system on the macroscopic scale $q_i^\varepsilon(t):=\varepsilon x_i(\varepsilon^{-2}t)$ as $\varepsilon\downarrow 0$. It is proved that the rescaled empirical distributions $\rho_t^\varepsilon:=\varepsilon\sum_i \delta_{q_i^\varepsilon(t)}$ converge weakly for $t\geq 0$ as $\varepsilon\downarrow 0$, if this holds for $t=0$. The limit distributions ρ_t $(t\geq 0)$ are absolutely continuous with respect to the Lebesgue measure and satisfy the nonlinear diffusion equation

$$\frac{\partial\rho_t(q)}{\partial t}=\frac{\partial^2}{\partial q^2}\left(F\left(\frac{1}{\rho_t(q)}\right)\right)$$

in a weak sense.

Part of the proof consists in the verification of local equilibrium, which means in this case, that locally, i.e. in microscopic neighbourhoods of macroscopic points, the particles are approximately equidistant.

The model is similar to Spitzer's model of unbounded spins, whose hydrodynamical behaviour was derived by J. Fritz (J. Statist. Phys. 38 (1985) 615–645) and E. Presutti and E. Scacciatelli (J. Statist. Phys. 38 (1985) 647–653). But because of the different interpretations, the assumptions on the potential, the form of the rescaling and hence the methods of the proof are different.

A limit theorem for a many-particle system with gradient dynamics

Karl Oelschlager, Universität Heidelberg, FR Germany

We study certain large systems of interacting particles, whose positions in $\mathbb{R}$ evolve according to

$$\frac{\mathrm{d}}{\mathrm{d}t}X_N^k(t)=-\frac{1}{N}\sum_{\substack{m=1\\m\neq k}}^{N} V_N'(X_N^k(t)-X_N^m(t)),\quad k=1,\dots,N.$$

The 'interaction potential' V_N is assumed to be obtained by the scaling

$$V_N(x)=\chi_N V_1(\chi_N x),\qquad \chi_N=N^\beta,\quad \beta\in(0,1],$$

where V_1 is some nice symmetric function. The empirical processes

$$X_N(t)=\frac{1}{N}\sum_{k=1}^{N}\delta_{X_N^k(t)}$$

are shown to converge as $N\to\infty$ to the solution of a porous medium equation. Depending on β different versions are obtained as limit dynamics.

(i) $\beta\in(0,1)$ ('moderate limit'):

$$\frac{\partial}{\partial t}p(x,t)=G[V_1]\left(\frac{\partial}{\partial x}\right)^2 p^2(x,t),\qquad G[V_1]=\int_{\mathbb{R}}V_1(x)\,\mathrm{d}x,$$

(ii) $\beta=1$ ('hydrodynamic limit'):

$$\frac{\partial}{\partial t}p(x,t)=\left(\frac{\partial}{\partial x}\right)^2(F[V_1,p(x,t)]p(x,t)),\qquad F[V_1,p]=\sum_{\substack{m=-\infty\\m\neq 0}}^{\infty}\left(\left(-\frac{m}{p}\right)V'\left(\frac{m}{p}\right)\right).$$

Hyperbolic conservation laws in infinite particle systems

Errico Presutti, Università di Roma "La Sapienza", Italy

The hydrodynamical behavior of interacting particle systems has been extensively studied theoretically, numerically and by use of computer simulations. The collective phenomena which appear in such an analysis are described by conservation laws. While mathematically rigorous derivations are available for many parabolic conservation laws and for reaction-diffusion equations, much less, indeed very little, is known in the hyperbolic case.

In my talk I wish to describe a recent result obtained in collaboration with S. Caprino, A. De Masi and M. Pulvirenti which concerns the Carlemann equation:

$$\frac{\partial}{\partial t} u(r, v, t) + v\frac{\partial}{\partial r} u(r, v, t) = \{u(r, -v, t)^2 - u(r, v, t)^2\}, \tag{1a}$$

$$u(r, v, 0) = u(r, v), \tag{1b}$$

where $r \in \mathbb{R}$, $v = \pm 1$ and, for each v, $u(r, v)$ is a uniformly bounded C^1 function (for such an initial value it is known that (1) has a unique, classical, solution).

We model (1) by introducing a Markov process on $\chi = (\mathbb{N}\times\mathbb{N})^{\mathbb{Z}}$ which depends on a positive parameter ε and whose generator $L_\varepsilon = \varepsilon^{-1}L + L_c$ is such that

$$Lf(\eta) = \sum_{q,\sigma} \eta(q, \sigma)[f(\eta - \delta_{q,\sigma} + \delta_{q+\sigma,\sigma}) - f(\eta)], \tag{2}$$

$$L_c f(\eta) = \sum_{q,\sigma} [\eta(q, \sigma)^2 - \eta(q, \sigma)][f(\eta - 2\delta_{q,\sigma} + 2\delta_{q,-\sigma}) - f(\eta)], \tag{3}$$

where $\eta \in \chi$ (i.e. $\eta \equiv \eta(q, \sigma)$, $q \in \mathbb{Z}$, $\sigma \in \{-1, 1\}$) and $\delta_{q,\sigma} \in \chi$, $\delta_{q,\sigma}(q', \sigma') = 1$ if $(q, \sigma) = (q', \sigma')$ and 0 otherwise. The function f in (2) and (3) depends only on finitely many entries of the configuration η.

We assume that the process starts from an initial distribution μ^ε such that the $\eta(q, \sigma)$ are mutually independent Poisson distributed random variables with parameter $u(\varepsilon q, \sigma)$, u being as in (1b). We prove that for all $t > 0$ and $r \in \mathbb{R}$,

$$\text{weak} \lim_{\varepsilon\to 0} \mu^\varepsilon S_{[\varepsilon^{-1} r]} T_t^\varepsilon = \nu_{u(r,\cdot,t)}, \tag{4}$$

where u solves (1), S is the space translation operator and T^ε the time evolution semigroup of the process defined above. The measure $\nu_{u(r,\cdot,t)}$ is the product of Poisson measures on χ such that the expectation of $\eta(q, \pm 1)$ equals $u(r, \pm 1, t)$ for all q.

The proof of the above result is based on correlation functions techniques and avoids the usual limitations to *short times* typical of these methods by establishing *strong ergodic properties* of the *free part* of the evolution. The procedure can be applied to other particle models of discrete velocity Boltzmann equations; results are in progress.

Remark on reversible particle systems on $\{0, 1\}^S$

Yan Shi-Jian, Beijing Normal University, China

In this talk, by the method which is proposed in Yan, Chen and Ding (1982) for discussing reversible spin systems, we proved that a probability measure on $\{0, 1\}^S$ (S countable set) is a Gibbs state with respect to some potential V (see Yan, Chen and Ding, 1982) if and only if for each $u \in S$,

$$\mu\{X \in \{0, 1\}^S\colon X(u) = Y(u) \mid X(x) = Y(x) \text{ for all } x \neq u\} > 0, \quad Y \text{ a.e. } (\mu)$$

(this notation follows Liggett, 1985) holds.

References

S. Yan, M. Chen and W. Ding, Potentiality and reversibility for general speed functions, Chinese Ann. Math. 3 (1982) I. 571-586, II. 705-720.

T.M. Liggett, Interacting Particle Systems (Springer, New York, 1985).

2.4. Martingales and stochastic calculus

Martingales and information process

Barbaini Franco, Università di Genova, Italy

Let $(\Omega, \mathscr{F}, P)$ be a probability space and let $(\mathscr{F}_n)_{n\in\mathbb{N}}$ be an increasing sequence of σ-fields generated by partitions of Ω. Define an 'information process' as the triple $(I_n, H_n, \mathscr{F}_n)$, where I_n is a 'quantity of information' on $\mathscr{F}_n$, given by

$$I_n(A) = \mu(A)(\varphi(P(A)) - \varphi(1)) \quad \forall A \in \mathscr{F}_n,$$

$\mu(A)$ is the utility of event A, $\varphi(t):(0,1]\to\mathbb{R}^+$ is decreasing. H_n is the entropy of $\mathscr{F}_n$, defined by $H_n = \sum_A P(A) I_n(A)$ where the sum runs over all the atoms of $\mathscr{F}_n$, for which $P(A) > 0$.

Let us define a sequence X_n of r.v. by

$$X_n = \sum_A \mathscr{X}_A I_n(A), \qquad \mathscr{X}_A = \begin{cases} 1, & \omega \in A, \\ 0, & \omega \notin A. \end{cases}$$

X_n is a submartingale with respect to $\mathscr{F}_n$, hence $H_n = E(X_n) \leqslant E(X_{n+1}) = H_{n+1}$. Using the Doob decomposition for submartingales, we prove the branching property $H_{n+1} = H_n + H(\mathscr{F}_{n+1}/\mathscr{F}_n)$, and the 'local' property, $H_n = \sum_k \Delta(H_k)$, where $\Delta(H_k)$ is the gain of information from time $k-1$ to time k. Define the entropy of $\mathscr{F}_T$, for a stopping time T, putting

$$H_T = \lim_n E(X_{T\wedge n}), \quad T\wedge n = \inf(T, n).$$

If T is 'regular', then $H_T = c_0 + \sum_{S_k<T} H(\mathscr{F}_{S_{k+1}}/\mathscr{F}_{S_k})$, where S_k are stopping times such that $S_k \leqslant S_{k+1}$.

If $T\in L^1$, $\Delta(H_n) = \text{const.}$, $\forall n\in\mathbb{N}$, then it may be proved as a theorem similar to Wald's identities that $H_T = E(T)\cdot\text{const.}$

Applying this approach to Markov processes, it can be shown that the expected permanence time T for a subset of the state space is finite if and only if the entropy of $\mathscr{F}_T$ is finite.

Biconcave functions and sharp inequalities for martingale transforms

K.P. Choi, National University of Singapore, Singapore

Let $(\Omega, \mathscr{A}_\infty, P)$ be a complete probability space and $\mathscr{A} = (\mathscr{A}_t)_{t\geqslant 0}$ a non-decreasing right-continuous family of sub-σ-fields of $\mathscr{A}_\infty$. Suppose $\{B\in\mathscr{A}_\infty: P(B)=0\}\subseteq\mathscr{A}_0$. Let $V=(V_t)_{t\geqslant 0}$ be a real predictable process taking values in $[0,1]$. Assume that $X=(X_t)_{t\geqslant 0}$ is a real martingale adapted to $\mathscr{A}$ and that almost all of the paths of X are right continuous on $[0,\infty)$ with left-hand limits on $(0,\infty)$. Define

$$Y_t = \int_{[0,t]} V\,\mathrm{d}X \quad \text{a.s.}$$

Let $\|X\|_p = \sup_{t\geqslant 0}\|X_t\|_p$. For $1<p<\infty$, Burkholder showed that there exists a constant c_p, such that $\|Y\|_p \leqslant c_p\|X\|_p$.

The contribution of this paper is to determine the optimal c_p in this inequality. Based on an idea of Burkholder, the problem is reduced to determine the smallest constant c, such that there exists a biconcave function $u:\mathbb{R}^2\to\mathbb{R}$ dominating $v(x,y) = |x|^p - c|x-y|^p$. We will also illustrate how this method of proof can be used to prove other sharp inequalities for martingale transforms.

Continuity properties of Itô stochastic integral

A. Jakubowski, Uniwersytet Mikołaja Kopernika, Toruń, Poland*
J. Memin, Université de Rennes I, France
G. Pages, Université P. et M. Curie, Paris, France

Let for each n, $X_n = (X_n(t))_{t\in\mathbb{R}^+}$ be a stochastic process with right-continuous trajectories with left-hand limits defined on a 'stochastic basis' $(\Omega_n, \mathscr{F}, (\mathscr{F}_n(t))_{t\in\mathbb{R}^+}, P_n)$.

We shall say that the sequence $\{X_n\}$ satisfies Condition (U.T) iff for each $t \in \mathbb{R}^+$ random variables

$$\sum_{k=0}^{k=m-1} |E((X_n(t_{k+1}) - X_n(t_k))\chi_{\{|X_n(t_{k+1})-X_n(t_k)|\leq 1\}} | \mathcal{F}_n(t_k))|,$$

where $n \in \mathbb{N}$ and $0 = t_0 < t_1 < \cdots < t_m \leq t$ is a partition of [0, 1], are uniformly tight.

It is well-known that if $X_n = X$, $n \in \mathbb{N}$, then Condition (U.T) holds if and only if X is a semimartingale. In any case Condition (U.T) implies all X_n are semimartingales.

Theorem. *Let for each $n \in \mathbb{N}$, K_n be a right-continuous process with left-hand limits adapted to the filtration $(\mathcal{F}_n(t))_{t\in\mathbb{R}^+}$. Assume that distributions of (K_n, X_n) on the Skorokhod space $D(\mathbb{R}^+ : \mathbb{R}^2)$ are weakly convergent,*

$$(K_n, X_n) \xrightarrow{\mathcal{D}} (K, X).$$

If $\{X_n\}$ satisfies Condition (U.T), *then X is a semimartingale with respect to the natural filtration generated by X and K and Itô stochastic integrals $K_{n-} \cdot X_n$ converge,*

$$K_{n-} \cdot X_n \xrightarrow{\mathcal{D}} K_- \cdot X.$$

Several applications of the above theorem are discussed.

Random time changes and convergence in distribution under the Meyer–Zheng conditions

Thomas G. Kurtz, University of Wisconsin, Madison, WI, USA

Meyer and Zheng (Ann. Inst. H. Poincaré 4 (1984) 353–372) gave conditions under which a sequence of stochastic processes is relatively compact (in the sense of convergence in distribution) when the space of sample paths is topologized by convergence in measure. An examination of their paper reveals that these conditions, involving boundedness of the conditional variations of the processes, imply much greater uniform regularity of the sample paths of the processes than is implied by convergence in measure. We formulate an analog of the Meyer–Zheng conditions for a general separable metric space, and capture the greater regularity of the convergence under these conditions by showing that they imply the existence of a sequence of random time transformations such that the sequence of transformed processes is relatively compact under the Skorohod topology.

Rapports entre adaptation et progressivité pour un processus mesurable

G. Letta, Università di Pisa, Italy

Sur un espace probabilisé, muni d'une filtration vérifiant les conditions habituelles, on considère un processus de la forme

$$X_t = \int_0^t H_s \, \mathrm{d}s,$$

où H est mesurable adapté.

Il est connu qu'on peut construire un processus progressif (même prévisible), dont chaque trajectoire soit équivalente (pour la mesure de Lebesgue) à la trajectoire correspondante de H; par conséquent, X est adapté.

On montre d'abord que ces résultats deviennent faux dans un cadre 'sans probabilité'. On se place ensuite dans un cadre 'avec probabilité', et l'on démontre une version affaiblie des résultats précédents, qui est valable sans les conditions habituelles (et qui se réduit à la version précédente lorsque ces conditions sont remplies).

The transformation formula of stochastic integrals with respect to semimartingales

Xuerong Mao, University of Warwick, Coventry, UK

Let $(\Omega, \mathscr{F}, P, \{\mathscr{F}_t\}_{t\geq 0})$ be a complete probability space with the filtration $\{\mathscr{F}_t\}$ satisfying the usual conditions. In this paper we first develop the transformation formula of stochastic integrals with respect to martingales as follows:

Theorem. *Let M be a continuous local martingale. Let X be a predictable process such that*

$$\int_0^t X_s^2 \, d\langle M\rangle_s < \infty \quad a.s. \quad \text{for all } 0 \leq t < \infty.$$

For $t \geq 0$, define

$$T_t = \inf\{s: \langle M\rangle_s > t\} \quad \text{and} \quad T_{t-} = \inf\{s: \langle M\rangle_s \geq t\},$$

then for any finite $\{\mathscr{F}_t\}$-stopping time τ, we have

$$\int_0^\tau X_t \, dM_t = \int_0^{\langle M\rangle_\tau} X_{T_t} \, dM_{T_t} = \int_0^{\langle M\rangle_\tau} X_{T_t} \, dM_{T_t} \quad a.s.$$

where the left integral and other two integrals are relative to $\{\mathscr{F}_t\}$ and $\{\mathscr{F}_{T_t}\}$ respectively.

As an application of this theorem, we prove that stochastic integral with respect to continuous local martingales can be transformed into Itô's stochastic integrals. These results, together with some well-known properties of Itô's stochastic integrals, will be used to study the properties of stochastic integrals with respect to continuous martingales. They are also to be applied to study the properties, for example, stochastic stability and boundedness, of the solutions to stochastic integral equations with respect to semimartingales. In particular, we discuss the Markov property of the solutions to stochastic integral equations with respect to continuous local martingales. Some examples are worked out to illustrate the usefulness of these results.

Restricted versions of martingale dominance between random variables, which imply demand dominance in a portfolio setting

Isaac Meilijson, Tel-Aviv University, Israel*
Michael Landsberger, University of Haifa, Israel

Let $X \sim F$ be the return of one unit of money invested in a risky asset, let $1-\alpha+\alpha X$ be a 'portfolio' based on X and on a safe return of one unit, and let U be a concave, nondecreasing utility function. The risk averse investor finds a best portfolio by evaluating $EU(1-\alpha_F+\alpha_F X) = \max_{0\leq\alpha\leq 1} EU(1-\alpha+\alpha X)$. Rothschild and Stiglitz have shown that if F is replaced by a riskier distribution G in the sense of second degree dominance (for some joint distribution of (X, Y) with $X\sim F$, $Y \sim G$, $E(Y|X) = X$ almost surely), then α_G may exceed α_F, contrary to intuitively appealing common sense. They analyse the subclass of concave utility functions for which $\alpha_G \leq \alpha_F$ whenever F dominates G as above.

We characterize the stochastic order between distributions with equal means under which $\alpha_G \leq \alpha_F$ for all concave, nondecreasing utility functions. Y is 'riskier' than X under this order if $E(Y-E(Y))I_{Y\leq x} \leq E(X-E(X))I_{X\leq x}$ for all x and $E(Y|Y\leq x) \leq E(X|X\leq x)$ for all x. Each one of these two requirements is stronger than second degree dominance. Each has further characterizations in terms of integral inequalities, permissible fair conditional distributions of Y given X, and the optional sampling schemes via Skorokhod-type embeddings in Brownian motion which generate such martingale pairs.

Stochastic integrals over independent increments bivariate symmetric α stable processes

A. Reza Soltani, Shiraz University, Iran

For an independent increments bivariate symmetric α stable process $\{X_t, 0 \leq t \leq 1\}$, we define the stochastic integral $\int_0^1 g(t)\,\mathrm{d}X_t$ along with the function space in which g lies. The univariate case is considered in M. Schilder, Some structure theorems for the symmetric stable laws, Ann. Math. Statist. 41 (1970) 412–421.

2.5. Measure and distributions

Intersection local times for the Brownian density process and an infinite system of planar Brownian motions

Robert J. Adler, Technion, Haifa, Israel

The Brownian density process is formally defined as the distribution valued Markov process η_t given by the solution of the stochastic partial differential equation

$$\frac{\partial \eta}{\partial t} = \tfrac{1}{2}\Delta\eta + \nabla \cdot W,$$

where W is a Gaussian white noise process on $\mathbb{R}_+ \times \mathbb{R}^d$.

It is more easily visualized, and understood, as a model describing the temporal development of an infinite system of independent Brownian motions on $\mathbb{R}^d$.

The 'self-intersection local time' of the Brownian density process is generally defined in a rather arbitrary fashion as an additive functional on the random distribution η. In this talk I shall present results obtained jointly with Raisa Epstein and Marika Lewin, that study this intersection local time (for $d = 2$) via intersection local times defined on an infinite system of planar Brownian motions.

There are two advantages to this approach. Firstly, it brings the rather abstract concept of intersection local time for random distributions down to the level of properties of planar Brownian motion, which helps considerably in developing intuition, and shows that the arbitrary definition referred to above is actually justifiable. Secondly, this approach enables the development of a Tanaka-like evolution equation for the Brownian density intersection local time, a result which, at the moment, seems to be unavailable by other methods.

On stable Markov processes

Robert J. Adler, Technion, Haifa, Israel
Stamatis Cambanis, University of North Carolina, Chapel Hill, NC, USA*
Gennady Samorodnitsky, Boston University, MA, USA

Necessary conditions are given for a symmetric α-stable (SαS) process, $1 < \alpha < 2$, to be Markov. These conditions are then applied to find Markov or weakly Markov processes within certain important classes of SαS processes: time changed Lévy motion, sub-Gaussian processes, moving averages and harmonizable processes. Two stationary SαS Markov processes are introduced, the right and the left SαS Ornstein–Uhlenbeck processes. Some of the results are in sharp contrast to the Gaussian case $\alpha = 2$.

Functional methods in the theory of stochastic processes with values in locally compact groups

T. Byczkowski, Wrocław Technical University, Poland

We discuss an application of functional methods for some problems concerning stochastic processes with values in locally compact groups.

We present solutions of the following problems: Invariance principle, zero-one law, and RKHS for Brownian motion on a Lie group.

Invariance principle: Suppose that $\{\xi_j^{(n)};\ j=1,\ldots,n,\ n\geq 1\}$ is a triangular array of i.i.d. r.v.'s with values in a locally compact separable group G. Assume that $S_n^{(n)}=\xi_1^{(n)}\cdots\xi_n^{(n)}$ converges in distribution to a Gaussian measure μ. Then the random walk $S_{[nt]}^{(n)}$ converges in distribution to the (left) Brownian motion on G generated by μ.

Zero-one law: Let $\xi=\{\xi(t);\ t\in T\}$ be a symmetric infinitely divisible stochastic process with values in G and let η be its Lévy measure. Suppose that H is a measurable subgroup of G^T. Then

(i) If $\eta(H^c)=\infty$ then $P\{\xi\in H\}=0$,

(ii) If $\eta(H^c)=0$ then either $P\{\xi\in H\}=0$ or 1.

RKHS: We discuss RKHS for Brownian motion on a Lie group in the context of admissible translates. Several applications are outlined.

Towards an axiomatic theory of distributions which are close to the normal distribution

H. Dinges, J.W. Goethe-Universität, Frankfurt, FR Germany

H. Daniels (Ann. Math. Statist., 1954) has shown that under suitable regularity conditions the density of

$$n^{-1}(Y_1+\cdots+Y_n),\quad \text{with } Y_j \text{ i.i.d.},$$

has the form

$$\sqrt{n/(2\pi)}\cdot[K''(x)]^{1/2}\exp(-nK(x))\exp(n^{-1}S(n^{-1},x))\,\mathrm{d}x \tag{1}$$

in some fixed neighborhood U of $x^*=\mathbf{E}Y$, where the *correcting term* has an asymptotic expansion

$$\varepsilon S(\varepsilon,x)=\varepsilon S_1(x)+\varepsilon^2S_2(x)+\cdots+\varepsilon^{m-1}S_m(x)+\mathrm{O}(\varepsilon^m), \tag{2}$$

uniformly in every compact subinterval of U.

In various estimation problems we meet families of densities of a similar type. It is our intention to develop a mathematical theory of such families. We should like to see these families as points with infinitesimal extension on a manifold.

Definition. Let U be an open neighborhood of a point x^* on a d-dimensional manifold. A family

$$\{f_\varepsilon(x)\,\mathrm{d}x:\varepsilon\to 0\}$$

of differential forms of degree d on U is called a *Wiener germ with center x^* of order m* if

$$f_\varepsilon(x)\,\mathrm{d}x=(2\pi\varepsilon)^{-d/2}\exp(-\varepsilon^{-1}K(x)+S_0(x)+\varepsilon S_1(x)+\cdots+\varepsilon^{m-1}S_{m-1}(x)+\mathrm{O}(\varepsilon^m))\,\mathrm{d}x, \tag{3}$$

uniformly on every compact subset of U with smooth functions $K(x)$, $S_j(x)$ which satisfy:

$$K(x^*)=0,\quad K(x)>0 \text{ for all } x\in U\backslash\{x^*\} \quad\text{and}\quad K''(x^*) \text{ is positive definite}, \tag{4}$$

$$\int_{U^*} f_\varepsilon(x)\,\mathrm{d}x=1-\mathrm{O}(\varepsilon^m), \tag{5}$$

for every neighborhood U^* of x^*. We call $K(x)$ the *entropy function* and $\exp(S_0(x))\,\mathrm{d}x$ the *modulating density* of the Wiener germ. S_j is called the jth order *correcting function*. Some basic properties of these mathematical objects will be pointed out.

Definition. Let $X^{(1)}, X^{(2)},\ldots,$ be random variables with values on a manifold. $\varepsilon_n\to 0$. We say that the distributions $\mathcal{L}(X^{(n)})$ follow a Wiener germ with center x^* along the sequence ε_n, if $\mathcal{L}(X^{(n)})$ restricted to some neighborhood U of x^* has a density $f_{\varepsilon_n}(x)\,\mathrm{d}x$ as described in (3).

Theorem. *Let x^* be a point on a d-dimensional manifold and y^* a point in $\mathbb{R}^d$. Let $T_0(x)$ be a diffeomorphism of a neighborhood U of x^* into $\mathbb{R}^d$ with $T_0(x^*) = y^*$. Let $T_1(x), T_2(x), \ldots, T_m(x)$ be smooth mappings of U into $\mathbb{R}^d$ and*

$$T(\varepsilon, x) = T_0(x) + \varepsilon T_1(x) + \cdots + \varepsilon^m T_m(x).$$

Consider a family of random variables $\{X_\varepsilon : \varepsilon \to 0\}$ whose distributions follow a Wiener germ with center x^* and put

$$Y_\varepsilon = T(\varepsilon, X_\varepsilon).$$

Then $\{\mathcal{L}(Y_\varepsilon) : \varepsilon \to 0\}$ follows a Wiener germ with center y^*.

Theorem. *Let $\{Z_\varepsilon : \varepsilon \to 0\}$ be a family of d_1-dimensional random vectors and $\{W_\varepsilon : \varepsilon \to 0\}$ be a family of d_2-dimensional random vectors such that the joint distributions $\mathcal{L}(Z_\varepsilon, W_\varepsilon)$ follow a $(d_1 + d_2)$-dimensional Wiener germ with center (z^*, w^*). Then we have:*

(a) *$\mathcal{L}(Z_\varepsilon)$ follows a Wiener germ with center z^*.*

(b) *For every fixed z in some neighborhood of z^* the conditional distributions $\mathcal{L}(W_\varepsilon \mid \{Z_\varepsilon = z\})$ follow a Wiener germ.*

A generalization of Kolmogorov's extension theorem

K.Y. Hu, National University of Singapore, Singapore

Let T be a set, $\mathbb{F}(T)$ be the set of all non-empty finite subsets of T, and S_t be a complete separable metric space for each $t \in T$. For any $F \in \mathbb{F}(T)$, let S_F be the product set $\Pi\{S_t : t \in F\}$, $\mathcal{B}_F$ be the Borel σ-algebra in S_F, and $\pi_F : S_T \to S_F$ be the projection map. Kolmogorov's extension theorem says that if P_F is a probability measure on $(S_T, \pi_F^{-1}[\mathcal{B}_F])$ for each $F \in \mathbb{F}(T)$ and if $\{P_F : F \in \mathbb{F}(T)\}$ satisfies the consistency condition, $A \in \pi_F^{-1}[\mathcal{B}_F] \cap \pi_{F'}^{-1}[\mathcal{B}_{F'}] \Rightarrow P_F(A) = P_{F'}(A)$, then there is a unique probability measure P on $(S_T, \mathcal{B}_T)$ such that $P|_{\pi_F^{-1}[\mathcal{B}_F]} = P_F$ for each $F \in \mathbb{F}(T)$, where $\mathcal{B}_T$ is the σ-algebra in S_T generated by $\bigcup_{\{\pi_F^{-1}[\mathcal{B}_F] : F \in \mathbb{F}(T)\}}$. In this way Kolmogorov's theorem randomizes the graph of a function in the function space S_T, which consists of functions with the same domain T. The objective of this talk is to show how we can extend Kolmogorov's theorem so that the domain of a function is also randomized, and to introduce an application of this extension theorem to the construction of stochastic processes with random time domains.

Reference

K.Y. Hu, A generalization of Kolmogorov's extension theorem and an application to the construction of stochastic processes with random time domains, Ann. Probab. 16 (1988) 222–230.

Exchangeable random measures in the plane

Olav Kallenberg, Auburn University, AL, USA

A random measure ξ on $[0, 1]^2$, $[0, 1] \times \mathbb{R}_+$ or $\mathbb{R}_+^2$ is said to be separately exchangeable, if its distribution is invariant under arbitrary Lebesgue measure preserving transformations in the two coordinates, and jointly exchangeable if ξ is defined on $[0, 1]^2$ or $\mathbb{R}_+^2$, and its distribution is invariant under mappings by a common measure preserving transformation in both directions. In each of the five cases, a de Finetti-type representation of ξ is obtained in terms of independent Poisson processes and i.i.d. random variables. In particular, a conjecture of David Aldous concerning the general structure of an ergodic and separately exchangeable counting random measure on $\mathbb{R}_+^2$ is essentially confirmed.

On nonlinear transformations in abstract Wiener spaces

S. Kusuoka, University of Tokyo, Japan

Let (μ, H, B) be an abstract Wiener space and G be a Lie group. Let $\sigma : G \to \mathcal{L}^\infty(H, H)$ be a unitary representation of G in H. Now let $g : B \to G$ and $F : B \to H$ be good measurable maps.

Then we can define a measurable map $\Phi : B \to B$ by $\Phi(z) = \tilde{\sigma}(g(z))(z + F(z))$, $z \in B$. We will show the change of variables formula and the Sard theorem relative to this map Φ under certain conditions.

2.6. Percolation

Singularity of the density of states for one-dimensional chains with random couplings

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Let H be the random operator on $l^2(\mathbb{Z})$,

$$Hu(n) = W_{n,n+1}u(n+1) + W_{n-1,n}u(n-1), \tag{1}$$

where $W_{i,i+1}$ are i.i.d. random variables with smooth distribution supported in some interval $[\alpha, \beta]$, $0 < \alpha < \beta < \infty$. The integrated density of states of H is the function $F(E)$ defined by the following limit (which exists and is constant a.e.),

$$F(E) = \lim_{N \to \infty} (2N+1) \# \{\lambda_i^{(N)} \leq E\}, \tag{2}$$

where the $\lambda_i^{(N)}$'s are the eigenvalues of the restriction of H to the interval $[-N, N]$ with Dirichlet boundary conditions. The density of states $\rho(E)$ is the derivative of $F(E)$. We prove that for H defined in (1) the density of states is singular at 0 and precisely

$$\rho(E) \geq C|E|^{-1}(\ln|E|)^{-4}. \tag{3}$$

This proves in the general case the singular behaviour which was found in some exactly solvable models by F.J. Dyson (Phys. Rev. 92 (1953) 1331). The result is established through the study of the recurrence properties of an associated Markov chain. Physical arguments and numerical computations supporting the existence of a singularity like (3) were given by T.P. Eggarter and R. Riedinger (Phys. Rev. B 18 (1978) 569).

Uniqueness of the infinite cluster in percolation theory

Alberto Gandolfi, Delft University of Technology, The Netherlands

This study is concerned with the structure of the infinite clusters of nearest neighbor equal spins in $\mathbb{Z}^d$, when the two states configurations of spins are described by a Gibbs state.

For some measures there are no infinite clusters w.p.1 and when there exists at least one infinite cluster w.p.1 we say there is percolation.

The following results are obtained for Gibbs states with a potential decreasing not too slowly with the distance:

(1) If the state is stationary then there is at most on infinite cluster of spins of a given sign. It can exist together with a unique cluster of opposite sign and it is dense in $\mathbb{Z}^d$.

(2) If the state is non-stationary the density of points in which the infinite clusters come near each other is zero.

(3) On the cluster together with the boundary the frequency of spins having a given local configuration around converges to the conditional probabilities exponentially in the volume of the cluster.

This generalizes previous results on Bernoulli percolation (Aizenman, Newman and Kesten, 1986) in dimension $d > 2$ and shows a different extension of known results for $d = 2$ (Gandolfi, Keane and Russo, 1986).

Many problems are still open in order to have a better description of the system: the occurrence of percolation for given interactions, its connection with phase transition, with the existence of non-stationary states and with the theory of large deviations.

Note: Since this talk was held new results have been obtained: (1) has been proved by B. Burton and M. Keane in a general finite energy model on $\mathbb{Z}^d$ and their proof can be extended to long range models.

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Symmetric exclusion on random sets and a related problem for random walks in random environment

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We study symmetric exclusion on a random set, where the underlying kernel is strictly positive. The random set is generated by Bernoulli experiments with success probability q.

We prove that for certain values of the involved parameters the transport of particles through the system is drastically different from the classical situation on $\mathbb{Z}$. In dimension one and $r := \liminf_{|x|\to\infty} |(|x|^{-1} \log p(0,x))| > |\log(1-q)|$ the transport of particles occurs on a nonclassical scale and is (on a macroscopic scale) *not* governed by the heat equation as in the case $r < |\log(1-q)|$ on a random set or in the classical situation on $\mathbb{Z}$.

The reason for this behaviour is that a random walk with jump rates $p(x,y)$ restricted to a random set converges to Brownian motion in the usual scaling if $r < |\log(1-q)|$ but yields nontrivial limit behaviour only in the scaling $x \to u^{-1}x$, $t \to u^{1+\alpha}t$ $(\alpha > 1)$ if $+\infty > r > |\log(1-q)|$. We calculate α and study the limiting processes for the various scalings for fixed random sets. We briefly discuss the case $r = +\infty$; here in general a great variety of scales yields nontrivial limits.

Finally we discuss the case of a 'stationary' random set.

Large dimensional behavior of percolation and Potts models

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Let $\theta(p, \mathbb{Z}^d, \text{bond})$ and $p_c(\mathbb{Z}^d, \text{bond})$ denote the percolation probability and critical probability for bond percolation on $\mathbb{Z}^d$. $\theta(p, \mathbb{Z}^d, \text{site})$ and $p_c(\mathbb{Z}^d, \text{site})$ are the analogues for site percolation.

Theorem 1.

$$\frac{1}{2d-1} \le p_c(\mathbb{Z}^d, \text{bond}) \le p_c(\mathbb{Z}^d, \text{site}) \le \frac{1}{2d} + O\left(\frac{(\log\log d)^2}{d \log d}\right).$$

Theorem 2. *For bond percolation on $\mathbb{Z}^d$, when $p = \beta/(2d)$ with $\beta > 1$ fixed,*

$$\lim_{d\to\infty} \theta\left(\frac{\beta}{2d}, \mathbb{Z}^d, \text{bond}\right) = y(\beta),$$

where $y(\beta)$ is the unique strictly positive solution of

$$y = 1 - \exp(-\beta y).$$

More generally

$$\lim_{d\to\infty} \sup_{A,B} |P_{\beta/(2d)}\{\textit{all sites in } A \textit{ belong to the infinite occupied cluster, but none of the sites in } \beta \textit{ do}\} - y(\beta)^{|A|}(1-y(\beta))^{|B|}| = 0.$$

Here A and B vary over pairs of disjoint finite sets of vertices of $\mathbb{Z}^d$, and $|A|$ denotes the cardinality of A.

We shall also discuss analogues of Theorems 1 and 2 for site percolation on $\mathbb{Z}^d$, the Potts model with q colors and the Fortuin-Kasteleyn model (see a forthcoming article by Aizenman, Chayes, Chayes and Newman in J. Statist. Phys. for a description of the latter). The work on the Potts model is joint with J. Bricmont, J.L. Lebowitz and R.H. Schonmann.

2.7. Point processes

Are stochastic Petri nets of interest in the theory of random processes?

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Petri nets (PN) are a widely used tool for the description of the logical behaviour of systems exhibiting concurrency, synchronization and conflicts (such as parallel computing systems and communication protocols). PN are based on a graphical notation, possibly enhanced by the use of types and predicates.

The introduction of random timing into PN leads to stochastic Petri nets (SPN), which have proved to be an effective tool for the analysis of the performance and reliability of various computer and communication systems.

SPN are isomorphic to continuous-time, discrete-state stochastic processes. In the simpler cases, SPN are isomorphic to continuous-time Markov chains.

SPN are presently used in the applied stochastic modeling field, although a deep theoretical foundation is lacking, from the point of view of both the general net theory, and the theory of stochastic processes.

The aim of the proposed presentation is introducing SPN to an audience composed of experts in the theory of stochastic processes, with the hope that the theoretical questions posed by this class of stochastic models may be of interest, thus spurring theoretical research activity in this field.

On prediction and state estimation of marked point processes, with applications to reliability theory

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Consider a marked point process (MPP), $(T, X)=(T_n, X_n)_{n\geqslant 1}$ defined on $(\Omega, \mathcal{F}, P)$ and with a countable marked space E. Let $N=(N_t(x);\, t\geqslant 0, x\in E)$ be the corresponding counting process and let (F_t) be the (completed) internal history. Such a framework can be conveniently used to describe part failures of a device, with F_t referring to the pre-t events. It can also be used for introducing natural definitions to describe, say, the ageing of the parts, or their dependence. A fundamental notion in such definitions is the prediction process $\mu=(\mu_t)_{t\geqslant 0}$, formed by the (F_t)-conditional distributions of the sample paths of (T, X).

Frequently the MPP (T, X) cannot be fully observed. Suppose, instead, that one can observe a 'derived MPP', say $(\hat{T}, \hat{X})=(\hat{T}_n, \hat{X}_n)$ with $\hat{X}_n\in\hat{E}$, such that the corresponding counting process $\hat{N}=(\hat{N}_t(\hat{x}), t\geqslant 0, \hat{x}\in\hat{E})$ satisfies $\hat{N}_t(\hat{x})=\sum_x\int_0^t C_s(x,\hat{x})\,\mathrm{d}N_s(x)$, with $C_s(x,\hat{x})$ (F_t)-predictable and taking values 0 and 1. In reliability applications such a representation will typically hold for (sub)system failures. Let $(\hat{F}_t)$ be the (completed) internal history of the observable process $(\hat{T}, \hat{X})$. Clearly $\hat{F}_t\subset F_t$.

It is often of interest to consider the conditional distribution of the original MPP (T, X), given the observable pre-t information $\hat{F}_t$. The corresponding $(\hat{F}_t)$-conditional prediction process $\hat{\mu}=(\hat{\mu}_t)$ can be viewed as consisting of two 'parts': the distribution $\hat{\pi}_t$ of the (partly unobservable) pre-t marked points, and the distribution of the post-t marked points. The latter has a similar role in reliability theory as (μ_t) in the (F_t)-conditional case, in defining distributional properties relating to future events. We shall consider the question: "To what extent are such properties inherited from μ to $\hat{\mu}$?" $\hat{\pi}_t$ has here the role of a mixing distribution. We also derive a filter formula for $\hat{\pi}=(\hat{\pi}_t)_{t\geqslant 0}$, which shows how the 'state estimate' of the unobservable points is updated.

Queueing simulation in heavy traffic

Søren Asmussen, University of California, Santa Barbara, USA, and Aalborg University, Denmark

The heavy traffic behavior of a number of standard queueing simulation procedures like sample mean estimation and regenerative simulation is studied. In particular, limit theorems based upon Brownian approximations are given in a double limit with both the sample size t tending to infinity and the traffic intensity ρ tending to one. The results provide a rigorous proof that the growth rate $t = t(\rho) \sim (1-\rho)^{-2}$ is critical, and it is shown both from theory and empirical illustrations that for this or smaller sample sizes negative bias presents a serious problem. Also the heavy traffic behaviour of the limiting variances in the standard large sample theory is found.

Further, a variance reduction method based upon a combination of importance sampling by means of exponential family likelihood ratio identities and control variates (regression) is surveyed and is found to present a substantial improvement over the standard methods in some simple cases (say a variance reduction by a factor of 2000 for the M/M/1 queue with $\rho = 0.9$).

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A counting process approach for age-dependent epidemics

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In [6] a non-Markovian type of epidemic was described in which the infectiousness of a diseased individual depends not only on the length of time he has been sick but also on the time at which he was infected with the disease. Thus the times at which each individual has got the infection are needed to define the behaviour of the system. The model is an extension of the general stochastic epidemic, in which the force of infection has been modified to take into account the above features.

In [7] the same model had been revisited with a point process approach by utilizing the information of the times of occurrences and removals of the infectives. Such an approach provides a more convenient framework for statistical analysis. In fact it refers to the nonparametric inference methods due to Aalen [1, 2] for counting processes based on martingale theory. Estimators for the parameters involved are easily suggested, and the central limit theorems for martingales [5] allow obtaining asymptotic distributions for the estimators themselves.

Chronological age had been ignored so far in the above epidemic models. But if we wish to consider the vital dynamics of the total population this parameter cannot be ignored. Recently the present author [3] has revisited an old model for age dependent population dynamics [4] with a marked point process approach, based on the information related to the times of occurrences of events of births and of deaths, in such a way that it leads to a natural dynamical description of the phenomenon, without imposing Markov properties. Age plays the role of a mark on the population process. Interesting properties of the process are shown, among which the classical renewal equation can be easily understood.

In this paper a superimposition of the above epidemic model to the age dependent birth-and-death process is discussed, adding an age dependence to the force of infection, which now will be depending on both age with respect to the disease, and chronological age, making the overall model more realistic.

Estimators for the force of infection are given, along with their asymptotic properties.

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Integration by parts for jump processes

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The Malliavin calculus is a calculus of variations in function space. One of its applications is to show the existence and smoothness of densities of processes defined by stochastic differential equations. Following the original work on the Malliavin calculus for diffusions there have been papers on the Malliavin calculus for equations driven by jump processes. See, for example, the papers of Bismut, Bichteler and Jacod, Bichteler, Gravereaux and Jacod, Leandre, and Bass and Cranston. A central result in these papers is an integration-by-parts formula in function space. This is often established using involved analysis to investigate a perturbation of the original process by a Girsanov change of measure. This note will show how, using stochastic flows, an integration-by-parts formula can be obtained from a classical integration-by-parts in the state space of the jump measure. The derivation is elementary and the Malliavin covariance matrix appears in a natural way.

Optimality in accelerated life tests

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We formulate a stochastic control problem which arises from the optimal design of accelerated life tests. The model is obtained, in a natural way, in the set up of point processes. Let N_t be the total number of items under test at time t; Y_t the observed number of items failed up to time t; S_t the stress level applied to the items under test at time t; $J(N, S)$ the cost functional given by

$$J(N,S)=E\left[\int_0^T cS_t(N_t-Y_t)\,\mathrm{d}t+J_T\right].$$

The optimal policy can be found via the dynamic programming equation. In particular, for each fixed S, we prove that the optimal policy to minimize $J(N, S)$ is a bang-bang one. This amounts to simply running the test with a constant number of items up to an optimal stopping time, which is the exit time of the state process from a given region.

Filtering Markov processes with pure jump observation: Reducing the dimension of the filter

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A solution of the filter equation for a Markov system with pure jump observation is constructed. The form of the equation suggests a new approach to the definition of dimension reduction in

filtering theory. This approach allows, for instance, the construction of an explicit algorithm for the case of Markov chains without feed-back from the observation and no correlation between jumps of the state and observation processes. More general results depend on a careful analysis of the algebraic structure of the operators involved.

Minimality of conditionally Poisson systems

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In this paper we discuss minimality of stochastic systems with a counting process output, that is conditionally Poisson and where the state process lives on a finite space. Hence we have for this system the forward representation

$$dX = AX\,dt + dM, \tag{1}$$

$$dN = CX\,dt + dm. \tag{2}$$

Here is $X_t \in \{0, 1\}^n$ the indicator process of an n-state Markov process, N the counting process and M and m are martingales.

The problems that we want to address are to find a characterization of minimality of the state space of X and, if this space is not minimal, to find an 'efficient' procedure to reduce the state space to a smaller (minimal) one, such that again a representation of the form (1), (2) holds. It turns out that the concept of stochastic observability plays a key role in finding solutions to the above stated problems. Also an algorithm will be presented that yields the minimal representation of the form (1), (2).

Calculation of the availability of a two unit parallel system with cold standby: an illustration of the embedding technique

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We consider a two unit parallel system with cold standby, Markovian degradation of the working unit and one repair facility. Kawai (1981) has shown that under certain conditions on the degradation and repair process, the optimal preventive maintenance policy is of control limit type, i.e. there exists a critical level such that a preventive maintenance on the working unit is carried out if and only if the repair facility is free and the condition of the working component is worse than the critical level. Moreover, Kawai provides an explicit expression for the availability of the system under such a control limit rule. In this paper we show how the embedding technique from Markov decision theory can be exploited to provide a more rigorous derivation of the availability under a control limit rule. Moreover, a recurrent computational scheme is proposed to numerically compute the availability in case of Erlang distributed repairtime distributions.

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2.8. Quantum probability and statistical physics

Squeezing noise and Bloch equations

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The general analysis of quantum noise in Accardi (1989) shows that a squeezing noise can produce quadratic nonlinearities in the Langevin equations leading to the Bloch equations. These quadratic nonlinearities are governed by the imaginary part of the off diagonal terms of the covariance of the noise (the squeezing terms) and imply a correction to the usual form of the Bloch equations.

Here we study numerically the case of spin one nuclei subjected to squeezing noises of particular type. We show that the corrections to the Bloch equations, suggested by the theory, to the behaviour of the macroscopic nuclear polarization in a scale of times of the order of the relaxation time can be quite substantial. In the equilibrium regime, even if the qualitative behaviour of the system is the same (exponential decay), the numerical equilibrium values predicted by the theory are different from those predicted by the usual Bloch equation and this difference can be tested experimentally.

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Quantum invariance principles and the weak coupling limits

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Some techniques and ideas, recently introduced in the proof of the quantum probabilistic generalizations of the De Moivre-Laplace theorem and the quantum invariance principle [1, 2], can be fruitfully applied to show that, in the weak coupling limit, a certain class of functionals of a finite temperature (resp. Fock) free Bose gas (the so-called 'collective Weyl operators') converges in law (in the sense of [1]) to a finite temperature (resp. Fock) quantum Brownian motion [3, 4].

Moreover, if this gas interacts with a system via a coupling of the type usually considered in the mathematical models of laser theory, then the corresponding interaction cocycle converges weakly in law to the solution of a quantum stochastic differential equation which is explicitly found.

This proves, among other things, a conjecture due to Spohn [12]. In the classical case this problem has been studied by several authors among which Kesten, Papanicolau, Varadhan, ... [9, 10, 12]. In the quantum case, even for the simplest interactions one was only able, up to now, to prove convergence of the reduced dynamics (cf. [5, 11] and reference therein), while for the full process only partial results were available [5, 8] and, for the limit quantum stochastic differential equation, no result at all.

The main new idea of the proof consists in the introduction of certain 'collective observables' and 'collective states' and in the study of the convergence of the matrix elements of the functionals of the process to the matrix elements of the corresponding functionals of the quantum Brownian motion.

For those choices of the parameters which reduce the quantum Brownian motion to the classical one, this notion of convergence coincides with the usual convergence in the law for some classes of functionals, but not for all.

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Convolution semigroups in quantum probability and quantum stochastic calculus

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In quantum theories certain operator-valued measures called instruments are used. They allow one to extract probabilities from the theory and to describe the state change of the system under measurement. Analogously to classical probability theory, it is possible to introduce convolution semigroups of instruments, which are the starting point for constructing certain quantum analogues of Markov processes with independent increments. These objects have applications, for instance, in the study of photon statistics (quantum counting processes).

Here the problem is considered of constructing convolution semigroups of instruments whose value space is some locally compact group, possibly non-Abelian. The technique used is the quantum stochastic calculus of Hudson and Parthasarathy and the theory of Fourier transforms of bounded measures on groups. First, it is shown how semigroups of positive-definite functions on the coefficient algebra of the group can be dilated to hemigroups of unitary operators. Then, by combining these dilations with stochastic unitary dynamics, certain objects are constructed, which are the candidates to be the Fourier transforms of convolution semigroups of instruments. By some quantum analogue of Bochner theorem, it is shown that indeed there is a one to one correspondence between convolution semigroups of instruments and their Fourier transforms (for a certain class of groups). A quantum analogue of the Lévy-Khintchine formula is obtained.

Canonical state extensions for von Neumann algebras and noncommutative stochastic processes

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In [1] the classical notion of conditional expectations was generalized to a von Neumann algebra M, on which a normal faithful state (n.f.s.) w is defined, with a von Neumann subalgebra N, by canonically contructing the w-conditional expectations $\varepsilon_w : M \to N$. Given an n.f.s. w' on N, in general there is no extension w' of w'_1 to an n.f.s. on M such that $\varepsilon_w = \varepsilon_{w'}$; there is however [4] a partial isometry u in M such that $\varepsilon_{w'}(a) = \varepsilon_w(u^+ a u)$ for all a in M. Thus [2], it is possible to give an abstract characterization of w conditional expectations as dual maps of canonical state extensions preserving the Connes' Radon-Nikodym derivatives from N to M, corrected with phase partial isometries to make them linear. As an application a von Neumann algebra M generated by two mutually commuting von Neumann subalgebras A_1 and A_2 is considered, in order to investigate their stochastic dependence, and processes satisfying a Markov property are constructed [3].

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Poisson point fields and coherent states of quantum particle systems

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In [4] Titulaer and Glauber deal with coherent states of photon systems. They discuss a characterization of these states by the property of local independence. In [3] a class of locally normal states of boson systems was introduced which can be considered as a generalization of normal coherent states. One can also prove that these states are characterized by local independence. More generally, quantum particle systems of this type can be decomposed into independent subsystems using some ideas of independent thinnings of point processes. On the other hand the position distribution (cf. [2]) of a coherent state is a Poisson process. For that reason independent decompositions of coherent states are closely related to independent decompositions and thinnings of Poisson processes (cf. [1]).

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Some applications of the quantum Poisson process

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The quantum (or noncommutative) Poisson process has been introduced independently by Kümmerer (1986) and by Frigerio and Maassen (1987), in two seemingly different fashions. In the present work we prove the equivalence of the two constructions, by elaborating on the representation of the classical Poisson process on Fock space given by Hudson and Parthasarathy (1984).

Taking advantage of this equivalence, we apply the quantum Poisson process to the following two problems:

(1) Approximation of the solution of a stochastic differential equation in Hilbert space of the form

$$\mathrm{d}u(t) = Ku(t)\,\mathrm{d}t + \sum_{j=1}^{n} L_j u(t)\,\mathrm{d}w_j(t),$$

under suitable conditions on the operators K and L_j.

(2) Construction of a unitary dilation of the nonlinear quantum dynamical semigroup associated with a quantum Boltzmann-like equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = -\mathrm{i}[H, \rho(t)] + \mathrm{tr}_1[S\rho(t)\otimes\rho(t)S^*] - \mathrm{Tr}[\rho(t)]\rho(t),$$

where S represents the (unitary) two-particle scattering operator.

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Time-ordered exponentials in quantum stochastic calculus
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In quantum stochastic calculus an important role is played by the solutions $U = U(t)$ of linear quantum stochastic differential equations

$$\mathrm{d}U = (L_0\,\mathrm{d}\Lambda + L_1\,\mathrm{d}A + L_2\,\mathrm{d}A^+ + L_3\,\mathrm{d}t)U, \quad t \geqslant 0, \qquad U(0) = I, \tag{1}$$

where $L_\alpha = L_\alpha(t)$ are operator-valued functions in initial Hilbert space $\mathscr{H}_0$; $\mathrm{d}\Lambda$, $\mathrm{d}A$ and $\mathrm{d}A^+$ are basic stochastic differentials in the Fock space $\Gamma = \Gamma(L^2(\mathbb{R}_+))$ (Hudson and Parthasarathy, 1984). We define and study the time-ordered exponentials

$$U(t) = \overleftarrow{\exp} \int_0^t (M_0\,\mathrm{d}\Lambda + M_1\,\mathrm{d}A + M_2\,\mathrm{d}A^+ + M_3\,\mathrm{d}\tau), \tag{2}$$

where $M_\alpha = M_\alpha(\tau)$ are functions with values in $\mathscr{L}(\mathscr{H}_0)$. Under some natural conditions on $M_\alpha(t)$ we show that the time-ordered exponential (2) is correctly defined and satisfies the equation (1) where L_α are related to M_α by the formulas

$$L_0 = \mathrm{e}^{M_0} - I, \qquad L_1 = M_1 \cdot \frac{\mathrm{e}^{M_0} - I}{M_0},$$

$$L_2 = \frac{\mathrm{e}^{M_0} - I}{M_0} \cdot M_2, \qquad L_3 = M_3 + M_1 \cdot \frac{\mathrm{e}^{M_0} - I - M_0}{M_0^2} \cdot M_2.$$

Applications to classical multiplicative stochastic integrals and to probabilistic representations of the solutions of the Cauchy problems are given.

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Quantum diffusions and non-Abelian cohomology
R.L. Hudson, University of Nottingham, UK

A quantum diffusion [3, 4] is a noncommutative analogue of a classical (Itô) diffusion in which the rôle of Itô calculus is played by the quantum stochastic calculus of [5].

Using the existence theorems of [1, 5] for quantum diffusions with bounded structure maps, and for unitary operator-valued processes driven by bounded operators in the initial space respectively, it is shown that such quantum diffusions are characterised by 1-cocycles, and those driven by unitary processes by 1-coboundaries, in the non-Abelian cohomology of [6], which is a generalisation of the Hochschild cohomology of associative algebras. A further existence theorem [2], for unitary processes driven by operators which themselves evolve under a quantum diffusion enables quantum diffusions corresponding in this way to equivalent 1-cocycles to be obtained from each other by a natural perturbation procedure.

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Reciprocal diffusion and quantum mechanics

Arthur J. Krener, University of California, Davis, CA, USA

In 1931 Schrödinger proposed a probabilistic approach to the wave equation based on forward and backward descriptions of Markov diffusion processes. The probabilistic implications of this were formalized in the concept of a reciprocal process by Bernstein in 1932. Recently we have introduced the concepts of a reciprocal diffusion and a stochastic differential equation of second order. In this talk we shall discuss the close connection of the latter concepts with the Schrödinger wave equation.

Stochastic geometry and Klein's projection procedure

Sisir Roy, Indian Statistical Institute, Calcutta, India

The idea of averaging over the tangent and higher order vectors must be regarded as a principle of general significance underlying the stochastic theory of space-time. In its rigorous sense, the principle of gauge-covariant averaging should read as follows. Given any object with the indices of pure external type, for example, the base metric tensor $g_{ij}(x, z, u, \ldots)$, the base curvature tensor $L^k_{nij}(x, z, u, \ldots)$, a mixed tensor $W^j_i(x, z, u, \ldots)$, etc. (in homogeneous approach for definiteness), then

$$a_{ij}(x) = \langle g_{ij}\rangle \overset{\text{def}}{=} \int g_{ij} G \, \mathrm{d}^{N-1}z \, \mathrm{d}^{N-1}u \cdots \Big/ \int G \, \mathrm{d}^{N-1}z \, \mathrm{d}^{N-1}u \cdots ,$$

where Jacobian G plays the role of the distribution function with respect to the 'internal' $z, u, \ldots$. We can treat the averaged entities as observable in a conventional sense and consider the differences, including

$$e_{ij} \overset{\text{def}}{=} g_{ij} - \langle g_{ij}\rangle$$

as small fluctuations for the purpose of making estimations. Within this framework fluctuation of the space-time metric we have used Klein's projection procedure to derive the relativistic Schrödinger equation with random potential.

Noncommutative central limits and applications in statistical mechanics

André Verbeure, Katholieke Universiteit Leuven, Belgium

Noncommutative central limit theorems are proved. The CCR-C*-algebra of fluctuations is analyzed in detail. The stability of the central limit is studied by means of the notion of relative entropy. This is applied to statistical mechanics yielding a stability property for equilibrium states against macroscopic fluctuations. Also a study is made of the small oscillations around equilibrium.

The behaviour of an atom in the radiation field

Wilhelm von Waldenfels, Universität Heidelberg, FR Germany

Interpreting the radiation field as a quantum heat bath, light emission and absorption may be considered as quantum stochastic effects. In concrete cases the problem of renormalization arises. Welton's treatment of the Lamb-shift yields a procedure for renormalizing the quantum stochastic process.

2.9. Random fields

Uniqueness of Gibbs measures and absorption probabilities for a simple Markov chain

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Using comparison with a simple Markov chain we study Gibbs measures on a countable lattice S and generalize a well known uniqueness criterion. Suppose $\emptyset = \Lambda_0 \subset \Lambda_1 \subset \cdots \uparrow S$. The energy difference $\Delta H(\sigma, \sigma')$ between states σ and σ' that coincide outside Λ_n can be written in the form

$$\Delta H(\sigma, \sigma') = \phi_{\Lambda_n}(\sigma) - \phi_{\Lambda_n}(\sigma').$$

Let $\phi_n := \phi_{\Lambda_n} - \phi_{\Lambda_{n-1}}$. Let us write $\sigma = (\sigma_1, \sigma_2, \ldots)$ where $\sigma_n = \sigma_{\Lambda_n \setminus \Lambda_{n-1}}$. Our main result uses a smoothness condition on ϕ_n. Define

$$r_k := \sup_{n \geq 1} \operatorname{var}_k \phi_n,$$

where

$$\operatorname{var}_k(\phi_n) = \sup_{\sigma_j = \sigma_j'; j > n+k} \phi_n(\sigma) - \phi_n(\sigma').$$

For $S = Z$ it is well known (see Ruelle, 1969) that $\sum r_k < \infty$ implies uniqueness of Gibbs measures. We strengthen this to a temperature sensitive result.

Theorem. *If*

$$\sum_{n \geq 1} \exp(-r_1 - \cdots - r_n) = \infty$$

then there is exactly one Gibbs measure.

The result is used to improve known results by, e.g., Dyson on uniqueness of Gibbs measures for models close to the $(1/r^2)$-model in one dimension. The proof of the theorem uses a Markov chain on the nonnegative integers. If this Markov chain is absorbed at 0 almost surely then the Gibbs measure is unique. This technique is possibly not known.

Random space fields

E. Nelson, Princeton University, NJ, USA

A *random space field* is a stochastic process indexed by test functions supported by space-like hyperplanes in Minkowski spacetime. A random space field does not necessarily give rise to a random field indexed by test functions on space time. It is argued that such fields are natural both mathematically and physically, and that they are the appropriate way of extending the ideas of stochastic mechanics to the realm of relativistic phenomena. In particular, the free field of Guerra and Ruggiero is reexamined from this point of view.

On the maximum of random fields arising in problems of vibrations

Enzo Orsingher, University of Salerno, Italy

Let an ideal, perfectly flexible membrane be given. If it is assumed that at time $t = 0$ it receives a diffused, white noise disturbance, then at time t its displacement over or below the zero level can be represented by the random field

$$Z(P, t) = \int_{C_P} G(P, t; P') \, \mathrm{d} W(P'), \tag{1}$$

where $G(P, t; P')$ is an appropriate response function and C_P is the circle where impulses acting at P at time t, originate. We construct upper bounds for the probability that $\max_{P\in A} Z(P, t)$ overpasses level β within a set A. This bound is interesting when breaking thresholds of vibrating membranes are studied. There are two major sources of difficulties in working out the desired bound, namely the fact that stochastic integrals over circles are involved and the presence of the response function G. To circumvent the last one we approximated G as follows

$$G(P, t; P') = \begin{cases} a, & \text{if } \operatorname{dist}(P', P) \leq R/2, \\ b, & \text{otherwise.} \end{cases}$$

Let now

$$U(P, t) = \int_{C_P} \mathrm{d}W(P').$$

We have been able to prove that

$$\frac{EU^2(P, t)\ \operatorname{Cov}(Z(P, t)Z(Q, t))}{EZ^2(P, t)\ \operatorname{Cov}(U(P, t)U(Q, t))}$$

is an increasing function of $\operatorname{dist}(P, Q)$ (as far as $\operatorname{dist}(P, Q) \leq R$) and, thus, in view of Slepian's lemma we can write

$$\operatorname{Prob}\left\{\max_{P\in A} \frac{Z(P, t)}{\sqrt{EZ^2(P, t)}} \geq \gamma\right\} \leq \operatorname{Prob}\left\{\max_{P\in A} \frac{U(P, t)}{\sqrt{EU^2(P, t)}} \geq \gamma\right\}. \tag{2}$$

The second difficulty mentioned (namely that stemming from integration over circles) is overcome by the following result. Let $W(P, t) = \int_{S_P} \mathrm{d}W(P')$ where S_P is the square inscribed in the circle C_P.

We have proved the following inequality

$$\operatorname{Prob}\left\{\max_{P\in A} \frac{U(P, t)}{\sqrt{EU^2(P, t)}} \geq \gamma\right\} < \operatorname{Prob}\left\{\max_{P\in A} \frac{W(P, t)}{\sqrt{EW^2(P, t)}} \geq \gamma\right\}. \tag{3}$$

Now bounding the right member of (3) corresponds to constructing upper bounds for the maximum of fields with pyramidal covariance (sometimes referred to as Slepian fields), a problem treated differently (and with different bounds) in some previous papers.

Parameter estimation for two-dimensional Ising fields corrupted by noise

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The Asymptotic properties of the maximum likelihood estimator of the temperature T in a 2-d ferromagnetic Ising model X without external field, which is exactly observed on a finite box $\Lambda_n \uparrow Z^2$, have been investigated in Pickard (1987).

Now assume that each spin is flipped with unknown probability ε, independently for each site and independently of X, and only the resulting field Y is observed. In this case iterative algorithms, such as EM, Newton, etc., have to be used in order to compute a local maximum of the likelihood function in (T, ε). However, two problems arise:

(i) at each step statistics of the field X, conditional on Y, using current estimates of the parameters, must be computed by means of a time consuming stochastic relaxation scheme, e.g. the Gibbs sampler (Geman and Geman, 1984);

(ii) even consistency is not guaranteed without any assumption on the initialization, due to the possible persistence of non-global but local maxima.

In this paper a possible answer to the above problems is attempted. Namely, estimators for T, ε and the weight λ of the pure positive phase (in the presence of phase transitions) are proposed. Their computation does not require any iterative evaluation of conditional statistics, thus solving (i). By using exact asymptotic results on the 2-d Ising model (Baxter, 1982), their consistency is proved as $\Lambda_n \uparrow Z^2$. As far as (ii) is concerned, these estimators appear to be favourite initial points for iterative estimation procedures, leading in some cases to asymptotic efficiency.

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Convex orderings for stochastic processes

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We consider some orderings for stochastic processes in discrete and continuous time. Processes are viewed as random variables with values in spaces of sequences or functions; convex (concave) and increasing convex (increasing concave) orders are considered for the laws of these random variables. Analogous orders are considered for all the possible finite dimensional marginals. We prove equivalence theorems between the two cases. We also tackle similar problems for random fields.

Our results may be compared with those in Kamae, Krengel and O'Brien (Ann. Probab. 5 (1977) 899-912), who proved the equivalence in the case of stochastic order.

Generalized Feynman-Kac functionals and Schrödinger equations with singular potentials

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The semigroup $(P_t^q)_{t>0}$ for the Schrödinger equation $(-\frac{1}{2}\Delta+q)u=0$ on $\mathbb{R}^d$ is given by the well known Feynman-Kac formula

$$P_t^q u(x)=E^x\left[u(X_t)\cdot\exp\left(-\int_0^t q(X_s)\,\mathrm{d}s\right)\right]. \tag{1}$$

If we consider a generalized Schrödinger equation

$$(-\tfrac{1}{2}\Delta+\mu-\nu)u=0 \tag{2}$$

with suitable measures μ and ν we obtain a semigroup of the form

$$P_t^{M-N}u(x)=E^x[u(X_t)\cdot\exp(-M_t+N_t)] \tag{3}$$

with right continuous, increasing additive functionals M and N, associated with μ and ν. We assume that $\sup_x E^x[\exp N_t]<\infty$. Then $(P_t^{M-N})_{t>0}$ turns out to be a semigroup of bounded operators from L^p to $L^{p'}$ for every $p\leqslant p'$. (Hence every L^p-eigenfunction for (2) is L^∞.) The Schrödinger equation (2) has compact L^∞-resolvent, i.e.

$$U^{M-N+\alpha}:=\int_0^\infty \exp(-\alpha t)\cdot P_t^{M-N}\,\mathrm{d}t \tag{4}$$

is a compact operator on L^∞ for α sufficiently large, if and only if U^M is compact on L^∞. This is shown to be equivalent to $\lim_{x\to\infty}U^M 1(x)=0$.

Examples. (a) Let $M_t=\int_0^t q^+(X_s)\,\mathrm{d}s$ and $\lim_{x\to\infty}q^+(x)=\infty$. Then U^M is compact on L^∞.

(b) Let $M_t=\infty\cdot 1_{[\tau(D),\infty[}(t)$ with some open set $D\subset\mathbb{R}^d$. Then U^M is compact on L^∞ if and only if there exists a $C>0$ such that for all $r>0$ there is a compact set K with

$$\operatorname{cap}(B_r(x)\backslash D;\, B_{2r}(x))\geqslant C\cdot r^{d-2}\quad\text{for all } x\notin K. \tag{5}$$

Assuming that (2) has compact L^∞-resolvent we discuss several assertions equivalent to $\inf\operatorname{spec}(-\frac{1}{2}\Delta+\mu-\nu)>0$. For example: existence of nonnegative supersolutions, validity of the minimum principle, finiteness of the gauge or conditional gauge function, ...

2.10. Statistics for random processes

A note on a lower bound to the negative correlation of stationary processes on a two dimensional lattice

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It is well known the asymmetry existing between positive and negative correlation in a stochastic process. While it is always possible to have any value less than 1 for the positive correlation, it is only possible to have very small (on the average) negative correlation. In particular we have the constraint that $\bar{\rho} \geq -1/(n-1)^{-1}$ where $\bar{\rho}$ is the average correlation and n the number of random variables of which the process is constituted. This result refers to a process of dependent random variables of any kind. In this paper we derive analogous constraint for the case of stationary spatial processes. Let us first consider the case of a stationary spatial process $\{X_i\}$, $i=1,\ldots,n$, recorded on a regular grid of n cells and such that $E(X_i)=0$, $E(X_i^2)=1$ and $E(X_iX_j)=\rho$ if $j\in N(i)$ or 0 otherwise, with $N(i)$ the set of neighbours of site i (Besag, 1974). Let us now consider the sum of the n random variables X and consider its variance $E(X^2)=\sum E(X_i^2)+\sum_{j\in N(i)} E(X_iX_j)$.

The second summation in the right hand side extends to all pair of random variables for which $\rho=0$. This equals the total connectedness of the graph associated with the lattice system, that is $A=\sum A_i$, where A_i is the cardinality of $N(i)$.

From the stationarity assumption we have that $E(X^2)=n+A\rho$. As a consequence from the non-negativity of the variance we derive the constraint that $\rho \geq -n/A$. Therefore in the spatial case a crucial role in determining the bound is played by the configuration of the cells in the lattice system. For instance, suppose that we are analysing a finite regular lattice of quadrat cells. Furthermore suppose that we define the neighbours in accordance with the rook's move. In this case neglecting *end-effect* each cell has four neighbours; it follows then that $\rho \geq -0.25$.

Let us now turn to the case of a stationary lattice process $\{X_i\}$ with $E(X_i)=0$, $E(X_i^2)=1$ and a continuous isotropic spatial covariance function obeying the simple negative power law $\rho(s)=-s^{-v}$, in which $s \geq 0$ is the distance between two points. The variance of a section of plane enclosed within a circle of radius 1 in such a process is (Whittle, 1956) $\sigma^2(1)=-\int_0^1 K(s)s^{-v}\,\mathrm{d}s$ where $K(s)$ is a function describing the distribution of distances between pairs of random variables. Let us further consider the particular hypothesis that $K(s)$ is proportional to the surface area of a circle of radius s. We have $\sigma^2(1)=-\int_0^1 a\pi s^2 s^{-v}\,\mathrm{d}s$ where a is a positive constant. It follows that $\sigma^2(1)=-a(3-v)^{-1}$.

From the non-negativity of the variance, we eventually obtain $v \geq 3$. This result implies that a negative correlation function obeying a power-law is forced to fall off very quickly. Notice that the threshold value depends on the particular hypothesis we made for the function $K(s)$. As a consequence in the case of a continuous correlation function the configuration of the lattice imposes again the constraint that the negative correlation at each distance cannot be indefinitely large.

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A confidence interval for Monte Carlo methods with an application to simulation of obliquely reflecting Brownian motion

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This paper deals with the estimate of errors introduced by finite sampling in Monte Carlo evaluation of functionals of stochastic processes. To this end we introduce a metric d over the space of probability measures which induces a topology finer than the weak topology. For any two measures μ, ν, this metric allows us to bound $|\langle\mu,f\rangle-\langle\nu,f\rangle|$, uniformly over a large class of C^1-functions f, by a quantity which can be computed by a finite number of calculations. In the case $\nu=\mu_n$, the empirical distribution of order n of μ, we can compute the minimum sample size that will ensure that this quantity will be smaller than any given ε, at any chosen confidence level. As an application we control the rate of convergence of an approximating scheme for obliquely reflecting Brownian motion on a half-plane by a Monte Carlo evaluation of two significant functionals on the path space.

Weak convergence of two sided stochastic integrals, with an application to models for left truncated survival data

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When using a Nelson–Aalen type estimator for estimating the hazard of an unknown survival distribution on $[0,\infty)$ from a sample of left truncated survival data, the estimator is well behaved away from 0, but not always close to 0. However, it is still possible to show asymptotic results for the estimator, valid everywhere on $(0,\infty)$, assuming only that the truncation distribution has mass arbitrarily close to 0. This is achieved by considering the estimator, which is a stochastic integral, as an additive function of two variables—the integral from s to t, $0<s\leqslant t$ arbitrary—and introducing a Skorohod type topology on the space of such additive functions.

On the χ-divergences in a filtered space

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For two probability measures P and P' define the χ-divergence of order α by

$$\chi^\alpha(P,P')=E_R|(\mathrm{d}P/\mathrm{d}R)^{1/\alpha}-(\mathrm{d}P'/\mathrm{d}R)^{1/\alpha}|^\alpha$$

(here R is a probability measure dominating P and P' and $\alpha\geqslant 2$).

If the probability measures P and P' are defined in a filtered space then the behaviour of the χ-divergence can be analysed in terms of the density processes $\mathrm{d}P/\mathrm{d}Q$ and $\mathrm{d}P'/\mathrm{d}Q$, where $Q=(P+P')/2$.

First we give bounds for the χ-divergence of order α in predictable terms. Then we give some applications of these bounds to the asymptotical theory of estimation (I.A. Ibragimov and R.Z. Has'minskii, Statistical Estimation: Asymptotic Theory (Springer, New York, 1981)).

Limiting behavior of U-statistics, V-statistics and one sample rank order statistics for non-stationary absolutely regular processes

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The weak convergence of U- and V-statistics were established by Yoshihara (1976) for stationary absolutely regular processes. We define the notion of U- and V-statistics for non-stationary processes and we prove the weak convergence of these statistics under conditions of absolute regularity. Subsequently we deduce the weak convergence of one sample rank order statistics under similar conditions.

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Interpolation by splines and kriging: a case study

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Scientists are frequently faced with problems which require estimating spatial variation of natural processes from a limiting observation network.

The study of spatial structures is required to derive good estimates of the variable from point measurements and for the development of geologic maps. Many interpolation techniques are based on fitting some curve between the data points and choosing the parameters of the curve to give continuity of a certain number of derivatives at each data point. These are usually called *spline methods* (Ahlberg et al., 1967) which have been widely used to reconstruct the spatial phenomenon. These techniques, however, do not need any structural information of the variable under study.

In recent years, the application of statistical methods for the study of spatially distributed phenomena has been motivated by the work of Matheron (1971) (kriging). The theory of *regionalized variables* and its application to the field known as *geostatistics* have received much attention in geology, mining and hydrology.

Before estimation can be applied, the structure of the field is identified. In the framework of *minimum variance unbiased estimation*, structure selection is equivalent to the estimation of the first two moments of the field of interest. The functional forms of the mean and the covariance function is selected and their parameters are statistically estimated from available data. In the geostatistical approach, the model is described by a *drift* (or *trend*) and a *variogram* (Journal and Huijbregts, 1978).

In this paper splines and kriging as interpolators are compared: spline interpolation is equivalent to kriging with fixed covariance and degree of polynomial trend.

No preliminary structural analysis is performed with splines; this should result in a loss of accuracy of splines compared to kriging. Both methods are compared by a practical example.

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Distribution functions of means of a Dirichlet process

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Let P be a random probability measure chosen from a Dirichlet process with parameter w on $(\mathbb{R}, \mathscr{B})$, where: $\mathbb{R}$ is the real line, $\mathscr{B}$ is the class of Borel subsets of $\mathbb{R}$, w is a non-null finite measure on $(\mathbb{R}, \mathscr{B})$.

By the use of generalized Stieltjes transform, an explicit expression for the distribution function of the random variable $Y = \int_{\mathbb{R}} xP(\mathrm{d}x)$ is given. As a consequence, it is shown that the distribution of Y is $Q(\cdot) = w(\cdot)/w(\mathbb{R})$ if and only if Q is a Cauchy distribution. A few asymptotical results are described in order to obtain Bayesian confidence intervals for Y.

These statements, which derive from previous papers of Cifarelli and Regazzini, complete some of the main results included in Hannum et al. (Ann. Probab. 9 (1981) 665–670) and Yamato (Ann. Probab. 12 (1984) 262–267).

On the joint distribution of some functionals of Brownian motion with quadratic drift

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Consider the Gaussian process $X = (X(t): 0 \leq t \leq T)$ defined by $X(t) = W(t) - t^2$, $0 \leq t \leq T$, where $W = (W(t): 0 \leq t \leq T)$ is a standard Brownian motion. We call X a Brownian motion with quadratic drift. We derive an explicit (albeit complicated) formula for the joint probability density function of (i) the location $\Theta(X) \in [0, T]$ of the maximum of X, (ii) the maximum value $Y(X) = \max(X(t): 0 \leq t \leq T)$ of X, and (iii) the terminal value $X(T)$. The derivation proceeds via a simple application of Girsanov's Theorem (change of measure) and some deep results of D.A. Darling (Ann. Probab. 11 (1983)). Distibutional results for $\Theta(X)$ are of interest in the asymptotic analysis of several statistical estimation problems, such as estimating the mode of a density (H. Chernoff, Ann. Inst. Statist. Math. 16 (1964)) and density estimation in the presence of a priori information such as unimodality (B.L.S. Prakasa Rao, Sankhyā Ser. A 31 (1969)).

2.11. Stochastic differential equations

Diffusion approximation for a class of physically reflecting transport processes

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Consider the pair of a velocity process and a position process, in a convex piecewise $\mathscr{L}_b^1$ domain in $\mathbb{R}^d$. In the interior of the domain the dynamics is determined by a potential and by random changes of the velocity—occurring at exponentially distributed times, according to a probability distribution which may depend on the current position and velocity, but such that the expected value is linear in the current velocity. On the boundary the process reflects physically (angle of reflection = angle of incidence).

A diffusion approximation for the position process, as the speed and the jump rate of the velocity diverge, is obtained. By observing that the process satisfies a Skorohod reflection equation and by means of a recent result by the author on the continuity properties of the Skorohod reflection problem, it is proved that the sequence of the position processes is tight. In the limit the dependence on the velocity averages out, and all accumulation points satisfy the same stochastic differential equation with (in general *oblique*) reflection on the boundary.

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On the Markov property of solutions of stochastic partial differential equations

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Let $D \subset \mathbb{R}^d$ $(d \geq 1)$ be a bounded domain with smooth boundary ∂D. We consider the following initial-boundary value problem:

$$\partial V/\partial t = \Delta V + \dot{M} \quad \text{on } D \times]0, \infty[, \qquad V = 0 \quad \text{on } \partial(D \times]0, \infty[),$$

where Δ is the Laplace-operator on $\mathbb{R}^d$ and $(M_t;\, t \geq 0)$ is a worthy martingale measure (Walsh, 1986). The solution of the corresponding integral equation is given by

$$V_t(\phi) = \int_{D \times [0,t]} G(\phi, t; y, s)\, \mathrm{d}M_{ys}, \quad \phi \in C_0^\infty(\bar{D}),$$

where $G(\phi, t; y, s) = \int \phi(x) G(x, t; y, s)\, \mathrm{d}x$ and $G(x, t; y, s)$ is the Green's function of the associated homogeneous problem (Walsh, 1986). Let $A \subset D \times]0, \infty[$ be a bounded domain with smooth boundary ∂A and suppose that M has the germfield Markov property with respect to A. Then V has the germfield Markov property with respect to A. We show in a preprint (Dozzi and Walsh, 1987) that the minimal splitting field can be represented by means of the trace of V on ∂A and the generalized normal derivative on V on ∂A.

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Viscosity solutions (VS) for Hamilton–Jacobi–Bellman (HJB) equations for controlled diffusion processes on finite-dimensional Riemannian manifolds with boundary

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The notion of VS is extended to manifolds for controlled diffusions

$$\mathrm{d}x_t = X(x_t, a_t)\,\mathrm{d}B_t + A(x_t, a_t)\,\mathrm{d}t$$

having generator $\bar{A}(p, u)$ and classical integral cost functional with function $f(p, u)$ as integrand. The usual HJB equation holds for the Optimal Control Problem, with $\bar{A}$ as operator and zero boundary data. Exploiting the geodesic ball around each point p of M and its connectedness by geodesics, just as in P.L. Lions' $\mathbb{R}^n$-case, the analogous classes of functions D^-g and D^+g are defined for g in $C^0(M)$. On the grounds of the Converse Poincaré's Lemma applied in geodesic balls B_p, one proves that each element (v, E) of such classes can be written as the P.D.'s $v = \mathrm{D}h$, $E = \mathrm{D}^2h$ of some infinitely differentiable function h given in B_p. The resulting notion of VS relying on classes '$\mathrm{D}g$' is analogous to P.L. Lions', when the HJB equation is the aforementioned one. Next, relying on the classical definition of value function V as the infimum of all the integral costs as the control action 'a' varies among the permissible ones (which lead x_t up to M's boundary), the fact that **if** V is $C^0(M)$ **then** V **is** a VS to this HJB equation is proven. It is therefore quite natural to seek sufficient conditions for V to be a VS. So, under the following hypotheses upon coefficients of X and A:

$$\max_{i,k=1,\dots,n}\ \sup_{M\times U}\ X^{ik}(p, u) > c, \quad \max_{i,j,k}\ \sup_{M\times U}\ \mathrm{D}_{x_i}X^{jk}(p, u) > c, \quad \max_{i=1,\dots,n}\ \sup_{M\times U}\ A^i(p, u) > a,$$

and $a > 0$, $c > 1$, stability of the sample functions w.r.t. perturbations of the initial state is achieved, essentially by the supermartingale inequality. Finally, on the grounds of this latter result and of f's being Lipschitz on M uniformly w.r.t. u in U (the control parameters set), one proves V's being $C^0(M)$ and therefore its being a VS to that HJB equation. Obviously, the next step should be the proof of the uniqueness of the VS. Besides, some results might be obtained in point of possibilities to 'transport' the HJB equation and the controlled diffusion on manifolds obtained from M by differentiable homotopies.

System of interacting particles and nonlinear diffusion reflecting in a domain with sticky boundary

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Our aim is to study a system of interacting particles reflecting in a domain θ with sticky boundary, with in particular an interaction in the 'sojourn' term, and the corresponding 'nonlinear' McKean–Vlasov process, which can be seen as its limit in statistical mechanics. This is a probabilistic model for a chromatography tube, where molecules in a gaseous phase pushed by a flow of neutral gas are absorbed and released by a liquid phase deposited on the tube interior face. What is measured is the mean time taken by a molecule to get through the tube. The chemists have experimentally established the nonlinearity of the phenomenon.

The construction of such an interacting system in itself is not a classical problem. The global process evolves in the angular domain θ^N, spends time in its corners, and obeys an unusual sojourn condition. Moreover we do not have strong trajectorial results. We construct our system of particles by a limiting procedure. Then we show existence and uniqueness for the nonlinear martingale problem, by a contraction argument on time-change. Finally we show propagation of chaos towards the nonlinear diffusion.

L denotes a diffusion operator and n the interior normal vector to θ. A law P on $\Omega = C(\mathbb{R}^+, \bar{\theta})$ solves the nonlinear martingale problem if there exists a continuous increasing adapted integrable process K, increasing only when $X_s \in \partial\theta$, $K_0 = 0$, such that for all $f \in C_b^2(\bar{\theta})$,

$$M_t^f = f(X_t) - f(X_0) - \int_0^t Lf(X_s, P_s)(\mathrm{d}s - \rho(X_s, P_s)\,\mathrm{d}K_s) - \int_0^t \langle n, \nabla f\rangle(X_s)\,\mathrm{d}K_s$$

is a martingale, P_s being the law of X_s, with $1_{\partial\theta}(X_s)\,\mathrm{d}s \geqslant \rho(X_s, P_s)\,\mathrm{d}K_s$.

Large deviations for weak solutions of stochastic differential equations

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This study was motivated by the analysis in the small noise limit of the process obtained as a weak solutions of the stochastic PDE,

$$\mathrm{d}\varphi_t = -\tfrac{1}{2}((-\Delta+1)^\rho \varphi_t + \lambda(-\Delta+1)^{-1+\rho} : \varphi_t^3 :)\,\mathrm{d}t + \varepsilon\,\mathrm{d}w_t,$$

where $\rho < 1$, $:\varphi^3 := \varphi^3 - 3\varepsilon^2 C(0)\varphi$,

$$C(x, y) = (-\Delta+1)^{-1}(x, y),$$

$$E(w(t, x)w(t', x')) = \min(t, t')(-\Delta+1)^{-1+\rho}(x, x') \quad x, x' \in \mathbb{R}^2.$$

Due to the presence of infinite renormalization ($C(0) = \infty$) the usual large deviation techniques do not apply immediately and a new strategy has to be developed. We prove some estimates analogous to the Freidlin–Ventzel inequalities from which it follows that the field trajectories suitably smeared in space over a scale r_0 are close in probability to the projection on the same scale of a field obeying a regularized equation. As a byproduct of our approach we obtain a general scheme to study large deviations for weak solutions of stochastic differential equations.

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Stochastic derivation of diffusions with reflecting barriers

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Let $h : \mathbb{R} \to]-\infty, +\infty]$ be a lower-semicontinuous, convex function, such that if $I = \{x \in \mathbb{R}, h(x) < +\infty\}$, $I \neq \emptyset$. Associated with h there is a multivalued operator ∂h defined by its graph in the following way:

$$(x, y) \in \operatorname{Gr} \partial h \quad \text{if and only if} \quad h(z) \geqslant h(x) + y(z - x)$$

for any $z \in \mathbb{R}$, whenever $(x, y) \in \mathbb{R}^2$.

Assume we are given two real Lipschitz functions σ and g, a Brownian motion W, defined on a filtered probability space $(\Omega, (F_t)_{t\in T}, P)$, and an F_0-measurable random variable η taking values in $\bar{I}$. A pair (Y, K) of real continuous, adapted processes is said to be the solution of the problem (h, σ, g, η) if the following conditions are satisfied:

(a) Y take its values in $\bar{I}$, and K is locally of bounded variation with $K_0 = 0$.

(b) $Y_t = \eta + \int_0^t [\sigma(Y_s)\,\mathrm{d}W_s + g(Y_s)\,\mathrm{d}s] - K_t$.

(c) For any pair (α_t, β_t) of optional processes belonging to $\mathrm{Gr}(\partial h)$, the measure

$$(Y_t - \alpha_t)(\mathrm{d}K_t - \beta_t\,\mathrm{d}t)$$

is positive a.s.

This kind of equation has been studied in D. Lépingle and C. Marois, Séminaire de Probabilités XXI, Lecture Notes in Math., Vol. 1247 (Springer, Berlin, 1986).

Our purpose is to prove that, if the initial condition η belongs to L^p, for some $p \geq 2$, then for any $t \geq 0$, the random variable Y_t belongs to the Sobolev space $D_{p,1}$, and an explicit expression for the Malliavin derivative DX_t can be obtained.

More precisely if $\bar{I} = [d, d']$, and

$$B = \left\{(r, \omega) \in [0, t] \times \Omega,\ d < \inf_{r \leq s \leq t} Y_s(\omega) \leq \sup_{r \leq s \leq t} Y_s(\omega) < d'\right\},$$

we show that for almost every $(r, \omega) \in [0, t] \times \Omega$, we have,

$$D_r Y_t = \sigma(Y_r) \exp\left\{\int_r^t [\sigma'(Y_s)\,\mathrm{d}W_s - \tfrac{1}{2}\sigma'(Y_s)^2\,\mathrm{d}s + g'(Y_s)\,\mathrm{d}s] - \int_{\mathbb{R}} \frac{L_t^x - L_r^x}{\sigma_2(x)}\,\mu(dx)\right\} \cdot 1_B,$$

where $\{L_t^x, t \geq 0\}$ is the local time of the process Y at x.

A fine topology criterion of vanishing mean oscillation

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Stochastic methods are used to obtain the following characterization of VMOA of the unit ball $\mathbb{B}^n$ in $\mathbb{C}^n$.

Theorem 1. *Let $f: \mathbb{B}^n \to \mathbb{C}$ be analytic such that*

$$\mathrm{Area}(f(\mathbb{B}^n)) < \infty.$$

For $\zeta \subset \partial\mathbb{B}^n$ and $\rho > 0$ define $D(\zeta, \rho) = \{z \in \mathbb{B}^n; |1 - \langle z, \zeta\rangle| < \rho\}$ and put

$$V_\zeta = \bigcap_{\rho > 0} f(D(\zeta, \rho)).$$

Assume that:

V_ζ has empty fine interior for all $\zeta \in \partial\mathbb{B}^n$.

Then $f \in$ VMOA (i.e. f has vanishing mean oscillation).

Stochastic two-point boundary value problems

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Let $\Omega = C(\mathbb{R}_+; \mathbb{R}^d)$, $\mathcal{F}$ = the Borel field on Ω, P = Wiener measure. We consider two classes of stochastic differential equations whose solution takes values in $\mathbb{R}^d$ and which both are of the following type:

$$\mathrm{d}X_t = f(X_t)\,\mathrm{d}t + \sum_{i=1}^k g_i(X_t) \circ \mathrm{d}W_t^i, \quad t \in [0, 1], \qquad h(X_0, X_1) = h_0,$$

where $f, g_1, \ldots, g_k$ map $\mathbb{R}^d$ into itself, h maps $\mathbb{R}^{2d}$ into $\mathbb{R}^d$ and $h_0 \in \mathbb{R}^d$. In the above equation, the stochastic integrals are interpreted as extended Stratonovich integrals (see D. Nualart and E. Pardoux, Stochastic calculus with anticipating integrands, to appear in: Probab. Theory Rel. Fields). We present some results concerning the existence and uniqueness of the solution to such an equation, as well as concerning its Markov property.

Two kinds of Markov properties are considered: the usual one, and the '*reciprocal property*' which is also called the Markov property in the sense of fields, which states that the values of $\{X_u\}$ inside and outside an interval $[s, t]$ (with $0 \leq s < t \leq 0$) are conditionally independent, given (X_s, X_t).

The two classes of equations which we consider are:

(a) The 'bilinear' case where $f, g_1, \ldots, g_k, h$ are assumed to be affine.

(b) The case where $k = n$, the matrix $(g_1, \ldots, g_n)$ is the identity matrix, and the pair (f, h) satisfies some hypotheses (e.g. a monotonicity type of condition).

Note that the intersection of the two above cases is the case where the solution is gaussian, which is essentially well known. Existence and uniqueness is proved in each of the above situations under appropriate conditions. Sufficient conditions are given in case (a) for the solution to be Markov, and to be reciprocal. In case (b), if $d = 1$, it is shown under appropriate smoothness conditions that the solution is reciprocal if and only if f is linear.

Case (a) is a joint work with D. Ocone from Rutgers University, and case (b) joint work with D. Nualart from the University of Barcelona.

The exterior Dirichlet problem for diffusions with small parameter drift

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Let $L_\varepsilon = L_0 + \varepsilon B \cdot \nabla$ be a diffusion generator in an exterior domain D. Assume that for each $\varepsilon > 0$, L_ε generates a recurrent diffusion and that L_0 generates a transient diffusion. Then, for each $\varepsilon > 0$ and each continuous function ψ on ∂D, it is well known that the exterior Dirichlet problem

$$L_\varepsilon u_\varepsilon = 0 \quad \text{in } D, \qquad u_\varepsilon = \psi \quad \text{on } \partial\Omega,$$

has a unique bounded solution. On the other hand, the Dirichlet problem

$$L_0 u_0 = 0 \quad \text{in } D, \qquad u_0 = \psi \quad \text{on } \partial D, \tag{1}$$

is not uniquely determined. Rather, a unique solution exists for each specification of data on the Martin boundary at infinity. Now it appears plausible that, as $\varepsilon \to 0$, $u_0 \equiv \lim_{\varepsilon \to 0} u_\varepsilon$ will solve (1). Under appropriate conditions on L_0, we will show that this is indeed true and we will identify the boundary condition at infinity that u satisfies.

Strong Markov continuous semimartingales and stochastic differential equations involving local time

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Let $(\Omega, F, F_t, X_t, \theta_t, P_x)$ be a continuous one-dimensional strong Markov process on an interval, say $[0, \infty)$ for simplicity. We assume that all points $x \in (0, \infty)$ are regular. The most interesting case to be treated is that the process X starting from $x = 0$ goes immediately into $(0, \infty)$ but starting from $x \in (0, \infty)$ the boundary 0 is not reachable. A typical example of such a process is the d-dimensional Bessel process ($d \geq 2$).

It is shown that every process of this kind is a space and time transformation of a 2-dimensional Bessel process.

If moreover X is a semimartingale one can compute the semimartingale decomposition as

$$X_t = X_0 + M_t(X) + \tfrac{1}{2} \int_0^\infty L^X(t, y)(\bar{g}(x))^{-1}\, \mathrm{d}g(x),$$

where $M(X)$ is a continuous local martingale, $L^X(\cdot, y)$ denotes the symmetric local time of X at y and $g \geq 0$ is a function of locally bounded variation, $\bar{g}(x) = \frac{1}{2}(g(x+) + g(x-))$. This leads to the study of a class of stochastic differential equations involving the local time of the unknown process X which generalize equations considered in [1, 2].

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Self-similar diffusions on $[0, \infty)$

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Let $(\tilde{X}(t), \tilde{P}^x)$ be an α-self-similar diffusion on $[0, \infty)$, $\alpha > 0$, such that $T_0 < \infty$ a.s. $(\tilde{P}^x)$, where T_0 is the first hitting time to 0. $(\tilde{X}(t), \tilde{P}^x)$ is on $(0, \infty)$ and generated by

$$A = \tfrac{1}{2}\delta^2 X^{2-1/\alpha} \frac{d^2}{dx^2} + \mu X^{1-1/\alpha} \frac{d}{dx}, \quad \delta^2 > 0, \ \mu \in \mathbb{R}, \ \delta^2 > 2\mu.$$

Let $(X(t), P^x)$ be $(\tilde{X}(t), \tilde{P}^x)$ killed at T_0. We consider the following problem: find all the α-self-similar, standard processes on $[0, \infty)$, which behave like $(X(t), P^x)$ on $(0, \infty)$. If 0 is a regular boundary point, then $(X(t), P^x)$ can be extended to $[0, \infty)$ such that either the extension is a completely reflecting or completely absorbing diffusion, or that immediately after reaching 0 it jumps into $(0, \infty)$ according to the "jumping in" measure $dx/X^{\beta+1}$, $0 < \beta < \min(1/\alpha, 1 - 2\mu/\delta^2)$. If 0 is an exit boundary point, then the process cannot leave 0 continuously. It may, however, jump according to the 'jumping in' measure $dx/x^{\beta+1}$, $0 < \beta < 1/\alpha$, and the resulting process is an α-self-similar, standard process on $[0, \infty)$.

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2.12. Stochastic models in science

Stochastic differential equation models of fisheries: Optimization and estimation problems

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Stochastic differential equation models of fisheries of the form $dN/dt = rf(N) + g(N) + \sigma h(N)\varepsilon(t)$, where $r > 0$ is a growth parameter, $\varepsilon(t)$ is standard white noise, $\sigma > 0$ is a noise intensity parameter, f is a density-dependence growth function, g represents the fishing yield and h is a noise intensity function, are studied. Stationary distributions, extinction, optimal fishing yield and parameter estimation problems are examined under reasonable assumptions on f, g and h.

On stochastic analysis of delta modulators for Ornstein-Uhlenbeck processes

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A real valued Ornstein-Uhlenbeck process $dx(t) = -\mu x(t)\,dt + dw(t)$, where $\mu > 0$ and the variables $w(t)$ form a standard Wiener process, is considered as the source of a delta modulator (DM). For $\delta > 0$ and $0 < c \leq 1$ the DM is defined (cf. [1, 2]) by the encoder $b_i = \text{sgn}(x(t_i) - cz_{i-1})$, with $\text{sgn}(x) = +1, 0, -1$, for $x > 0$, $x = 0$, $x < 0$, respectively, and the decoder $z_i = cz_{i-1} + \delta b_i$, with $\{t_i \mid i = 1, 2, \ldots\}$ designating a set of equidistant sampling points $(\Delta = t_i - t_{i-1})$.

The encoder variables b_i are square integrable functionals of the source process, which implies the following result (see [3]).

Proposition 1.

$$b_i = F(x_{i-1}, cz_{i-1})^{-1} + \int_{t_{i-1}}^{t_i} \frac{\partial}{\partial x} V(s, x(s); cz_{i-1}, t_i)\,\mathrm{d}w(s),$$

where

$$F(x_{i-1}, cz_{i-1}) = \operatorname{erfc}\left(\sqrt{\mu}\left(\frac{cz_{i-1} - x_{i-1}\,\mathrm{e}^{-\mu\Delta}}{\sqrt{1-\mathrm{e}^{-\mu\Delta}}}\right)\right),$$

with

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} \mathrm{e}^{-r^2}\,\mathrm{d}r$$

and

$$\frac{\partial}{\partial x} V(s, x(s), cz_{i-1}, t_i) = \frac{2\mu}{\sqrt{\pi}} \frac{\mathrm{e}^{-\mu(t_i - s)}}{\sqrt{1-\mathrm{e}^{-2\mu(t_i-s)}}} \exp(-\mu\theta(x(s), cz_{i-1}, s))$$

where $\theta(x, z, s) = (x\,\mathrm{e}^{-\mu(t_i-s)} - z)^2/(1-\mathrm{e}^{-2\mu(t_i-s)})$.

The proof of the result is based on Itô's formula. Using the fact that the probability of b_i being zero, conditioned on $x(t_{i-1})$ and z_{i-1}, is equal to zero as well as the properties of the stochastic integral in Proposition 1 above we obtain:

Proposition 2. $D(b_i|x(t_{i-1}), z_{i-1})$, *the conditional variance of* b_i *given* $x(t_{i-1})$ *and* z_{i-1}, *is equal to* $2F(x_{i-1}, cz_{i-1}) - F^2(x_{i-1}, cz_{i-1})$.

These results are obviously useful in the analysis of a number of engineering properties of the DM. In particular, the stochastic stability and the signal-to-noise ratio of the DM can be studied by these formulae, see [4, 5]. Asymptotic normal approximations of the distribution of the output variable z_i are also obtainable. Furthermore, the idea of the representation in Proposition 1 can be extended to some more general DPCM-systems.

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Random walks, harmonic functions, and uniqueness of electric currents in infinite networks

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An *infinite electric network* is an unoriented, infinite, locally finite graph $G(V, E)$ together with an assignment $e \mapsto R(e)$ of a positive *resistance* to each edge in E. We choose an (arbitrary) orientation of the edges and write e^-, e^+ for the vertices which constitute origin and endpoint of edge e. The *boundary operator* ∂ maps functions on E to functions on V:

$$\partial F(v) = \sum_{e^+ = v} F(e) - \sum_{e^- = v} F(e).$$

A *finite cycle* is a finitely supported function C on E with $\partial C \equiv 0$. Given a *voltage source* U (a finitely supported function on E) and a *current source* i (a finitely supported function on V whose values sum up to zero), an *electric current* is an assignment $e \mapsto I(e)$ of a real number to each edge, such that Kirchhoff's laws are satisfied and I has finite energy:

$$\partial I + i \equiv 0, \tag{1}$$

$$\langle U - R \cdot I, C\rangle = 0 \quad \text{for every finite cycle } C, \tag{2}$$

$$\langle R \cdot I, I\rangle < \infty. \tag{3}$$

Here, $\langle F, G\rangle$ denotes the usual inner product in $\ell^2(E)$.

Flanders (1971) has shown that such a current always exists. The question of *uniqueness* of the current with respect to (1), (2) and (3) has been considered e.g. by Thomassen (1987) for some types of networks. We present uniqueness results for two large classes of networks.

It turns out that existence and construction of currents is closely related with the stochastic transition matrix P of a random walk (time-homogeneous Markov chain) on V, which is defined by

$$p(u, v) = \begin{cases} 1/a(u)R([u, v]), & \text{if } [u, v] \in E, \\ 0, & \text{otherwise,} \end{cases}$$

where $a(u)$ is the appropriate norming constant. In fact, I is unique if and only if the only solutions h of $Ph = h$ with finite 'Dirichlet sum'

$$\sum_{e \in E} (h(e^+) - h(e^-))^2/R(e)$$

are the constants. For vertex-transitive networks with *polynomial growth*, this yields uniqueness of I. On the other hand, for vertex-transitive networks with more than two *ends* (i.e. deletion of some finite set of vertices disconnects G into at least three components), I is never unique under conditions (1), (2) and (3) (with a few possible exceptions). This result is obtained by studying the boundary behaviour at 'infinity' ($\equiv$ the Poisson boundary) of the associated random walk.

A limit model for a system of particles with interaction on the boundary

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This work describes a probabilistic model for a chromatographic tube where molecules can only interact on a sticky support. More precisely the sojourn time of each particle on the support involves the average number of particles on this boundary. Through a phenomenon of propagation of chaos, this stochastic system leads to a nonlinear stochastic differential equation with local time, when the number of particles tends to infinity.

In the one-dimensional case when the boundary is just the origin, this equation is

$$X_t = x + \int_0^t 1_{(X_s \neq 0)}\, \mathrm{d}B_s + \frac{(a^+ - a^-)}{2a^+} L_t^0, \qquad \int_0^t 1_{(X_s = 0)}\, \mathrm{d}s = \frac{1}{2a^+} \int_0^t \rho(h_s^x)\, \mathrm{d}L_s^0, \tag{1}$$

where $B.$ is a one dimensional Brownian motion, L_t^0 is the local time at O of $X.$, in Tanaka's sense, and $h_t^x = P_x(X_t = 0)$. We prove a result of weak and strong uniqueness for equation (1) provided that ρ is a strictly positive Lipschitz function.

A stochastic model of traffic flow on freeways

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Deterministic partial differential equations have been used in traffic theory to describe the evolution of the mean velocity and the density of a traffic flow along a stretch of a freeway. It has been recognized that some form of randomness should be introduced into this deterministic model.

We will discuss a stochastic model for traffic flow obtained by adding space-correlated noise to the deterministic model. The resulting stochastic model is solved elaborating on techniques introduced by e.g. D.A. Dawson.

2.13. Time series and filtering

Highest posterior density intervals for the Prony's spectral estimator of a normal stationary process

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Prony's method of spectral analysis [4] models a stationary time series as a sum of a white noise and the output of a linear system with zero input, stochastic initial conditions and complex conjugate characteristic roots with absolute value less than one. Equivalently the time series is modelled as a linear combination of damped sinusoids plus white noise. Estimates of the frequencies, damping factors, amplitudes and phases of each sinusoid provide the required spectral information.

Although the method is, often, the only one suited for computing the spectrum of short time series, it is very sensitive to noise [2]. From a practical point of view it is therefore necessary to have a measure of the reliability of the estimates produced by that method.

The asymptotic distribution of the frequency estimators is obtained in [1] when no damping is present. Moreover this distribution depends on the amplitudes. This is not satisfactory because the frequencies are characteristics of the linear system while the amplitudes depend only on initial values.

In order to overcome these problems we consider the Prony's method in a Bayesian framework. the characteristic polynomial is therefore stochastic and the distribution of its roots can be approximated, as in [3], by making use of the distribution of the system coefficients. The marginal posterior distribution, for each frequency, can be derived and the highest posterior density interval computed.

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Bilateral autoregressive models for data validation in time series

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Bilateral autoregressive models describe the current value of a time series as a linear function of its preceding and following observations.

Such models are well known but they have received little attention in applications, partly because of some difficulties in parameter estimation. Recent developments in the estimation of the inverse correlation allow such problems to be overcome.

In this paper a methodology for building bilateral models is outlined. Two main uses of the bilateral models: outlier detection and missing values recovery, are suggested and contrasted with methods based on autoregressive moving average models through an application.

Adaptive linear techniques in nonlinear filtering

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We consider a discrete-time nonlinear filtering problem described by the partially observable process $\{(x_t, y_t), t = 1, 2, \ldots, T\}$ given by

$$x_{t+1} = a_t(x_t) + \beta_{t+1}, \qquad y_t = c_t(x_t) + \gamma_{t+1},$$

where a_t and c_t are piecewise linear functions,

$$a_t(x) = \sum_{i=1}^{N} [A_t(i)x + B_t(i)]I_{\pi_i}(x), \qquad c_t(x) = \sum_{i=1}^{N} [C_t(i)x + D_t(i)]I_{\pi_i}(x),$$

with $\pi_i (i = 1, 2, \ldots, N)$ a finite partition of the state space. The initial condition x_0 as well as the disturbances β_t and γ_t are distributed according to finite mixtures of normal distributions. Furthermore the variances of the normal distributions relative to the initial condition and to the process disturbance β_t are infinitesimal with a parameter ε.

It is shown that the original nonlinear filtering model can be approximated by an adaptive linear model of the type

$$x_{t+1} = A_t(\theta)x_t + B_t(\theta) + \beta_{t+1}, \qquad y_t = C_t(\theta)x_t + D_t(\theta) + \gamma_t,$$

with θ a suitably chosen random parameter.

More precisely, it is shown that the adaptive linear problem admits a finite-dimensional, explicitly computable solution. Furthermore, as ε converges to 0, the original and the approximate models have the same conditional moments and, in particular, the same conditional conditional mean square errors.

Approximate stochastic models

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The problem we consider here occurs quite frequently in the construction of Markovian representations of a stochastic process y. There is a complete and general theory due to Lindquist, Picci and Ruckebusch [1, 3] to construct an exact stochastic realization of y, which makes use of the factorization of the spectral density $\mathcal{S}(e^{i\omega})$. Nevertheless, if the degree of the density is very high, the realization we obtain is not very useful for computational purposes. This sometimes happens because, in presence of noise in the data, the estimation procedure for the density generates some unnecessary poles. The question we try to answer in this paper is whether these redundant poles can be eliminated through some reasonable approximation procedure. The idea is to start with an exact realization (no matter how large) of the process y, and then seek an approximate submodel (in the sense explained below) within the state space of this original model. The submodel which best represents y within all submodels of a fixed degree k is our approximant. This yields an interesting representation of a Markovian space through a Markovian basis, and sets up a correspondence of these basis with the Malmquist basis in the spectral domain [2]. Then it is shown that a minimal realization can be obtained with a very simple algorithm of polynomial from a nonminimal one, and that this algorithm can be extended to give an approximate realization of fixed degree k. We consider here the case of internal finite dimensional realizations of a scalar process y.

References

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Approximate filtering and Lie algebras

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Associated to a (nonlinear) filtering problem there is a Lie algebra of differential operator generated by the operators occurring in the evolution equation for an unnormalized version of the conditional density. (The Duncan-Zakai equation.) This can be used to prove the impossibility of exact recursive finite dimensional filters in some cases such as the cubic sensor. This approach also makes it plausible that as a rule finite dimensional recursive exact filters will not exist, though a precise version of this conjecture remains to be formulated and proved.

In this talk I want to outline some approaches based on the Lie-algebraic analysis to construct approximate filters. This is more in the nature of a research program than a finished piece of research but some initial results are available.

Approximation of filters by sampling and quantization

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Approximation procedures based on sampling and quantization are considered, for the computation of the conditional probability of x_t given $\{y_s; s \leq t\}$ when the state process x and the observation process y are given by

$$x_t = x_0 + \int_0^t f(s, x_s)\,\mathrm{d}s + \int_0^t g(s, x_s)\,\mathrm{d}b_s, \qquad y_t = \int_0^t h(s, x_s)\,\mathrm{d}s + w_t,$$

where x, y, b, w are multidimensional processes and b and w are independent Brownian motions. It is supposed that x_0 is an L^2 random variable, independent of (b, w) and f, g and h satisfy the Lipschitz and linear growth conditions. The time parameter is restricted to the finite interval $[0, T]$.

The successive approximation steps are the following: sampling of y, sampling of h, Euler approximation of x, quantization of x, quantization of y. The filtering formula is given and the approximation degree estimated for each step.

For a bounded function F on the space of values of x the approximate unnormalized filter σ'_t is given by

$$\sigma'_t(F) = E^x\left\{ F(x'_t) \exp \sum_0^{t(\delta)} \left(h_k \Delta y_k - \frac{\delta}{2} \|h_k\|^2 \right) \right\}$$

where E^x is the expectation under the law of x, x' the approximation of x, δ the sampling period, $t(\delta)$ the integer part of t/δ, h_k the approximation of $h(t, x_t)$ on $[k\delta, (k+1)\delta[$, $\Delta y_k = y_{k\delta} - y_{(k-1)\delta}$ and $\|\cdot\|$ denotes the Euclidean norm.

Let π denote the normalized exact filter and π' the normalized approximate filter corresponding to σ'. Then for each approximation step, one gets

$$E|\pi_t(F) - \pi'_t(F)| \leq K\|F\|\sqrt{\delta}$$

where $\|F\|$ is the supnorm of F, supposed to have the Lipschitz property, and K is a constant.

These results, derived in [2], are similar to those given in [1, 3] under the boundedness condition of h and the homogeneity condition of f, g and h. The release of these conditions improves the degree of the approximations and widens their domain of applications.

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Finite dimensional realizations of bilinear stochastic PDE's

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We address the problem of finding finite dimensional realizations, or shortly FDR, for bilinear stochastic partial differential equations:

$$\begin{aligned}&\langle \mathrm{d}\nu_t, \phi\rangle = \langle \nu_t, M_0\phi\rangle\,\mathrm{d}t + \sum_{i=1}^{p} \langle \nu_t, M_i\phi\rangle\,\mathrm{d}y_t^i \quad \forall\phi\in C_K^\infty(\mathbb{R}^n),\\ &\nu_0 = \mu_0 \in \mathbb{M}_+(\mathbb{R}^n),\end{aligned} \tag{1}$$

where M_0 is a linear second order differential operator with non-negative second order part, $M_1, \ldots, M_p$ are linear zeroth order operators and where y_t is a standard Brownian motion.

Precisely, we say that there exists an FDR if there exists an output function σ and a diffusion η_t on a finite dimensional manifold, driven by the process y_t, such that $\nu_t = \sigma(\eta_t)$ (Marcus, 1984).

The classical finite dimensional filtering problem can be recovered from this realization problem by means of transformations such as immersions and gauge transformations.

Following the geometric approach, the estimation Lie algebra $\mathscr{E}$ generated by the operators $M_0, \ldots, M_p$ is introduced and we prove a general necessary condition on $\mathscr{E}$ for the existence of FDR and an easy criterion of non existence.

In the particular case of stochastic linear-quadratic PDE's (linear operators with coefficients at most quadratic in the state variables) we find an explicit derivation of the minimal realization. It turns out that all the known examples of finite dimensional filters belong to this class, up to immersions and gauge transformations.

Our definition of FDR depends on the initial measure and this allows to build a general class of systems for which $\mathscr{E}$ is not finite dimensional even if it is finite dimensional at every point. This is why we focus on the role of the initial measure to define different classes of FDR's, yielding a classification of estimation Lie algebras based on the fact that a realization holds for a single initial measure (Brockett's homomorphism principle), for every Dirac initial measure ($\mathscr{E}$ contains only operators of order less than two) or finally is the same for all Dirac initial measures ($\mathscr{E}$ is finite dimensional and is made of operators of order less than two). These results are used to compute formal uniform realizations which appear to be global when $\mathscr{E}$ is solvable, thanks to a generalization of the Baker-Campbell-Hausdorff formula.

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Wong-Zakai approximations for nonlinear filtering equations

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Assume that the vector-valued process (X, Y) is solution of the Itô equation

$$\mathrm{d}X_t = b(X_t)\,\mathrm{d}t + \tilde{g}(X_t)\,\mathrm{d}W_t + g(X_t)\,\mathrm{d}B_t, \qquad \mathrm{d}Y_t = h(X_t)\,\mathrm{d}t + \mathrm{d}B_t,$$

where W and B are two independent Brownian motions. Then it is well known that under some regularity and boundedness conditions, the conditional law Π_t of X_t given $(Y_s, s\leq t)$ is given by a stochastic partial differential equation (SPDE) driven by Y. Now write this equation in the Stratonovich form, replace Y_t by a family of absolutely continuous processes Y_t^ε which converge in probability as $\varepsilon\to 0$ to Y_t and consider the resulting filters Π_t^ε; the most classical example for Y_t^ε is the polygonal interpolation associated to some subdivision with step ε but more general cases can also be dealt with. In the past years, many studies have been devoted to this type of approximations for ordinary stochastic differential equations (OSDEs), and in this work, we generalize some of these results to the framework of nonlinear filtering. We check the convergence of Π_t^ε to Π_t for polygonal interpolations and we show that for more general approximations, a corrective term may appear; moreover, we estimate the rate of convergence; it turns out that for instance in the case of polygonal interpolations, the difference $\Pi_t^\varepsilon - \Pi_t$ is of order $\sqrt{\varepsilon}$; with an additional assumption concerning some vector fields (which is for instance satisfied when $g=0$ and more generally which means that there exists a version of the filter which is continuous with respect to the observation paths), an approximate filter of order ε can be deduced from Π_t^ε. Our method does not rely on an asymptotic study of SPDEs; we rather use the reference probability method (which leads to the Kallianpur–Striebel formula) in order to reduce this study to the case of OSDEs and then apply the method of (Picard, 1986); some stochastic calculus of variations is involved in order to get the rate of convergence.

Reference

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Simulation of nonlinear filters with application to time series analysis

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The object is to study systems in which the state may 'jump' suddenly. Given an observed process, $(Y(t), -\infty < t < \infty)$, the model is

$$dX = A(X)\,dt + dM, \qquad dY = g(X)\,dt + \sigma\,dW,$$

where X is a state vector, M is a martingale, and W is a standard Wiener process, independent of M. The problem can be thought of as one of observing a piecewise smooth curve, $g = g(X(t))$, with measurement error. The model might be of use in discrete time problems—known as innovation outliers problems—as a limiting case where the variance of the measurement error is proportional to the sampling frequency when the latter tends to infinity.

In the present study the simpler situation where X is a scalar and $g(X) = X$ is treated. Two special cases are considered, one where X is a 'random telegraph signal', i.e. a Markov process with two states, the other where X has independent discrete increments. In these situations exact solutions to the filter equations can be obtained numerically both in continuous and in discrete time. The resulting filters are simulated and compared with each other and with the optimal linear filter.

Processes with independent increments in discrete-time stochastic filtering

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We consider a stationary discrete-time stochastic process

$$(\{X_n\}, \{Y_n\})_{n=1,2,\ldots} \quad (X_n \in \mathscr{X} \subset \mathbb{R}^v,\ Y_n \in \mathscr{Y} \subset \mathbb{R}^w),$$

whose dynamics is described by the following properties:

- $\{X_n\}$ is a Markov chain with transition density $p(x \mid x')$;
- Y_n is conditionally independent on $X^{(n-1)} \equiv (X_1, X_2, \ldots, X_{n-1})$ and $Y^{(n-1)} \equiv (Y_1, Y_2, \ldots, Y_{n-1})$, given X_n $(n = 2, 3, \ldots)$;
- The family of 'observation densities', formed by the conditional densities

$$f(y \mid x) = f_{Y_n}(y \mid X^{(n-1)} = x^{(n-1)};\ Y^{(n-1)} = y^{(n-1)};\ X_n = x),$$

$x \in \mathscr{X}$, is given.

Let p_1 denote the probability density function of X_1, $p_n(x \mid y^{(n-1)})$ denote the conditional density of X_n given $Y^{(n-1)} = y^{(n-1)}$ and $\pi_n(x \mid y^{(n)})$ denote the conditional density of X_n given $Y^{(n)} = y^{(n)}$.

A family $P \equiv \{\xi_u(x);\ x \in \mathscr{X}\}$ of probability density functions on $\mathscr{X}$, indexed by $u \in U$, is *filter-conjugate* if there exists a measurable function $\varphi : U \times \mathscr{Y} \to U$ such that, $\forall u \in U, \forall y \in \mathscr{Y}$,

$$\left[\int_{\mathscr{X}} \xi_u(x')p(x \mid x')\,dx'\right] \cdot f(y \mid x) \propto \xi_{\varphi(u,y)}(x).$$

By 'Dynamical Bayes Formula',

$$p_1(x) = \int_{\mathscr{X}} \xi_{u_0}(x')p(x \mid x')\,dx',$$

$\xi_{u_0} \in P$, and P filter conjugate imply $\pi_n(x \mid y^{(n)}) \in P\ \forall y^{(n)} \in \mathscr{Y}^n$.

We show that, when the family of 'observation densities' is an exponential family, the theory of processes with independent increments can be of help for building 'compatible' transition densities $p(x \mid x')$ which allow for the existence of filter-conjugate families with dim $U = \dim \mathscr{X}$.

Weak convergence of sums of moving averages in the α-stable domain of attraction

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Skorohod has shown that the convergence of sums of i.i.d random variables to an α-stable Lévy motion, with $0 < \alpha < 2$, holds in the weak-J_1 sense. J_1 is the commonly used Skorohod topology. We show that for sums of moving averages with at least two non-zero coefficients, weak-J_1 convergence does not hold in general; however, if the moving average coefficients are positive, then one can have weak-M_1 convergence. M_1 is weaker than J_1, but it is strong enough for the sup and inf functionals to be continuous.

This is joint work with Florin Avram.

On sample path properties of nonlinear stochastic realizations

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The linear-Gaussian stochastic realization problem has been considered in some detail and largely solved by Lindquist and Picci and by Ruckelbusch, as well as co-workers. The corresponding nonlinear and non-Gaussian problem has been considered by in various forms by various authors, with various degrees of partial success. In particular, Van Schuppen and co-workers have considered a sigma algebraic form of the nonlinear stochastic realization problem, while Lindquist, Mitter and Picci were able to solve certain special nonlinear realization problems. Recently, Taylor and Pavon have given a solution of the full nonlinear realization problem. That is, let $y(t)$ be a purely nondeterministic process taking values in a locally compact Hausdorff space Y, with certain nice statistical and analytic properties. Taylor and Pavon show how to construct a locally compact Hausdorff state space X, an output induced right continuous strong Markov process $x(t)$ taking values in X and a continuous function $\phi : X \to Y$ such that $\phi(x(t)) = y(t)$. Some incomplete aspects of the theory developed by Taylor and Pavon have been that the topological structure of the space X and the relationship between the sample path properties of $y(t)$ and those of the realization $x(t)$ have not been well understood.

In this paper we describe the theory developed by Taylor and Pavon, and also announce new results concerning conditions under which the realization $x(t)$ has continuous sample paths. Corresponding to this new result there are also results about the topological structure of the state space X. These results follow from results of Pavon and Taylor concerning connections between the theory of nonlinear filtering and output induced nonlinear stochastic realizations.