



Cosmological constant from gauge fields on extra dimensions

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Abstract

We present a new model of dark energy which could explain the observed accelerated expansion of our Universe. We show that a five-dimensional Einstein–Yang–Mills theory defined in a flat Friedmann–Robertson–Walker universe compactified on a circle possesses degenerate vacua in four dimensions. The present Universe could be trapped in one of these degenerate vacua. With the natural requirement that the size of the extra dimension could be of the GUT scale or smaller, the energy density difference between the degenerate vacua and the true ground state can provide us with just the right amount of dark energy to account for the observed expansion rate of our Universe.

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It is now generally agreed that the biggest unsolved problem in astronomy and cosmology is the newly discovered fact that our Universe is undergoing a stage of accelerated expansion. Recent astrophysical and cosmological observations such as structure formation, type Ia supernovae, gravitational lensing, and cosmic microwave background anisotropies have concordantly predicted a spatially flat universe containing a mixture of matter and a dominant smooth component, dubbed the “dark energy”, which provides an anti-gravity force to accelerate the cosmic expansion [1]. The simplest candidate for this invisible component carrying a sufficiently large negative pressure for the anti-gravity is a true cosmological constant. Current data, however, are consistent with a somewhat broader diversity of such dark energy as long as its equation of state approaches that of the cosmological constant at a recent epoch. Many models have been proposed to account for this dynamical dark energy, including quintessential and k-essential models, modified gravity and scalar–tensor theories, and so on [2]. In these models, dark energy is characterized by an equation of state $w(z)$ which is the ratio of its pressure and energy density

at redshift z . The cosmological constant is indeed vacuum energy with $w(z) = -1$. Current observational data, when fitted to dark energy models with a static w , favor the so-called Λ CDM model with vacuum energy $\Omega_\Lambda = 0.7$ and cold dark matter $\Omega_{\text{CDM}} = 0.3$, while constraining $w = -1.02^{+0.13}_{-0.19}$ at the 95% confidence level. Furthermore, joint constraints on both $w(z)$ and its time evolution $dw(z)/dz$ at $z = 0$ are consistent with the value of the equation of state expected of a static cosmological constant [3]. However, the data are not precise enough to pinpoint whether the dark energy is truly static or dynamical. Most likely, new kinds of measurements or next-generation experiments are needed to reveal the nature of dark energy. In this Letter, we will address the dark energy problem by considering non-Abelian gauge theories on compact extra dimensions.

Interest in theories of extra dimensions has been quite immense in recent years [4]. New models such as brane scenarios, large extra dimensions, and warped extra dimensions have not only revolutionized the Kaluza–Klein theory, but also shed new light on some long-standing problems in particle physics and cosmology. Interestingly, theories with large extra dimensions can be even tested by future collider experiments. Recently cosmological models involving extra dimensions have been constructed to account for the current cosmic acceleration or to accommodate the dark energy [5]. Here we shall show how a

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five-dimensional (5d) gauge theory based on the Hosotani symmetry breaking mechanism [6] could naturally give rise to a small but finite cosmological constant that plays the role of dark energy.

The Hosotani mechanism is a non-Higgs-type symmetry-breaking mechanism which has been widely discussed in the literature [7]. Its main idea is that in a multiply connected space-time manifold, the vanishing of the field strength $F_{MN} = 0$ of gauge fields A_M in a vacuum does not necessarily imply the vanishing of the gauge fields, and $A_M \neq 0$ will imply gauge symmetry breaking in general. The relevant order parameter is the path-ordered Wilson lines $U_n = P \exp(\oint_{C_n} A_M dx^M)$, where C_n represent non-contractible loops in the manifold. If U_n do not belong to the center of the group, then the original gauge group is broken to some subgroup that commutes with all U_n . This mechanism has been extensively employed in superstring phenomenology [8], and applied to Kaluza–Klein cosmology in connection with the problem of vacuum stability [9]. Recently, a new extranatural inflation model in which the inflaton is the fifth component of a gauge field in a 5d theory compactified on a circle was presented [10], and it was shown that the fifth component may also be a good candidate for quintessence if the quintessential potential is provided by massive bulk fields with bare masses of order of the GUT scale [11].

In Ref. [9], the authors considered a 5d Einstein–Yang–Mills theory, with massless fermions, defined in a flat Friedmann–Robertson–Walker universe compactified on a circle. The spacetime metric is given by

$$ds^2 = g_{MN} dx^M dx^N \quad (M = 0, 1, 2, 3, 5) \\ = dt^2 - a^2(t) d\vec{x}^2 - b^2(t) dx_5^2, \quad (1)$$

and the action is consisted of gravity, $SU(2)$ gauge fields, and a fermionic sector L_f which contains N_f massless gauged fermions and n_f massless free fermions:

$$S = - \int d^5x \sqrt{|g_{MN}|} \left(\frac{R}{16\pi G} + \frac{1}{2} \text{Tr} F_{MN} F^{MN} + L_f \right). \quad (2)$$

Here the gauge bosons and the fermions can be considered as fields in some hidden sector of a grand unified theory. Dynamics of the gauge fields and the fermion fields here determines the structure of the vacuum. The Casimir energy of the system was computed by evaluating the one-loop effective potentials of the gauge fields and the fermions in the backgrounds defined by the metric (1) and the classical gauge fields of the form

$$A_\mu = 0, \quad A_5 = \phi(t)\sigma^3, \quad \mu = 0, 1, 2, 3, \quad (3)$$

where A_5 is the fifth component along the circle and σ is the Pauli matrix. This amounts to a total effective potential of the system denoted by $V(bR_0, \phi)$, where R_0 is the final radius of the circle. It was shown that the Einstein's equations for $a(t)$, $b(t)$ and $\phi(t)$, derived from the action (2), admit static vacuum solutions with stable compactification. These solutions with $\dot{a}_0 = \dot{b}_0 = \dot{\phi}_0 = 0$ are given by the global minima of $V(bR_0, \phi)$ determined by

$$2g_5 b_0 R_0 \phi_0 = \frac{r}{2} \pmod{r}, \quad (4)$$

where g_5 is the 5d $SU(2)$ gauge coupling and $r = 1$ and $r = 2$ correspond respectively to periodic fermions in the adjoint and fundamental representations. In the case of adjoint fermions, the vacuum states correspond to $U(1)$ symmetry, hence $SU(2)$ gauge symmetry is dynamically broken. For fundamental fermions the gauge symmetry is unbroken as the Wilson line is an element of Z_2 . To obtain a zero cosmological constant for these vacuum states, an appropriate number (n_f) of free fermions with assigned boundary conditions along the circle has been chosen such that $V(b_0 R_0, \phi_0) = 0$. A salient feature of this model is that only a small number of fermions is required to stabilize the vacuum. In fact, $N_f = 1$ gauged fermion will suffice to do the job. On the contrary, in the other Kaluza–Klein-type theories one usually needs to add a large number (of the order of 10^4) of matter fields for the same purpose.

Now we turn to the dark energy problem. Naively, one would expect that the vacuum energy density ρ_Λ is of order M_P^4 , where the reduced Planck mass is given by $M_P = (8\pi G)^{-1/2} = 2.44 \times 10^{18}$ GeV, since the Planck scale is the natural cutoff scale of zero-point energies of each quantum field. But the observed value for the vacuum-like energy density is $\rho_\Lambda \simeq 0.7\rho_c \simeq 1.6h^2 \times 10^{-120} M_P^4$, where $h \simeq 0.7$ is the present Hubble parameter defined by $H_0 = 8.76h \times 10^{-61} M_P$ [1], and so the naive estimate is larger than the observed value by a factor of 10^{120} . Many solutions have been proposed about a vanishingly small cosmological constant in some ultimate ground state [12]. Here we assume that the cosmological constant absolutely vanishes in a true ground state with lowest possible energy density. Then, we will show that the vacuum states in the model considered above are indeed metastable due to quantum tunneling effects and will eventually settle down to this true ground state. If the present Universe is still in one of these quasiground states, then the energy density difference above the true ground state can provide us with a small but finite cosmological constant. This idea has already been put forth by considering the topological vacua in a 4d $SU(2)$ Yang–Mills–Higgs theory which is spontaneously broken to $U(1)$ via the Higgs mechanism with a Higgs potential [13]. Unfortunately, the existence of massless or light gauged fermions would suppress the tunneling [14] and thus spoil the idea. Although the fermions can get masses via the Higgs mechanism, the Yukawa couplings are quite arbitrary and may be very small. In contrast, our scenario has new merits. Firstly, we do not need to introduce an *ad hoc* Higgs field, which is here replaced by the fifth component of the gauge field. Secondly, the gauge symmetry breaking needs not be introduced arbitrarily, but instead is determined dynamically by the Casimir energy of the non-integrable phase of the gauge field on the compact dimension through the Hosotani mechanism. Thirdly, the gauged fermions naturally get huge masses of order of the inverse of the size of the compact extra dimension. Lastly, the cosmological constant in our model, as we will show below, is naturally related to the size of the compact extra dimension.

Without loss of generality, let us consider a universe corresponding to the vacuum state in the case of periodic fundamental fermions with $2g_5 R_0 \phi_0 = 1$, where we have set $a_0 = b_0 = 1$ and $r = 2$ in Eq. (4). Presumably, the universe has undergone

the compactification of the circle with initial conditions at some early time t_i (for instance, given by $a(t_i)$, $b(t_i)$, and $\phi(t_i) \simeq 0$), and rolls down the effective potential V to this vacuum state. The actual evolution is very interesting and it warrants a detailed study of the Einstein's equations. In fact, this scenario has been discussed in the context of the extranatural inflationary model [10]. Here we only concern about the vacuum state and the dark energy problem. At energies below $1/R_0$, the 4d effective action for the zero Fourier modes of gauge fields in Eq. (2) is given by

$$S_{\text{eff}}^{\text{gauge}} = - \int d^4x \left(\frac{1}{2} \text{Tr} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \text{Tr} \tilde{F}_{\mu 5} \tilde{F}^{\mu 5} \right). \quad (5)$$

Note that we have rescaled $A_M = \tilde{A}_M / \sqrt{2\pi R_0}$ and $g_5 = g_4 \sqrt{2\pi R_0}$, where g_4 is the dimensionless 4d $SU(2)$ gauge coupling. Since \tilde{A}_μ is independent of x_5 in the effective action, so $\partial_5 \tilde{A}_\mu = 0$ and $\tilde{F}_{\mu 5}$ reduces to the covariant derivative of \tilde{A}_5 . Hence, by rewriting $\tilde{A}_5 = \Phi$, we have

$$S_{\text{eff}}^{\text{gauge}} = \int d^4x \left(-\frac{1}{2} \text{Tr} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \text{Tr} (D_\mu \Phi) (D^\mu \Phi) \right). \quad (6)$$

This resembles the $SU(2)$ Yang–Mills–Higgs system without a Higgs potential. Although the 5d gauge symmetry is unbroken for the vacuum states given by Eq. (4), the fifth component obtains a non-zero vacuum expectation value $\langle \Phi \rangle = \tilde{\phi}_0 \sigma^3$, where $\tilde{\phi}_0 = 1/(2g_4 R_0)$ [9]. Thus, the zero-mode gauge fields and gauged fermions acquire huge mass of the mass of $1/(2R_0)$ [15]. In the case of periodic adjoint fermions, the system is spontaneously broken to $U(1)$ by the Wilson loop along the small circle with $\langle \Phi \rangle = \tilde{\phi}_0 \sigma^3$, where $\tilde{\phi}_0 = 1/(4g_4 R_0)$ [9]. Nevertheless, some fermion modes still remain massless [15]. Since these massless fermions suppress the vacuum tunneling [14], we will not consider this case.

It is well known [14,16] that a non-Abelian gauge theory, such as that described by Eq. (6), bears degenerate perturbative vacua classified in terms of the winding number n and denoted by $|n\rangle$. Before we go on, we should emphasize that there are no instantons nor topological numbers for the 5d gauge fields in Eq. (2). Even in the massless sector (given by Eq. (6)), we still do not have exact instantons but only “constrained ones”. However, these constrained instantons are sufficient in describing the tunneling phenomenon [16,17] as we shall do in due course. Thus let us consider the 4d $SU(2)$ Yang–Mills theory with gauge coupling g . While the degenerate vacua are each separated by an energy barrier, the change of the winding number can take place through quantum tunneling from one vacuum to another. As such, the true ground state, so-called the θ vacuum, can be constructed as

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{in\theta} |n\rangle, \quad (7)$$

where θ is a real parameter. Using the dilute instanton gas approximation [14], it was found that the expectation value of the Hamiltonian \mathcal{H} over the Euclidean spacetime volume VT is

given by

$$\begin{aligned} \langle \theta | e^{-\mathcal{H}T} | \theta \rangle &\propto \sum_{n, n'} \langle n' | e^{-\mathcal{H}T} | n \rangle e^{i(n-n')\theta} \\ &= \sum_{n, n'} \frac{1}{n!} \frac{1}{n'!} (KVT e^{-S_0})^{n+n'} e^{i(n-n')\theta} \\ &= \exp(2KVT e^{-S_0} \cos \theta), \end{aligned} \quad (8)$$

where $S_0 = 8\pi^2/g^2$ is the Euclidean action for an instanton solution which corresponds to the quantum tunneling from $|n\rangle$ to $|n \pm 1\rangle$. Here K is a positive determinantal prefactor. Eq. (8) shows that the energy density difference between each θ vacuum and the perturbative vacuum is given by

$$\langle \theta | \rho_\Lambda | \theta \rangle - \langle n | \rho_\Lambda | n \rangle = -2K e^{-S_0} \cos \theta. \quad (9)$$

Therefore, the θ vacuum with the lowest energy is given by $\theta = 0$. Let us assume that the $\theta = 0$ vacuum is the true ground state where the vacuum energy vanishes. In fact, it has been proposed [18] that the effect of wormholes [19] drives the Universe to this CP-symmetric $\theta = 0$ vacuum state with zero cosmological constant. (More discussions about this can be found in Ref. [13] and references therein.) Then, after normalizing $\langle \theta = 0 | \rho_\Lambda | \theta = 0 \rangle = 0$, we find that the vacuum energy density in each perturbative vacuum is

$$\langle n | \rho_\Lambda | n \rangle = 2K e^{-S_0}. \quad (10)$$

This vacuum energy will manifest as a cosmological constant provided that we still live in one of these perturbative vacua. To guarantee this condition, the tunneling probability from any one of these perturbative vacua to the true ground state in the current horizon volume in the cosmic age must satisfy $\Gamma H_0^{-4} \leq 1$, where Γ is the tunneling rate per unit volume per unit time given by $\Gamma \simeq \rho_\Lambda e^{-S_0}$.¹

Unfortunately, for a pure $SU(2)$ gauge theory which is scale-invariant, the size of the instanton ρ is arbitrary, and so the prefactor K which involves an integral over instanton sizes diverges as ρ goes to infinity. However, if the $SU(2)$ gauge field is coupled to a Higgs scalar with isospin q and vacuum expectation value v , the quantum tunneling will proceed via a constrained instanton whose size is cut off at a scale v^{-1} [16,17]. The Higgs contribution to the Euclidean action of the constrained instanton is approximately given by $S_H = 4\pi^2 q \rho^2 v^2$, rendering K a finite quantity. Including N_f gauged fermions with mass m_f , we find that [16]

$$\begin{aligned} K &= \frac{4\pi^2}{\alpha^4} \int_0^\infty \frac{d\rho}{\rho^5} (m_f \rho)^{N_f} \exp[-S_H + c_1 \ln(\rho v) + c_2] \\ &= 2\pi^2 e^{c_2} \frac{\Gamma[(N_f + c_1 - 4)/2]}{(4\pi^2 q)^{(N_f + c_1 - 4)/2}} \left(\frac{m_f}{v}\right)^{N_f} \left(\frac{v}{\alpha}\right)^4, \end{aligned} \quad (11)$$

¹ It has been proved [20] that the $|n\rangle$ vacua are not physical because they do not satisfy cluster decomposition: $\langle n | e^{-\mathcal{H}(T_1+T_2)} | n + v \rangle = \sum_p \langle n | e^{-\mathcal{H}T_1} | p \rangle \langle p | e^{-\mathcal{H}T_2} | n + v \rangle$ for large times T_1 and T_2 . Here, however, the tunneling time is longer than the age of the Universe. So, observationally we have $\langle n | e^{-\mathcal{H}T} | p \rangle \simeq \delta_{np}$, which implies the cluster decomposition, $\langle n | e^{-\mathcal{H}(T_1+T_2)} | n \rangle = \langle n | e^{-\mathcal{H}T_1} | n \rangle \langle n | e^{-\mathcal{H}T_2} | n \rangle$.

where $\alpha = g^2/(4\pi)$ is the gauge coupling strength at the energy scale v , $c_1 = 20/3 - 2N_f/3$, and $c_2 = 5.96 - 0.36N_f$. Therefore (cf. [13]), if $m_f = gv$, we will have

$$\rho_\Lambda \simeq c^2 \left(\frac{v}{\alpha}\right)^4 \alpha^{\frac{N_f}{2}} e^{-2\pi/\alpha}, \quad (12)$$

$$\Gamma \simeq c^2 \left(\frac{v}{\alpha}\right)^4 \alpha^{\frac{N_f}{2}} e^{-4\pi/\alpha}, \quad (13)$$

where

$$c^2 = 4\pi^2 e^{c_2} (4\pi)^{\frac{N_f}{2}} \frac{\Gamma[(N_f + c_1 - 4)/2]}{(4\pi^2 q)^{(N_f + c_1 - 4)/2}}. \quad (14)$$

The requirements that $\rho_\Lambda = 1.6h^2 \times 10^{-120} M_P^4$ and $\Gamma H_0^{-4} \leq 1$ give

$$\frac{\pi}{2\alpha} + \left(1 - \frac{N_f}{8}\right) \ln \alpha = \ln\left(\frac{v}{M_P}\right) + 30 \ln 10 - \frac{1}{2} \ln\left(\frac{1.27h}{c}\right), \quad (15)$$

$$\frac{v}{M_P} \geq 1.45\alpha^{1 - \frac{N_f}{8}} / \sqrt{c}. \quad (16)$$

For the minimal value of v in Eq. (16) and $h = 0.7$, we obtain $\alpha \simeq 1/44.25$. Let us assume that $q = 1$. Then, we have $v \simeq 3.8 \times 10^{16}$ GeV for $N_f = 1$ and $v \simeq 8.3 \times 10^{16}$ GeV for $N_f = 3$. If we take $v = M_P$ in Eq. (15), we will obtain $\alpha = 1/46.9$ for $N_f = 1$ and $\alpha = 1/46.4$ for $N_f = 3$.

To apply the above results to our model (6), we make the replacements $g = g_4$ and $v = \phi_0 = 1/(2g_4 R_0)$. Thus, we find that $R_0^{-1} \geq 4.1 \times 10^{16}$ GeV, which indicates that the maximal size of the compact extra dimension is of the order of the GUT scale. This result is obtained without fine-tuning. The smallness of the cosmological constant is related to the tunneling rate between the various vacua of the theory which is also very tiny. This idea was first put forth generically in Ref. [13] based on the Higgs mechanism, but Eq. (11) shows that the existence of massless or light fermions with $m_f \ll v$ will suppress the prefactor K . Consequently, the $|n\rangle$ vacua do not satisfy cluster decomposition due to the chirality conservation [14]. Here we revolutionize the idea by replacing the Higgs field with the gauge field in the extra dimension through the Hosotani mechanism. The vacuum state is chosen by the minimum of the Casimir energy instead of the rather *ad hoc* Higgs potential. An important advancement is that the gauged fermions in our model naturally obtain a huge and definite mass given by $m_f = gv$. To our knowledge, the Hosotani mechanism is the only way that does the job.

In summary, we have constructed a new model of dark energy based on a 5d Einstein–Yang–Mills theory. The presence of dark energy comes out rather naturally in our model just from the assumption that there exist extra dimensions. Symmetry breaking by the Hosotani mechanism and the constrained instantons related to the vacuum structure of the gauge field are both immediate consequences. As long as the size of the extra dimension is between the GUT and the Planck scales, the resulting cosmological constant will then be just of the right amount

to account for the dark energy content in our Universe. No other ingredients are needed to achieve this goal.

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