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Some exact and new solutions of the Nizhnik–Novikov–Vesselov equation using the Exp-function method

Byeong-Chun Shin [a], M.T. Darvishi [b,*], A. Barati [b]

[a] Department of Mathematics, Chonnam National University, Gwangju 500-757, South Korea

[b] Department of Mathematics, Razi University, Kermanshah 67149, Iran

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ABSTRACT

In this paper, using the Exp-function method, we give some explicit formulas of exact traveling wave solutions for the Nizhnik–Novikov–Vesselov equation.

1. Introduction

It is well known that many important phenomena and dynamic processes in physics, mechanics, chemistry, biology can be represented by nonlinear partial differential equations. For decades, mathematicians and physicists have devoted considerable effort to the study of solutions of nonlinear partial differential equations. The study of exact solutions of nonlinear evolution equations plays an important role in soliton theory and explicit formulas of such exact solutions play an essential role in the nonlinear science. Also, the explicit formulas may provide physical information and help us to understand the mechanism of related physical models.

In recent years, many kinds of powerful methods have been proposed to find solutions of nonlinear partial differential equations, e.g., the inverse scattering method [1], the variational iteration method [2–4], the homotopy perturbation method [5–7], the Bäcklund transformation method [8,9], the tanh-method [10], the sinh-method [11], the homogeneous balance method [12], the F-expansion method [13], and the algebraic geometric method [14]. One may find a complete review in [15].

In [16], He suggested a novel method, the so-called Exp-function method, to search for solitary solutions, compact-like solutions and periodic solutions of various nonlinear wave equations. The basic idea of the Exp-function method was provided in [17] and one may find several applications of the Exp-function method over various areas in [16,18–23].

In this paper, we investigate explicit formulas of solutions of the following (2+1)-dimensional Nizhnik–Novikov–Vesselov (NNV) equation given in [24]

$$\begin{aligned} &u_t = Au_{xxx} + Bu_{yyy} - 3Av_xu - 3Avu_x - 3Bw_yu - 3Bwu_y, \\ &u_x = v_y, \qquad u_y = w_x, \end{aligned} \tag{1}$$

where A and B are given constants satisfying $A+B \neq 0$. In recent years, (1+1)- and (2+1)-dimensional soliton equations have been studied over several areas of physics including condense matter physics [25], fluid mechanics [26], plasma physics [27] and optics [28]. The (2+1)-dimensional NNV equation is an isotropic extension of the well-known (1+1)-dimensional KdV equation. We apply the Exp-function method to derive some explicit formulas of the solutions of NNV equation (1).

The outline of this paper is as follows. In the following section we review the Exp-function method and then we apply the method to find explicit formulas of solutions of the NNV equation in Section 3. We present a brief conclusion in Section 4.

* Corresponding author.
E-mail address: darvishimt@yahoo.com (M.T. Darvishi).

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2. The Exp-function method

Consider the following nonlinear partial differential equation

$$N(\chi, \chi_x, \chi_y, \chi_z, \chi_t, \chi_{xx}, \chi_{yy}, \chi_{zz}, \chi_{xy}, \chi_{xt}, \chi_{yt}, \ldots) = 0. \tag{2}$$

Using a transformation

$$\eta = ax + by + cz + dt + \gamma$$

where a, b, c, d and γ are constants, we can convert (2) to the following nonlinear ordinary differential equation

$$M(\chi, \chi', \chi'', \chi''', \ldots) = 0, \tag{3}$$

where the prime denotes the differentiation symbol with respect to η.

Adopting the Exp-function method given in [16], we assume that the traveling wave solution can be expressed in the following form

$$\chi(\eta) = \frac{\sum_{n=-N_a}^{N_b} a_n \exp(n\eta)}{\sum_{m=-M_a}^{M_b} b_m \exp(m\eta)}, \tag{4}$$

where M_a, M_b, N_a and N_b are positive integers which could be freely chosen, and a_n and b_m are unknown coefficients to be determined. The formula (4) can be rewritten in the expanded form such as

$$\chi(\eta) = \frac{a_{N_b} \exp(N_b\eta) + \cdots + a_{-N_a} \exp(-N_a\eta)}{b_{M_b} \exp(M_b\eta) + \cdots + b_{-M_a} \exp(-M_a\eta)}. \tag{5}$$

In order to determine the values of N_a and M_a, we balance the linear terms of the highest order in Eq. (3) with the highest order nonlinearity. Similarly, to determine the values of N_b and M_b, we balance the linear terms of the lowest order in Eq. (3) with the lowest order nonlinear terms. For more details see [16,22].

3. Explicit formulas of solutions of the NNV equation

To solve the (2+1)-dimensional NNV equation (1), with the following linear transformation

$$\eta = \lambda(x + y + kt + \gamma), \tag{6}$$

define

$$\phi(\eta) = u(x, y, t), \qquad \psi(\eta) = v(x, y, t), \qquad \tau(\eta) = w(x, y, t), \tag{7}$$

where λ and k are constants which will be determined later, and γ is an arbitrary given constant. Substituting Eqs. (7) into Eqs. (1), we have the following ordinary nonlinear differential equations for ϕ, ψ and τ such as

$$(A + B)\lambda^2\phi''' - 3A(\psi\phi)' - 3B(\tau\phi)' - k\phi' = 0 \tag{8}$$

$$\phi' = \psi', \qquad \psi' = \tau'. \tag{9}$$

From Eq. (9), we easily obtain that

$$\psi = \phi + C \quad \text{and} \quad \tau = \phi + D \tag{10}$$

where C and D are constants. Therefore, by substituting Eqs. (10) into (8) we lead to the following equation

$$\lambda^2\phi''' - 6\phi\phi' - n\phi' = 0 \tag{11}$$

where

$$n = \frac{3AC + 3BD + k}{A + B}.$$

Using Eq. (5) and according to the homogeneous balance principle, we have that

$$M_a = N_a \quad \text{and} \quad M_b = N_b.$$

In the following subsections, we consider some arbitrary values of the numbers N_a and N_b to derive explicit analytic solutions of (11). One may choose the numbers N_a and N_b freely, but the resultant solutions do not strongly depend upon such choice (see [16,22]).

3.1. Case 1: $N_a = 1$ and $N_b = 1$

For the simple case of these choices, the trial function (5) becomes

$$\phi(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)}. \tag{12}$$

For convenience, set $b_1 = 1$. Substituting Eq. (12) into Eq. (11) and using some symbolic calculations we can derive the following relations:

$$a_1 = 0, \qquad a_0 = -\frac{b_0(3BD+k)}{A+B}, \qquad a_{-1} = 0,$$
$$b_1 = 1, \qquad b_{-1} = \frac{b_0^2}{4}, \qquad C = 0,$$
$$\lambda = \frac{\sqrt{(A+B)(3BD+k)}}{A+B},$$

provided that $(A+B)(3BD+k) > 0$.

Consequently the solitonary solution $\phi(\eta)$ is given by

$$\phi(\eta) = \frac{-b_0\lambda^2}{\exp(\eta) + b_0 + \frac{b_0^2}{4}\exp(-\eta)}$$

where D, b_0 and k are free parameters. The other solutions $\psi(\eta)$ and $\tau(\eta)$ are given by the relation (10).

3.2. Case 2: $N_a = 2$ and $N_b = 2$

In this case, we set $N_a = M_a = 2$ and $N_b = M_b = 2$, then the trial function (5) becomes

$$\phi(\eta) = \frac{a_2 \exp(2\eta) + a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta) + a_{-2} \exp(-2\eta)}{b_2 \exp(2\eta) + b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta) + b_{-2} \exp(-2\eta)}. \tag{13}$$

There are some free parameters in the above equation. We also set $b_{-2} = b_2 = 0$ for convenience, then the trial function (13) is simplified as:

$$\phi(\eta) = \frac{a_2 \exp(2\eta) + a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta) + a_{-2} \exp(-2\eta)}{b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)}. \tag{14}$$

Substituting Eq. (14) into Eq. (11), we can derive the following relations:

$$a_2 = 0, \qquad a_1 = 0, \qquad a_0 = -\frac{b_0(3AC+k)}{A}, \qquad a_{-1} = 0,$$
$$b_2 = 0, \qquad b_1 = \frac{b_0^2}{4b_{-1}}, \qquad b_{-2} = 0,$$
$$B = 0, \qquad \lambda = \frac{\sqrt{A(3AC+k)}}{A},$$

provided that $A(3AC+k) > 0$. Then, we have the following solitonary solution $\phi(\eta)$

$$\phi(\eta) = \frac{-b_0\lambda^2}{\frac{b_0^2}{4b_{-1}}\exp(\eta) + b_0 + b_{-1}\exp(-\eta)}$$

where C, D, b_0, b_{-1} and k are free parameters. The other solutions $\psi(\eta)$ and $\tau(\eta)$ are also given by the relation (10).

3.3. Case 3: $N_a = 3$, $N_b = 1$

In this case, Eq. (5) can be expressed as

$$\phi(\eta) = \frac{a_3 \exp(3\eta) + a_2 \exp(2\eta) + a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{b_3 \exp(3\eta) + b_2 \exp(2\eta) + b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)}. \tag{15}$$

Set $a_2 = 0$ and $b_2 = 0$ for simplicity. By the similar arguments given in the previous cases, we derive the following relations

$$a_3 = 0, \qquad a_2 = 0, \qquad a_1 = \frac{4a_{-1}b_1^2}{b_0^2}, \qquad a_0 = \frac{-(-4a_{-1}b_1 + \lambda^2 b_0^2)}{b_0},$$

$$b_3 = 0, \qquad b_2 = 0, \qquad b_{-1} = \frac{b_0^2}{4b_1},$$

$$k = \frac{-3ACb_0^2 + \lambda^2 b_0^2 A + \lambda^2 b_0^2 B - 3BDb_0^2 - 24a_{-1}b_1 B - 24a_{-1}b_1 A}{b_0^2}.$$

Then we have the following solitonary solution $\phi(\eta)$

$$\phi(\eta) = \frac{\frac{4a_{-1}b_1^2}{b_0^2}\exp(\eta) - \frac{(-4a_{-1}b_1+\lambda^2 b_0^2)}{b_0} + a_{-1}\exp(-\eta)}{b_1\exp(\eta) + b_0 + \frac{b_0^2}{4b_1}\exp(-\eta)}$$

where C, D, a_{-1}, b_1, b_0 and λ are free parameters. The other solutions $\psi(\eta)$ and $\tau(\eta)$ are also given by the relation (10).

In this case one may derive other relations such as

$$a_3 = \frac{a_{-1}b_1^2}{4b_{-1}^2}, \qquad a_2 = 0, \qquad a_1 = \frac{-b_1(4\lambda^2 b_{-1} - a_{-1})}{b_{-1}}, \qquad a_0 = 0,$$

$$b_3 = \frac{b_1^2}{4b_{-1}}, \qquad b_2 = 0, \qquad b_0 = 0,$$

$$k = \frac{-3ACb_{-1} + 4\lambda^2 b_{-1}A + 4\lambda^2 b_{-1}B - 3BDb_{-1} - 6a_{-1}B - 6a_{-1}A}{b_{-1}}.$$

Then we have the following solitonary solution $\phi(\eta)$

$$\phi(\eta) = \frac{\frac{a_{-1}b_1^2}{4b_{-1}^2}\exp(3\eta) - \frac{b_1(4\lambda^2 b_{-1} - a_{-1})}{b_{-1}}\exp(\eta) + a_{-1}\exp(-\eta)}{\frac{b_1^2}{4b_{-1}}\exp(3\eta) + b_1\exp(\eta) + b_{-1}\exp(-\eta)}$$

where C, D, a_{-1}, b_1, b_{-1} and λ are free parameters. The other solutions $\psi(\eta)$ and $\tau(\eta)$ are also given by the relation (10). In the above solution, if we set $b_1 = 2b_{-1}$, we have the compact form of solution

$$\phi(\eta) = \frac{a_{-1}}{b_{-1}} - \frac{2\lambda^2}{\cosh^2(\eta)}.$$

4. Conclusions

In this paper, we have applied the Exp-function method to find some explicit formulas of solutions for the (2+1)-dimensional Nizhnik–Novikov–Vesselov equation. The solution procedure is very simple and straightforward. Also the obtained solutions have very concise explicit form.

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