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Soft BL-algebras based on fuzzy sets

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ABSTRACT

By means of $\in$-soft sets and q-soft sets, some characterizations of (implicative, positive implicative and fantastic) filteristic soft BL-algebras are investigated. Finally, we prove that a soft set is an implicative filteristic soft BL-algebra if and only if it is both a positive implicative filteristic soft BL-algebra and a fantastic filteristic soft BL-algebra.

1. Introduction

To solve complicated problems in economics, engineering, and environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. Uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which have been pointed out in [1]. Maji et al. [2] and Molodtsov [1] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov [1] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, research on the soft set theory is progressing rapidly. Maji et al. [3] described the application of soft set theory to a decision making problem. They also studied several operations on the theory of soft sets. Chen et al. [4] presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh [5,6]. The second author of this paper [7] applied the notion of soft sets by Molodtsov to the theory of BCK/BCI-algebras, and introduced the notions of soft BCK/BCI-algebras, and then investigated their basic properties [8]. Aktas et al. [9] studied the basic concepts of soft set theory, and compared soft sets to fuzzy and rough sets, providing some examples to clarify their differences. They also discussed the notion of soft groups.

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The concept of *BL*-algebra was introduced by Hájek's as the algebraic structures for his Basic Logic [10]. A well known example of a *BL*-algebra is the interval [0, 1] endowed with the structure induced by a continuous t-norm. On the other hand, the *MV*-algebras, introduced by Chang in 1958 (see [11]), are one of the most well known classes of *BL*-algebras. In order to investigate the logic system whose semantic truth value is given by a lattice, Xu [12] proposed the concept of lattice implication algebras and studied the properties of filters in such algebras [13]. Later on, Wang [14] proved that the lattice implication algebras are categorically equivalent to the *MV*-algebras. Furthermore, in order to provide an algebraic proof of the completeness theorem of a formal deductive system [15], Wang [16] introduced the concept of R_0-algebras. In fact, the *MV*-algebras, Gödel algebras and product algebras are the most known classes of *BL*-algebras. *BL*-algebras are further discussed by many researchers, see [17–23].

The idea of quasi-coincidence of a fuzzy point with a fuzzy set, which was mentioned in [24], played a vital role to generate some different types of fuzzy subsets. It is worth pointing out that Bhakat and Das [25] initiated the concepts of (α, β)-fuzzy subgroups by using the "belongs to" relation $(\in)$ and "quasi-coincident with" relation (q) between a fuzzy point and a fuzzy subgroup, and introduced the concept of an $(\in, \in \vee q)$-fuzzy subgroup. In fact, the $(\in, \in \vee q)$-fuzzy subgroup is an important generalization of Rosenfeld's fuzzy subgroup. It is now natural to investigate similar type of generalizations of the existing fuzzy subsystems of other algebraic structures. With this objective in view, Ma et al. [26–30] discussed some kind of generalized fuzzy filters of *BL*-algebras.

Based on [26,27], we continue to study the characterization of *BL*-algebras in this paper. We deal with soft *BL*-algebras based on fuzzy sets. By means of $\in$-soft sets and q-soft sets, we investigate some characterizations of (implicative, positive implicative and fantastic) filteristic soft *BL*-algebras. Finally, we prove that a soft set is an implicative filteristic soft *BL*-algebra if and only if it is both a positive implicative filteristic soft *BL*-algebra and a fantastic filteristic soft *BL*-algebra.

2. Preliminaries

Recall that an algebra $L = (L, \leq, \wedge, \vee, \odot, \to, 0, 1)$ is a *BL*-algebra [10] if it is a bounded lattice such that the following conditions are satisfied:

(i) $(L, \odot, 1)$ is a commutative monoid,
(ii) $\odot$ and $\to$ form an adjoin pair, i.e., $z \leq x \to y$ if and only if $x \odot z \leq y$ for all $x, y, z \in L$,
(iii) $x \wedge y = x \odot (x \to y)$,
(iv) $(x \to y) \vee (y \to x) = 1$.

In what follows, L is a *BL*-algebra unless otherwise specified.

In any *BL*-algebra L, the following statements are true (see [17,18]):

(1) $x \leq y \Leftrightarrow x \to y = 1$,
(2) $x \to (y \to z) = (x \odot y) \to z = y \to (x \to z)$,
(3) $x \odot y \leq x \wedge y$,
(4) $x \to y \leq (z \to x) \to (z \to y), x \to y \leq (y \to z) \to (x \to z)$.
(5) $x \to x' = x'' \to x$,
(6) $x \vee x' = 1 \Rightarrow x \wedge x' = 0$,
(7) $x \vee y = ((x \to y) \to y) \wedge ((y \to x) \to x)$,

where $x' = x \to 0$.

A non-empty subset A of L is called a *filter* of L if it satisfies the following conditions: (i) $1 \in A$; (ii) $\forall x \in A, y \in L, x \to y \in A \Rightarrow y \in A$. It is easy to check that a non-empty subset A of L is a filter of L if and only if it satisfies: (i) $\forall x, y \in L, x \odot y \in A$; (ii) $\forall x \in A, x \leq y \Rightarrow y \in A$ (see [17,21]).

Now, we call a filter A of L an *implicative filter* of L if it satisfies $x \to (z' \to y) \in A, y \to z \in A \Rightarrow x \to z \in A$. A filter A of L is said to be a *positive implicative filter* of L if it satisfies $x \to (y \to z) \in A, x \to y \in A \Rightarrow x \to z \in A$. A filter A of L is called a *fantastic filter* of L if it satisfies $z \to (y \to x) \in A, z \in A \Rightarrow ((x \to y) \to y) \to x \in A$ (see [17,19,20]).

Definition 2.1 (*[19]*). A fuzzy set μ of L is called a *fuzzy filter* of L if it satisfies:
(F1) $\forall x, y \in L, \mu(x \odot y) \geq \min\{\mu(x), \mu(y)\}$;
(F2) μ is order-preserving, that is, $\forall x, y \in L, x \leq y \Rightarrow \mu(x) \leq \mu(y)$.

Theorem 2.2 (*[19]*). *A fuzzy set μ of L is a fuzzy filter of L if and only if it satisfies:*

(F3) $\forall x \in L, \mu(1) \geq \mu(x)$;
(F4) $\forall x, y \in L, \mu(y) \geq \min\{\mu(x \to y), \mu(x)\}$.

Definition 2.3 (*[20,26]*). (i) A fuzzy filter μ of L is called a *fuzzy implicative filter* of L if it satisfies:
(F5) $\mu(x \to z) \geq \min\{\mu(x \to (z' \to y)), \mu(y \to z)\}$, for all $x, y, z \in L$.
(ii) A fuzzy filter μ of L is called a *fuzzy positive implicative filter* of L if it satisfies:
(F6) $\mu(x \to z) \geq \min\{\mu(x \to (y \to z)), \mu(x \to y)\}$, for all $x, y, z \in L$.
(iii) A fuzzy filter μ of L is called a *fuzzy fantastic filter* of L if it satisfies:
(F7) $\mu(((x \to y) \to y) \to x) \geq \min\{\mu(z \to (y \to x)), \mu(z)\}$, for all $x, y, z \in L$.

Definition 2.4 ([26]). A fuzzy set μ of L is said to be an $(\in, \in \vee q)$-*fuzzy filter* of L if for all $t, r \in (0, 1]$ and $x, y \in L$,
(F8) $x_t \in \mu$ and $y_r \in \mu$ imply $(x \odot y)_{\min\{t,r\}} \in \vee q\mu$;
(F9) $x_t \in \mu$ implies $y_t \in \vee q\mu$ with $x \le y$.

Equivalently, we have

Definition 2.4′ ([26]). A fuzzy set μ of L is said to be an $(\in, \in \vee q)$-*fuzzy filter* of L if for all $t, r \in (0, 1]$ and $x, y \in L$,
(F10) $x_t \in \mu$ implies $1_t \in \vee q\mu$;
(F11) $(x \to y)_t \in \mu$ and $x_r \in \mu$ imply $y_{\min\{t,r\}} \in \vee q\mu$.

Theorem 2.5 ([26]). *A fuzzy set μ of L is an $(\in, \in \vee q)$-fuzzy filter of L if it satisfies:*

(F12) $\mu(1) \ge \min\{\mu(x),\ 0.5\}$;
(F13) $\mu(y) \ge \min\{\mu(x \to y), \mu(x),\ 0.5\}$.

Definition 2.6 ([26]). (i) An $(\in, \in \vee q)$-fuzzy filter μ of L is called an $(\in, \in \vee q)$-*fuzzy implicative filter* of L if it satisfies:
(F14) $\mu(x \to z) \ge \min\{\mu(x \to (z' \to y)), \mu(y \to z), 0.5\}$, for all $x, y, z \in L$.
(ii) An $(\in, \in \vee q)$-fuzzy filter μ of L is called an $(\in, \in \vee q)$-*fuzzy positive implicative filter* of L if it satisfies:
(F15) $\mu(x \to z) \ge \min\{\mu(x \to (y \to z)), \mu(x \to y), 0.5\}$, for all $x, y, z \in L$.
(iii) An $(\in, \in \vee q)$-fuzzy filter μ of L is called an $(\in, \in \vee q)$-*fuzzy fantastic filter* of L if it satisfies:
(F16) $\mu(((x \to y) \to y) \to x) \ge \min\{\mu(z \to (y \to x)), \mu(z), 0.5\}$, for all $x, y, z \in L$.

Definition 2.7 ([27]). A fuzzy set μ of L is called an $(\overline{\in}, \overline{\in} \vee \overline{q})$-*fuzzy filter* of L if for all $t, r \in (0, 1]$ and for all $x, y \in L$,
(F17) $(x \odot y)_{\min\{t,r\}} \overline{\in} \mu$ implies $x_t \overline{\in} \vee \overline{q}\mu$ or $y_r \overline{\in} \vee \overline{q}\mu$;
(F18) $y_t \overline{\in} \mu$ implies $x_t \overline{\in} \vee \overline{q}\mu$ with $x \le y$.

Equivalently, we have

Definition 2.7′ ([27]). A fuzzy set μ of L is called an $(\overline{\in}, \overline{\in} \vee \overline{q})$-*fuzzy filter* of L if for all $t, r \in (0, 1]$ and for all $x, y \in L$,
(F19) $1_t \overline{\in} \mu$ implies $x_t \overline{\in} \vee \overline{q}\mu$;
(F20) $y_{\min\{t,r\}} \overline{\in} \mu$ implies $(x \to y)_t \overline{\in} \vee \overline{q}\mu$ or $x_r \overline{\in} \vee \overline{q}\mu$.

Theorem 2.8 ([27]). *A fuzzy set μ of L is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy filter of L if and only if it satisfies:*

(F21) $\forall x \in L,\ \ \max\{\mu(1), 0.5\} \ge \mu(x)$;
(F22) $\forall x, y \in L,\ \ \max\{\mu(y), 0.5\} \ge \min\{\mu(x \to y), \mu(x)\}$.

Definition 2.9 ([27]). Given $\alpha, \beta \in (0, 1]$ and $\alpha < \beta$, we call a fuzzy set μ of L a *fuzzy filter with thresholds* $(\alpha, \beta]$ of L if for all $x, y \in L$, the following conditions are satisfied:
(F23) $\max\{\mu(1), \alpha\} \ge \min\{\mu(x), \beta\}$;
(F24) $\max\{\mu(y), \alpha\} \ge \min\{\mu(x \to y), \mu(x), \beta\}$.

Definition 2.10 ([27]). (i) An $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy filter of L is called an $(\overline{\in}, \overline{\in} \vee \overline{q})$-*fuzzy implicative filter* of L if it satisfies:
(F25) $\max\{\mu(x \to z), 0.5\} \ge \min\{\mu(x \to (z' \to y)), \mu(y \to z)\}$, for all $x, y, z \in L$.
(ii) An $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy filter μ of L is called an $(\overline{\in}, \overline{\in} \vee \overline{q})$-*fuzzy positive implicative filter* of L if it satisfies:
(F26) $\max\{\mu(x \to z), 0.5\} \ge \min\{\mu(x \to (y \to z)), \mu(x \to y)\}$, for all $x, y, z \in L$.
(iii) An $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy filter μ of L is called an $(\overline{\in}, \overline{\in} \vee \overline{q})$-*fuzzy fantastic filter* of L if it satisfies:
(F27) $\max\{\mu(((x \to y) \to y) \to x), 0.5\} \ge \min\{\mu(z \to (y \to x)), \mu(z)\}$, for all $x, y, z \in L$.

Definition 2.11 ([27]). Given $\alpha, \beta \in (0, 1]$ and $\alpha < \beta$.
(i) A fuzzy set μ of L is called a *fuzzy implicative filter with thresholds* $(\alpha, \beta]$ of L if it satisfies (F23), (F24) and
(F28) $\max\{\mu(x \to z), \alpha\} \ge \min\{\mu(x \to (z' \to y)), \mu(y \to z), \beta\}$, for all $x, y, z \in L$.
(ii) A fuzzy set μ of L is called a *fuzzy positive implicative filter with thresholds* $(\alpha, \beta]$ of L if it satisfies (F23), (F24) and
(F29) $\max\{F(x \to z), \alpha\} \ge \min\{\mu(x \to (y \to z)), \mu(x \to y), \beta\}$, for all $x, y, z \in L$.
(iii) A fuzzy set μ of L is called a *fuzzy fantastic filter with thresholds* $(\alpha, \beta]$ of L if it satisfies (F23), (F24) and
(F30) $\max\{\mu(((x \to y) \to y) \to x), \alpha\} \ge \min\{\mu(z \to (y \to x)), \mu(z), \beta\}$, for all $x, y, z \in L$.

3. Filteristic soft *BL*-algebras

Molodtsov [1] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $\mathcal{P}(U)$ denote the power set of U and $A \subset E$.

A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A).

Definition 3.1. Let (F, A) be a soft set over L. Then (F, A) is called a *filteristic soft BL-algebra over L* if $F(x)$ is a filter of L for all $x \in A$. For our convenience, the empty set $\emptyset$ is regarded as a filter of L.

Example 3.2. Let $L = \{0, a, b, 1\}$, where $0 < a < b < 1$. Then we define $x \wedge y = \min\{x, y\}$, $x \vee y = \max\{x, y\}$, and $\odot$ and $\rightarrow$ as follows:

$\odot$	0	a	b	1
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
1	0	a	b	1

$\rightarrow$	0	a	b	1
0	1	1	1	1
a	a	1	1	1
b	0	a	1	1
1	0	a	b	1

It is clear that $(L, \wedge, \vee, \odot, \rightarrow, 1)$ is now a BL-algebra. Let (F, A) be a soft set over L, where $A = (0, 1]$ and $F : A \rightarrow \mathscr{P}(L)$ is a set-valued function defined by

$$F(x) = \begin{cases} \{0, a, b, 1\} & \text{if } 0 < x \leq 0.3, \\ \{1, b\} & \text{if } 0.3 < x \leq 0.6, \\ \{1\} & \text{if } 0.6 < x \leq 0.8, \\ \emptyset & \text{if } 0.8 < t \leq 1. \end{cases}$$

Thus, $F(x)$ is a filter of L for all $x \in A$, and so (F, A) is a filteristic soft BL-algebra over L.

Given a fuzzy set μ in any BL-algebra L and $A \subseteq [0, 1]$, consider two set-valued functions

$$F : A \rightarrow \mathscr{P}(L), \quad t \mapsto \{x \in L \mid x_t \in \mu\}$$

and

$$F_q : A \rightarrow \mathscr{P}(L), \quad t \mapsto \{x \in L \mid x_t q\mu\}.$$

Then (F, A) and (F_q, A) are called an $\in$-soft *set* and q-*soft set* over L, respectively.

Theorem 3.3. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0, 1]$. Then (F, A) is a filteristic soft BL-algebra over L if and only if μ is a fuzzy filter of L.*

Proof. Let μ be a fuzzy filter of L and $t \in A$. If $x \in F(t)$, then $x_t \in \mu$, and so $1_t \in \mu$, i.e., $1 \in F(t)$. Let $x, y \in L$ be such that $x, x \rightarrow y \in F(t)$. Then $x_t \in \mu$ and $(x \rightarrow y)_t \in \mu$, and so $y_{\min\{t,t\}} = y_t \in \mu$. Hence $y \in F(t)$. Hence (F, A) is a filteristic soft BL-algebra over L.

Conversely, assume that (F, A) is a filteristic soft BL-algebra over L. If there exists $a \in L$ such that $\mu(1) < \mu(a)$, then we can choose $t \in A$ such that $\mu(1) < t \leq \mu(a)$. Thus, $1_t \overline{\in} \mu$, i.e., $1 \overline{\in} F(t)$. This is a contradiction. Thus, $\mu(1) \geq \mu(x)$, for all $x \in L$. If there exist $a, b \in L$ such that $\mu(b) < s \leq \min\{\mu(a \rightarrow b), \mu(a)\}$, then $(a \rightarrow b)_s \in \mu$ and $a_s \in \mu$, but $b_s \overline{\in} \mu$, that is, $a \rightarrow b \in F(s)$ and $a \in F(s)$, but $b \overline{\in} F(s)$, contradiction, and so, $\mu(y) \geq \min\{\mu(x \rightarrow y), \mu(x)\}$, for all $x, y \in L$. Therefore, μ is a fuzzy filter of L. □

Theorem 3.4. *Let μ be a fuzzy set of L and (F_q, A) a q-soft set over L with $A = (0, 1]$. Then the following are equivalent:*

(i) *μ is a fuzzy filter of L;*
(ii) *$\forall\, t \in A$, $F_q(t) \neq \emptyset \Rightarrow F_q(t)$ is a filter of L.*

Proof. Let μ be a fuzzy filter of L. For any $t \in A$ be such that $F_q(t) \neq \emptyset$. If $1 \overline{\in} F_q(t)$, then $1_t \overline{q} \mu$, and so $\mu(1) + t < 1$. Then $\mu(x) + t \leq \mu(1) + t < 1$ for all $x \in L$, and so $F_q(t) = \emptyset$, contradiction. Hence $1 \in F_q(t)$. Let $x, y \in L$ be such that $x \rightarrow y \in F_q(t)$ and $x \in F_q(t)$. Then $(x \rightarrow y)_t q\mu$ and $x_t q\mu$, or equivalently, $\mu(x \rightarrow y) + t > 1$ and $\mu(x) + t > 1$. Thus,

$$\begin{aligned} \mu(y) + t &\geq \min\{\mu(x \rightarrow y), \mu(x)\} + t \\ &= \min\{\mu(x \rightarrow y) + t, \mu(x) + t\} \\ &> 1, \end{aligned}$$

and so $y_t q\mu$, i.e., $y \in F_q(t)$. Hence $F_q(t)$ is a filter of L.

Conversely, assume that the condition (ii) holds. If $\mu(1) < \mu(a)$ for some $a \in L$, then $\mu(1) + t \leq 1 < \mu(a) + t$ for some $t \in A$. Thus, $a_t q\mu$, and so $F_q(t) \neq \emptyset$. Hence $1 \in F_q(t)$, and so $1_t q\mu$, i.e.,$\mu(1) + t > 1$, contradiction. Hence $\mu(1) \geq \mu(x)$ for all $x \in L$.

If there exist $a, b \in L$ such that $\mu(b) < \min\{\mu(a \rightarrow b), \mu(a)\}$. Then $\mu(b) + s \leq 1 < \min\{\mu(a \rightarrow b), \mu(a)\} + s$ for some $s \in A$. Hence $(a \rightarrow b)_s q\mu$ and $a_s q\mu$, i.e., $a \rightarrow b \in F_q(s)$ and $a \in F_q(s)$. Since $F_q(s)$ is a filter of L, we have $b \in F_q(s)$, and so $b_s q\mu$, that is, $\mu(b) + s > 1$, contradiction. Hence $\mu(y) \geq \min\{\mu(x \rightarrow y), \mu(x)\}$, for all $x, y \in L$. Therefore μ is a fuzzy filter of L. □

Theorem 3.5. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0, 0.5]$. Then the following are equivalent:*

(i) *μ is an $(\in, \in \vee q)$-fuzzy filter of L;*
(ii) *(F, A) is a filteristic soft BL-algebra over L.*

Proof. Let μ be an $(\in, \in \vee q)$-fuzzy filter of L. For any $t \in A$, we have $\mu(1) \geq \min\{\mu(x), 0.5\}$ for all $x \in F(t)$ by Theorem 2.5(F12). Hence $\mu(1) \geq \min\{\mu(x), 0.5\} \geq \min\{t, 0.5\} = t$, which implies, $1_t \in \mu$, and so $1 \in F(t)$. If $x \to y \in F(t)$ and $x \in F(t)$, then $(x \to y)_t \in \mu$ and $x_t \in \mu$, that is, $\mu(x \to y) \geq t$ and $\mu(x) \geq t$. By Theorem 2.5(F13), we have

$$\begin{aligned}\mu(y) &\geq \min\{\mu(x \to y), \mu(x), 0.5\}\\ &\geq \min\{t, 0.5\}\\ &= t,\end{aligned}$$

which implies, $y_t \in \mu$, and so $y \in F(t)$. Thus, (F, A) is a filteristic soft BL-algebra over L.

Conversely, assume that the condition (ii) holds. If there exists $a \in L$ such that $\mu(1) < \min\{\mu(a), 0.5\}$, then $\mu(1) < t \leq \min\{\mu(a), 0.5\}$ for some $t \in A$. It follows that $1_t \overline{\in} \mu$, i.e., $1 \overline{\in} F(t)$, contradiction. Hence $\mu(1) \geq \min\{\mu(x), 0.5\}$ for all $x \in L$. If there exist $a, b \in L$ such that $\mu(b) < \min\{\mu(a \to b), \mu(a), 0.5\}$. Taking $t = \frac{1}{2}(\mu(b) + \min\{\mu(a \to b), \mu(a), 0.5\})$, we have $t \in A$ and

$$\mu(b) < t < \min\{\mu(a \to b), \mu(a), 0.5\},$$

which implies, $a \to b \in F(t)$, $a \in F(t)$, but $b \overline{\in} F(t)$, contradiction. It follows from Theorem 2.5 that μ is an $(\in, \in \vee q)$-fuzzy filter of L. □

Theorem 3.6. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0.5, 1]$. Then the following are equivalent:*

(i) *μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy filter of L;*
(ii) *(F, A) is a filteristic soft BL-algebra over L.*

Proof. Let μ be an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy filter of L. For any $t \in A$. By Theorem 2.8(F21), we have $\mu(x) \leq \max\{\mu(1), 0.5\}$ for all $x \in F(t)$. Thus, $t \leq \mu(x) \leq \max\{\mu(x), 0.5\} = \mu(x)$, which implies $1_t \in \mu$, i.e., $1 \in F(t)$. Let $x, y \in L$ be such that $x \to y \in F(t)$ and $x \in F(t)$, then $(x \to y)_t \in \mu$ and $x_t \in \mu$, i.e., $\mu(x \to y) \geq t$ and $\mu(x) \geq t$. It follows from Theorem 2.8(F22) that

$$t \leq \min\{\mu(x \to y), \mu(x)\} \leq \max\{\mu(y), 0.5\} = \mu(y),$$

which implies, $y_t \in \mu$, i.e., $y \in F(t)$. Hence $F(t)$ is a filter of L for all $t \in A$, and so (F, A) is a filteristic soft BL-algebra over L.

Conversely, assume that (F, A) is a filteristic soft BL-algebra over L. If there exists $a \in L$ such that $\mu(a) \geq \max\{\mu(1), 0.5\}$, then $\mu(a) \geq t > \max\{\mu(1), 0.5\}$ for some $t \in A$ and so $\mu(1) < t$, and so $1 \overline{\in} F(a)$. Contradiction. Hence $\mu(x) \leq \max\{\mu(1), 0.5\}$ for all $x \in L$. If these exist $a, b \in L$ such that $\min\{\mu(a \to b), \mu(a)\} \geq t > \max\{\mu(b), 0.5\}$ for some $t \in A$. Thus $(a \to b)_t \in \mu$, $a_t \in \mu$, but $b_t \overline{\in} \mu$, which implies, $a \to b \in F(t)$, $a \in F(t)$, but $b \overline{\in} F(t)$. Contradiction. It follows from Theorem 2.8 that μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy filter of L. □

Theorem 3.7. *Given $\alpha, \beta \in (0, 1]$ and $\alpha < \beta$, let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (\alpha, \beta]$. Then the following are equivalent:*

(i) *μ is a fuzzy filter with thresholds $(\alpha, \beta]$ of L;*
(ii) *(F, A) is a filteristic soft BL-algebra over L.*

Proof. Let μ be a fuzzy filter with thresholds $(\alpha, \beta]$ of L. Set $t \in A$. Then by Definition 2.9(F23), we have $\max\{\mu(1), \alpha\} \geq \min\{\mu(x), \beta\}$ for all $x \in F(t)$. Thus, $\max\{\mu(1), \alpha\} \geq \min\{\mu(x), \beta\} \geq \min\{t, \beta\} = t > \alpha$, which implies, $\mu(1) \geq t$, i.e., $1_t \in \mu$. Hence $1 \in F(t)$. Let $x, y \in L$ be such that $x \to y \in F(t)$ and $x \in F(t)$. Thus, $(x \to y)_t \in \mu$ and $x_t \in \mu$, i.e., $\mu(x \to y) \geq t$ and $\mu(x) \geq t$. By Definition 2.9(F24), we have $\max\{\mu(y), \alpha\} \geq \min\{\mu(x \to y), \mu(x), \beta\} \geq \min\{t, \beta\} = t > \alpha$, and so $\mu(y) \geq t$, i.e., $y_t \in \mu$, and so $y \in F(t)$. Therefore, (F, A) is a filteristic soft BL-algebra over L.

Conversely, assume that (F, A) is a filteristic soft BL-algebra over L. If there exists $a \in L$ such that $\max\{\mu(1), \alpha\} < \min\{\mu(a), \beta\}$, then $\max\{\mu(1), \alpha\} < t \leq \min\{\mu(a), \beta\}$ for some $t \in (\alpha, \beta]$. It follows that $1 \overline{\in} F(t)$. Contradiction. If there exist $a, b \in L$ such that $\max\{\mu(b), \alpha\} < t \leq \min\{\mu(a \to b), \mu(a), \beta\}$, then $(a \to b)_t \in \mu$, $a_t \in \mu$, but $b_t \overline{\in} \mu$, and so $a \to b \in F(t)$, $a \in F(t)$, but $b \overline{\in} F(t)$, contradiction. Therefore, μ is a fuzzy filter with thresholds $(\alpha, \beta]$ of L. □

4. Implicative (positive implicative, fantastic) filteristic soft *BL*-algebras

In this section, we divide the results into three parts. In Section 4.1, we describe implicative filteristic soft *BL*-algebras. In Section 4.2, we investigate positive implicative filteristic soft *BL*-algebras. In Section 4.3, we discuss fantastic filteristic soft *BL*-algebras. Finally, we give the relationship between implicative filteristic soft *BL*-algebras, positive implicative filteristic soft *BL*-algebras and fantastic filteristic soft *BL*-algebras.

4.1. Implicative filteristic soft BL-algebras

In this subsection, we describe implicative filteristic soft *BL*-algebras.

Definition 4.1.1. Let (F, A) be a soft set over L. Then (F, A) is called an *implicative filteristic soft BL-algebra over L* if $F(x)$ is an implicative filter of L for all $x \in A$. For our convenience, the empty set $\emptyset$ is regarded as an implicative filter of L.

Example 4.1.2. Let $L = \{0, a, b, 1\}$, where $0 < a < b < 1$. Then we define $x \wedge y = \min\{x, y\}$, $x \vee y = \max\{x, y\}$, and $\odot$ and $\to$ as follows:

$\odot$	0	a	b	1
0	0	0	0	0
a	0	a	a	a
b	0	a	a	b
1	0	a	b	1

$\to$	0	a	b	1
0	1	1	1	1
a	0	1	1	1
b	0	b	1	1
1	0	a	b	1

It is clear that $(L, \wedge, \vee, \odot, \to, 1)$ is now a *BL*-algebra. Let (F, A) be a soft set over L, where $A = (0, 1]$ and $F : A \to \mathscr{P}(L)$ is a set-valued function defined by

$$F(x) = \begin{cases} \{0, a, b, 1\} & \text{if } 0 < x \le 0.4, \\ \{1, a, b\} & \text{if } 0.4 < x \le 0.8, \\ \emptyset & \text{if } 0.8 < x \le 1. \end{cases}$$

Thus, $F(x)$ is an implicative filter of L for all $x \in A$, and so (F, A) is an implicative filteristic soft BL-algebra over L.

From the above definitions, we can get the following:

Proposition 4.1.3. *Every implicative filteristic BL-algebra is a filteristic BL-algebra, but the converse may not be true.*

Theorem 4.1.4. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0, 1]$. Then (F, A) is an implicative filteristic soft BL-algebra over L if and only if μ is a fuzzy implicative filter of L.*

Proof. Let μ be a fuzzy implicative filter of L and $t \in A$. Then μ is also a fuzzy filter of L. It follows from Theorem 3.3 that (F, A) is a filteristic soft *BL*-algebra over L. Let $x, y, z \in L$ be such that $x \to (z' \to y), y \to z \in F(t)$, then $(x \to (z' \to y))_t \in \mu$ and $(y \to z)_t \in \mu$. Hence $\mu(x \to z) \ge \min\{\mu(x \to (z' \to y)), \mu(y \to z)\} \ge t$, and so $x \to z \in F(t)$. Hence (F, A) is an implicative filteristic soft *BL*-algebra over L.

Conversely, assume that (F, A) is an implicative filteristic soft *BL*-algebra over L, then, (F, A) is a filteristic soft *BL*-algebra over L, and so μ is a fuzzy filter of L by Theorem 3.3. If there exist $a, b, c \in L$ such that $\mu(a \to c) < s \le \min\{\mu(a \to (c' \to b)), \mu(b \to c)\}$ for some $s \in A$. Then $(a \to (c' \to b))_s\mu$ and $(b \to c)_s \in \mu$, but $(a \to c)_s \overline{\in} \mu$, that is, $a \to (c' \to b) \in F(s)$ and $b \to c \in F(s)$, but $a \to c \overline{\in} F(s)$, contradiction. Therefore, μ is a fuzzy implicative filter of L. □

Theorem 4.1.5. *Let μ be a fuzzy set of L and (F_q, A) a q-soft set over L with $A = (0, 1]$. Then the following are equivalent:*

(i) *μ is a fuzzy implicative filter of L;*
(ii) *$\forall\, t \in A$, $F_q(t) \neq \emptyset \Rightarrow F_q(t)$ is an implicative filter of L.*

Proof. Let μ be a fuzzy implicative filter of L, then μ is also a fuzzy filter of L. By Theorem 3.4, $F_q(t)$ is a filter of L. Let $x, y, z \in L$ be such that $x \to (z' \to y) \in F_q(t)$ and $y \to z \in F_q(t)$. Then $(x \to (z' \to y))_t q\mu$ and $(y \to z)_t q\mu$, or equivalently, $\mu(x \to (z' \to y)) + t > 1$ and $\mu(y \to z) + t > 1$. Since μ is a fuzzy implicative of L, we have

$$\begin{aligned} \mu(x \to z) + t &\ge \min\{\mu(x \to (z' \to y)), \mu(y \to z)\} + t \\ &= \min\{\mu(x \to (z' \to z)) + t, \mu(y \to z) + t\} \\ &> 1, \end{aligned}$$

and so $(x \to z)_t q\mu$, i.e., $x \to z \in F_q(t)$. Hence $F_q(t)$ is an implicative filter of L.

Conversely, assume that the condition (ii) holds. Then μ is a fuzzy filter of L by Theorem 3.4. If there exist $a, b, c \in L$ such that $\mu(a \to c) < \min\{\mu(a \to (c' \to b)), \mu(b \to c)\}$. Then $\mu(a \to c) + s \le 1 < \min\{\mu(a \to (c' \to b)), \mu(b \to c)\} + s$ for some $s \in A$. Hence $(a \to (c' \to b))_s q\mu$ and $(b \to c)_s q\mu$, but $(a \to c)_s \overline{q}\mu$, i.e., $a \to (c' \to b) \in F_q(s)$ and $b \to c \in F_q(s)$, but $a \to c \overline{\in} F_q(s)$, contradiction. Therefore μ is a fuzzy implicative filter of L. □

Theorem 4.1.6. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0, 0.5]$. Then the following are equivalent:*

(i) *μ is an $(\in, \in \vee q)$-fuzzy implicative filter of L;*
(ii) *(F, A) is an implicative filteristic soft BL-algebra over L.*

Proof. Let μ be an $(\in, \in \vee q)$-fuzzy implicative filter of L. Then μ is also an $(\in, \in \vee q)$-fuzzy filter of L. By Theorem 3.5, (F, A) is a filteristic soft *BL*-algebra. For any $t \in A$, let $x, y, z \in L$ be such that $x \to (z' \to y) \in F(t)$ and $y \to z \in F(t)$, that is, $\mu(x \to (z' \to y)) \ge t$ and $\mu(y \to z) \ge t$. Thus,

$$\begin{aligned} \mu(x \to z) &\ge \min\{\mu(x \to (z' \to y)), \mu(y \to z), 0.5\} \\ &\ge \min\{t, 0.5\} \\ &= t, \end{aligned}$$

which implies, $(x \to z)_t \in \mu$, and so $x \to z \in F(t)$. Thus, (F, A) is an implicative filteristic soft *BL*-algebra over L.

Conversely, assume that the condition (ii) holds. Then (F, A) is also a filteristic soft BL-algebra over L. If there exist $a, b, c \in L$ such that $\mu(a \to c) < \min\{\mu(a \to (c' \to b)), \mu(b \to c), 0.5\}$. Taking $t = \frac{1}{2}(\mu(a \to c) + \min\{\mu(a \to (c' \to b)), \mu(b \to c), 0.5\})$, we have $t \in A$ and

$$\mu(a \to c) < t < \min\{\mu(a \to (c' \to b)), \mu(b \to c), 0.5\},$$

which implies, $a \to (c' \to b) \in F(t)$, $b \to c \in F(t)$, but $a \to c \overline{\in} F(t)$, contradiction. It follows from Definition 2.6 that μ is an $(\in, \in \vee q)$-fuzzy implicative filter of L. □

Theorem 4.1.7. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0.5, 1]$. Then the following are equivalent:*

(i) *μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy implicative filter of L;*
(ii) *(F, A) is an implicative filteristic soft BL-algebra over L.*

Proof. Let μ be an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy implicative filter of L, then μ is also an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy filter of L. By Theorem 3.6, (F, A) is a filteristic soft BL-algebra. For any $t \in A$, let $x, y, z \in L$ be such that $x \to (z' \to y) \in F(t)$ and $y \to z \in F(t)$, then $(x \to (z' \to y))_t \in \mu$ and $(y \to z)_t \in \mu$, i.e., $\mu(x \to (z' \to y)) \geq t$ and $\mu(y \to z) \geq t$. Thus,

$$t \leq \min\{\mu(x \to (z' \to y)), \mu(y \to z)\} \leq \max\{\mu(x \to z), 0.5\} = \mu(x \to z),$$

which implies, $(x \to z)_t \in \mu$, i.e., $x \to z \in F(t)$. Hence $F(t)$ is an implicative filter of L for all $t \in A$, and so (F, A) is an implicative filteristic soft BL-algebra over L.

Conversely, assume that (F, A) is an implicative filteristic soft BL-algebra over L. If there exist $a, b, c \in L$ such that $\min\{\mu(a \to (c' \to b)), \mu(b \to c)\} \geq t > \max\{\mu(a \to c), 0.5\}$ for some $t \in A$. Thus, $(a \to (c' \to b))_t \in \mu$, $(b \to c)_t \in \mu$, but $(a \to c)_t \overline{\in} \mu$, which implies, $a \to (c' \to b) \in F(t)$, $b \to c \in F(t)$, but $a \to c \overline{\in} F(t)$, contradiction. Therefore, μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy implicative filter of L. □

The following is a consequence of Theorems 3.7, 4.1.6 and 4.1.7.

Theorem 4.1.8. *Given $\alpha, \beta \in (0, 1]$ and $\alpha < \beta$, let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (\alpha, \beta]$. Then the following are equivalent:*

(i) *μ is a fuzzy implicative filter with thresholds $(\alpha, \beta]$ of L;*
(ii) *(F, A) is an implicative filteristic soft BL-algebra over L.*

4.2. Positive implicative filteristic soft BL-algebras

In this subsection, we describe positive implicative filteristic soft BL-algebras.

Definition 4.2.1. Let (F, A) be a soft set over L. Then (F, A) is called a *positive implicative filteristic soft BL-algebra over L* if $F(x)$ is a positive implicative filter of L for all $x \in A$. For our convenience, the empty set $\emptyset$ is regarded as a positive implicative filter of L.

Example 4.2.2. Let $L = \{0, a, b, 1\}$, where $0 < a < b < 1$. Then we define $x \wedge y = \min\{x, y\}$, $x \vee y = \max\{x, y\}$, and $\odot$ and $\to$ as follows:

$\odot$	0	a	b	1
0	0	0	0	0
a	0	a	a	a
b	0	a	b	b
1	0	a	b	1

$\to$	0	a	b	1
0	1	1	1	1
a	0	1	1	1
b	0	a	1	1
1	0	a	b	1

It is clear that $(L, \wedge, \vee, \odot, \to, 1)$ is now a BL-algebra. Let (F, A) be a soft set over L, where $A = (0, 1]$ and $F : A \to \mathcal{P}(L)$ is a set-valued function defined by

$$F(x) = \begin{cases} \{0, a, b, 1\} & \text{if } 0 < x \leq 0.4, \\ \{1\} & \text{if } 0.4 < x \leq 0.8, \\ \emptyset & \text{if } 0.8 < x \leq 1. \end{cases}$$

Thus, $F(x)$ is a positive implicative filter of L for all $x \in A$, and so (F, A) is a positive implicative filteristic soft BL-algebra over L.

From the above definitions, we can get the following:

Proposition 4.2.3. *Every positive implicative filteristic BL-algebra is a filteristic BL-algebra, but the converse may not be true.*

Theorem 4.2.4. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0, 1]$. Then (F, A) is a positive implicative filteristic soft BL-algebra over L if and only if μ is a fuzzy positive implicative filter of L.*

Proof. It is similar to the proof of Theorem 4.1.4. □

Theorem 4.2.5. *Let μ be a fuzzy set of L and (F_q, A) a q-soft set over L with $A = (0, 1]$. Then the following are equivalent:*

(i) *μ is a fuzzy positive implicative filter of L;*
(ii) *$\forall t \in A, F_q(t) \neq \emptyset \Rightarrow F_q(t)$ is a positive implicative filter of L.*

Proof. It is similar to the proof of Theorem 4.1.5. □

Theorem 4.2.6. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0, 0.5]$. Then the following are equivalent:*

(i) *μ is an $(\in, \in \vee q)$-fuzzy positive implicative filter of L;*
(ii) *(F, A) is a positive implicative filteristic soft BL-algebra over L.*

Proof. It is similar to the proof of Theorem 4.1.6. □

Theorem 4.2.7. *Let μ be a fuzzy set of L and (F, A) be an $\in$-soft set over L with $A = (0.5, 1]$. Then the following are equivalent:*

(i) *μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy positive implicative filter of L;*
(ii) *(F, A) is a positive implicative filteristic soft BL-algebra over L.*

Proof. It is similar to the proof of Theorem 4.1.7. □

The following is a consequence of Theorems 3.7, 4.2.6 and 4.2.7.

Theorem 4.2.8. *Given $\alpha, \beta \in (0, 1]$ and $\alpha < \beta$, let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (\alpha, \beta]$. Then the following are equivalent:*

(i) *μ is a fuzzy positive implicative filter with thresholds $(\alpha, \beta]$ of L;*
(ii) *(F, A) is a positive implicative filteristic soft BL-algebra over L.*

Finally, we give the relation between positive implicative filteristic soft *BL*-algebras and implicative filteristic soft *BL*-algebras.

Theorem 4.2.9. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0, 1]((0, 0.5], [0.5, 1]$ and $(\alpha, \beta])$, respectively. If (F, A) is an implicative filteristic soft BL-algebra, then it is a positive implicative filteristic soft BL-algebra, but the converse may not be true.*

Proof. It is a consequence of Theorem 4.1.6, 4.1.7, 4.1.8, 4.2.6, 4.2.7, 4.2.8 and 4.5 in [26]. □

4.3. Fantastic filteristic soft BL-algebras

In this subsection, we describe fantastic filteristic soft *BL*-algebras.

Definition 4.3.1. Let (F, A) be a soft set over L. Then (F, A) is called a *fantastic filteristic soft BL-algebra over L* if $F(x)$ is a fantastic filter of L for all $x \in A$. For our convenience, the empty set $\emptyset$ is regarded as a fantastic filter of L.

Example 4.3.2. Let $L = \{0, a, b, 1\}$, where $0 < a < b < 1$. Then we define $x \wedge y = \min\{x, y\}$, $x \vee y = \max\{x, y\}$, and $\odot$ and $\rightarrow$ as follows:

$\odot$	0	a	b	1
0	0	0	0	0
a	0	0	0	a
b	0	0	a	b
1	0	a	b	1

$\rightarrow$	0	a	b	1
0	1	1	1	1
a	b	1	1	1
b	a	b	1	1
1	0	a	b	1

It is clear that $(L, \wedge, \vee, \odot, \rightarrow, 1)$ is now a *BL*-algebra. Let (F, A) be a soft set over L, where $A = (0, 1]$ and $F : A \rightarrow \mathcal{P}(L)$ is a set-valued function defined by

$$F(x) = \begin{cases} \{0, a, b, 1\} & \text{if } 0 < x \leq 0.4, \\ \{1\} & \text{if } 0.4 < x \leq 0.8, \\ \emptyset & \text{if } 0.8 < x \leq 1. \end{cases}$$

Thus, $F(x)$ is a fantastic filter of L for all $x \in A$, and so (F, A) is a fantastic filteristic soft *BL*-algebra over L.

From the above definitions, we can get the following:

Proposition 4.3.3. *Every fantastic filteristic BL-algebra is a filteristic BL-algebra, but the converse may not be true.*

Theorem 4.3.4. *Let μ be a fuzzy set of L and (F, A) be an $\in$-soft set over L with $A = (0, 1]$. Then (F, A) is a fantastic filteristic soft BL-algebra over L if and only if μ is a fuzzy fantastic filter of L.*

Proof. It is similar to the proof of Theorem 4.1.4. □

Theorem 4.3.5. *Let μ be a fuzzy set of L and (F_q, A) a q-soft set over L with $A = (0, 1]$. Then the following are equivalent:*

(i) *μ is a fuzzy fantastic filter of L;*
(ii) *$\forall\, t \in A, F_q(t) \neq \emptyset \Rightarrow F_q(t)$ is a fantastic filter of L.*

Proof. It is similar to the proof of Theorem 4.1.5. □

Theorem 4.3.6. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0, 0.5]$. Then the following are equivalent:*

(i) *μ is an $(\in, \in \vee q)$-fuzzy fantastic filter of L;*
(ii) *(F, A) is a fantastic filteristic soft BL-algebra over L.*

Proof. It is similar to the proof of Theorem 4.1.6. □

Theorem 4.3.7. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0.5, 1]$. Then the following are equivalent:*

(i) *μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy fantastic filter of L;*
(ii) *(F, A) is a fantastic filteristic soft BL-algebra over L.*

Proof. It is similar to the proof of Theorem 4.1.7. □

The following is a consequence of Theorems 3.7, 4.3.6 and 4.3.7.

Theorem 4.3.8. *Given $\alpha, \beta \in (0, 1]$ and $\alpha < \beta$, let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (\alpha, \beta]$. Then the following are equivalent:*

(i) *μ is a fuzzy fantastic filter with thresholds $(\alpha, \beta]$ of L;*
(ii) *(F, A) is a fantastic filteristic soft BL-algebra over L.*

Next, we give the relation between fantastic filteristic soft *BL*-algebras and implicative filteristic soft *BL*-algebras.

Theorem 4.3.9. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0, 1]((0, 0.5], [0.5, 1]$ and $(\alpha, \beta])$, respectively. If (F, A) is an implicative filteristic soft BL-algebra, then it is a fantastic filteristic soft BL-algebra, but the converse may not be true.*

Proof. It is a consequence of Theorem 4.1.6, 4.1.7, 4.1.8, 4.3.6, 4.3.7, 4.3.8 and 5.5 in [26]. □

Finally, we give the relation between positive implicative, implicative and fantastic filteristic soft *BL*-algebras.

Theorem 4.3.10. *Let μ be a fuzzy set of L and (F, A) an $\in$-soft set over L with $A = (0, 1]((0, 0.5], [0.5, 1]$ and $(\alpha, \beta])$, respectively. Then (F, A) is an implicative filteristic soft BL-algebra if and only if it is both a fantastic filteristic soft BL-algebra and a positive implicative filteristic soft BL-algebra.*

Proof. It is a consequence of Theorem 4.2.9, 4.3.9 and 5.6 in [26]. □

5. Conclusion

As a continuation of [26,27], we apply fuzzy and soft set theory to *BL*-algebras. We hope that the research along this direction can be continued, and in fact, some results in this paper have already constituted a platform for further discussion concerning the future development of soft *BL*-algebras and other algebraic structures.

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References

[1] D. Molodtsov, Soft set theory—First results, Comput. Math. Appl. 37 (1999) 19–31.
[2] P.K. Maji, A.R. Roy, R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44 (2002) 1077–1083.
[3] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[4] D. Chen, E.C.C. Tsang, D.S. Yeung, X. Wang, The parametrization reduction of soft sets and its applications, Comput. Math. Appl. 49 (2005) 757–763.
[5] L.A. Zadeh, From circuit theory to system theory, Proc. Inst. Radio Eng. 50 (1962) 856–865.
[6] L.A. Zadeh, Toward a generalized theory of uncertainty (GTU)-an outline, Inform. Sci. 172 (2005) 1–40.
[7] Y.B. Jun, Soft BCK/BCI-algebras, Comput. Math. Appl. 56 (2008) 1408–1413.
[8] Y.B. Jun, C.H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, Inform. Sci. 178 (2008) 2466–2475.
[9] H. Aktas, N. Cagman, Soft sets and soft groups, Inform. Sci. 177 (2007) 2726–2735.

[10] P. Hájek, Metamathematics of Fuzzy Logic, Kluwer Academic Press, Dordrecht, 1998.
[11] C.C. Chang, Algebraic analysis of many valued logics, Trans. Amer. Math. Soc. 88 (1958) 467–490.
[12] Y. Xu, Lattice implication algebras, J. Southeast Jiaotong Univ. 1 (1993) 20–27.
[13] Y. Xu, K.Y. Qin, On fuzzy filters of lattice implication algebras, J. Fuzzy Math. 1 (1993) 251–260.
[14] G.J. Wang, *MV*-algebras, *BL*-algebras, R_0-algebras and multiple-valued logic, Fuzzy Systems Math. 3 (2002) 1–5.
[15] G.J. Wang, Non-classical Mathematical Logic and Approximate Reasoning, Science Press, Beijing, 2000.
[16] G.J. Wang, On the logic foundation of fuzzy reasoning, Inform. Sci. 117 (1999) 47–88.
[17] A. Di Nola, G. Georgescu, L. Leustean, Boolean products of *BL*-algebras, J. Math. Anal. Appl. 251 (2000) 106–131.
[18] A. Di Nola, L. Leustean, Compact representations of *BL*-algebras, University Aarhuy, BRICS Report Series, 2002.
[19] L. Liu, K. Li, Fuzzy filters of *BL*-algebras, Inform. Sci. 173 (2005) 141–154.
[20] L. Liu, K. Li, Fuzzy boolean and positive implicative filters of *BL*-algebras, Fuzzy Sets Syst. 152 (2005) 333–348.
[21] E. Turunen, Boolean deductive systems of *BL*-algebras, Arch. Math. Logic 40 (2001) 467–473.
[22] X.H. Zhang, Y.B. Jun, M.I. Doh, On fuzzy filters and fuzzy ideals of *BL*-algebras, Fuzzy Systems Math. 3 (2006) 8–20.
[23] X.H. Zhang, K. Qin, W.A. Dudek, Ultra *LI*-ideals in lattice implication algebras and *MTL*-algebras, Czechoslovak Math. J. 57 (132) (2007) 591–605.
[24] P.M. Pu, Y.M. Liu, Fuzzy topology I, Neighborhood structure of a fuzzy point and Moore–Smith convergence, J. Math. Anal. Appl. 76 (1980) 571–599.
[25] S.K. Bhakat, P. Das, $(\in, \in \vee \mathrm{q})$-fuzzy subgroup, Fuzzy Sets and Systems 80 (1996) 359–368.
[26] X. Ma, J. Zhan, On $(\in, \in \vee q)$-fuzzy filters of *BL*-algebras, J. Syst. Sci. Complexity 21 (2008) 144–158.
[27] X. Ma, J. Zhan, W.A. Dudek, Some kinds of $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy filters of *BL*-algebras, Comput. Math. Appl. 58 (2009) 248–256.
[28] X. Ma, J. Zhan, Y. Xu, Generalized fuzzy filters of *BL*-algebras, Appl. Math. J. Chinese Univ. Ser. B 22 (2007) 490–496.
[29] J. Zhan, W.A. Dudek, Y.B. Jun, Interval valued $(\in, \in \vee q)$-fuzzy filters of pseudo *BL*-algebras, Soft Comput. 13 (2009) 13–21.
[30] J. Zhan, Y. Xu, Some types of generalized fuzzy filters of *BL*-algebras, Comput. Math. Appl. 56 (2008) 1604–1616.