Applied Mathematical Modelling 37 (2013) 398–405

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Cross-ranking of Decision Making Units in Data Envelopment Analysis

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ARTICLE INFO

Article history:
Received 30 March 2011
Received in revised form 17 February 2012
Accepted 29 February 2012
Available online 8 March 2012

Keywords:
Data Envelopment Analysis
Preference aggregation
Group decision making

ABSTRACT

Data Envelopment Analysis (DEA) is a mathematical model that evaluates the relative efficiency of Decision Making Units (DMUs) with multiple input and output. In some applications of DEA, ranking of the DMUs are important. For this purpose, a number of approaches have been introduced. Among them is the cross-efficiency method. The method utilizes the result of the cross-efficiency matrix and averages the cross-efficiency scores of each DMU. Ranking is then performed based on the average efficiency scores. In this paper, we proposed a new way of handling the information from the cross-efficiency matrix. Based on the notion that the ranking order is more important than individual efficiency score, the cross-efficiency matrix is converted to a *cross-ranking matrix*. A cross-ranking matrix is basically a cross-efficiency matrix with the efficiency score of each element being replaced with the ranking order of that efficiency score with respect to the other efficiency scores in a column. By so doing, each DMU assume the role of a decision maker and how they voted or ranked the other DMUs are reflected in their respective column of the cross-ranking matrix. These votes are then aggregated using a preference aggregation method to determine the overall ranking of the DMUs. Comparison with an existing cross-efficiency method indicates a relatively better result through usage of the proposed method.

1. Introduction

Data Envelopment Analysis (DEA) is a linear programming technique that was first introduced in Charnes et al. paper [1]. This technique evaluates the relative efficiency of Decision Making Units (DMUs) which contain some nonhomogeneous input and output. In the traditional DEA models like the CCR model [1] and the BCC model [2], efficiency scores of efficient DMUs are 1 and efficiency scores of inefficient DMUs are less than 1. A frequently discussed problem involving the said models is that some of the efficient DMUs cannot be discriminated. A number of approaches have been introduced to overcome this problem and to improve the discrimination power of DEA. Among them are the super-efficiency methods, the multi-criteria DEA methods and the cross-efficiency methods [3].

The super-efficiency method was introduced by Andersen and Petersen [4] and is known as the AP model. In this method, the traditional DEA models are reformulated by excluding the input and output of the DMU that is being ranked from the models. However, this exclusion might cause unfeasibility of the DEA models [4,5]. To overcome this situation, Saati et al. [6] modified the non-radial model presented in Mehrabian et al. [5] and converted it to an input–output orientation model

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0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2012.02.038

that caused the LP model to be always feasible. The infeasibility of super-efficiency DEA model in a variable return to scale environment was dealt with by Chen [7] in which both input-oriented and output-oriented super-efficiency DEA models were proposed to fully characterize super-efficiency.

One of the methods which used the multi-criteria decision-making (MCDM) models for ranking DMUs is the method proposed by Li and Reeves [8]. The method introduced a Multi-Criteria Data Envelopment Analysis (MCDEA) model with three objective functions in which the first objective is to obtain the optimal solution of the CCR or the BCC models. The other two objective functions are to minimize the maximum quantity among all deviation variables and to minimize the summation of deviations. Recently, Bal et al. [9] combined the coefficients of variation for the input–output weights to the objective function of the CCR model. However, as noted by Wang and Luo [10], Bal's proposed method has its shortcomings. For instance, since the input and output weights are derived from different dimensions and units, they cannot be simply added. Also, since the model is nonlinear, multiple local optimal solutions may be obtained. Furthermore, Bal et al. [11] converted the MCDEA model proposed by Li and Reeves [8] to a goal programming model in order to discriminate the efficient DMUs.

Cross-efficiency method, pioneered by Sexton et al. [12], evaluates the performance of a DMU by comparing it to the optimal input and output weights of other DMUs. Doyle and Green [13] developed a set of formulations in order to remedy the shortcoming of the Sexton' method which is the non-uniqueness of the factor weights obtained from the DEA models. These models, known as aggressive and benevolent methods, obtain the robust factor weights for use in the construction of the cross-efficiencies method. In the aggressive and benevolent formulations the cross-efficiencies of the other DMUs are minimized and maximized, respectively. Recently, Wang and Chin [14] proposed an alternative cross-efficiency model known as the neutral DEA model to obtain a different set of input and output weights from aggressive and benevolent formulations.

A number of other methods have been integrated with DEA models. Liang et al. [15] integrated game theory and cross-efficiency in order to rank the Decision Making Units in DEA. In the model, DMUs are considered as players. Before this, Sinuany-Stern et al. [16] proposed an approach based on the relationship between DEA and AHP (analytic hierarchy process) to rank DMUs. Recently, Wu [17] utilized the concept of fuzzy for ranking of DMUs in the traditional DEA models. For this purpose, firstly the DMUs are evaluated by the CCR and the cross-efficiency models. Secondly, a fuzzy preference relation is established. Finally, a priority vector of the preference is constructed and is used for ranking DMUs. Zerafat Angiz et al. [18] also utilized the fuzzy concept to completely rank efficient DMUs. On this aspect, this paper integrates the preference aggregation method with DEA to determine the ranking of DMUs.

Preference aggregation problem, in the context of a ranked voting system is a group decision making problem of selecting m alternatives from a set of n alternatives ($n > m$). Hence, each decision maker ranks the alternatives from the most preferred (rank = 1) to the least preferred (rank = n). Obviously, due to different opinions of the decision makers, each alternative may be placed in a different ranking position. Some studies suggest a simple aggregation method by finding the total score of each alternative as the weighted sum of the votes that each alternative received by different decision makers. In this method, the best alternative is the one with the largest total score. The key issue of the preference aggregation is how to determine the weights associated with different ranking positions. Perhaps, Borda–Kendall method [19] is the most commonly used approach for determining the weights due to its computational simplicity. Pioneered by Cook and Kress [20], a number of DEA-based preference aggregation methods have been proposed in recent years. Analysis of these methods and their drawbacks can be found in Llamazares and Peña [21]. Recently, Zerafat Angiz et al. [22,23] also proposed DEA-based methods for handling preference aggregation. The first work was on the use of multi-objective linear programming approach and the second was on the use of fuzzy numbers.

In some DEA applications, pursuing the best ranking is more important than maximizing the individual efficiency score. Wu et al. [24] incorporated this notion into their DEA model by proposing a mixed integer programming model with a secondary goal of ranking order minimization. In this paper, we incorporated this notion by converting the cross-efficiency matrix into a *cross-ranking matrix*. A cross-ranking matrix is basically a cross-efficiency matrix with the efficiency score of each element being replaced with the ranking order of that efficiency score with respect to the other efficiency scores in a column. By so doing, each DMU has now become a decision maker and how they voted or ranked the other DMUs are reflected in their respective column of the cross-ranking matrix. The aggregation of these votes is a preference aggregation problem and a modified Cook and Kress method is used to handle the situation.

The rest of this paper is organized in the following manner. Some mathematical models used in this paper are introduced in Section 2. The proposed method is presented in Section 3. Section 4 illustrates the new method and conclusions are given in Section 4.

2. Background

2.1. CCR model

In mathematical terms, consider a set of n DMUs, in which x_{ij} $(i = 1, 2, \ldots, m)$ and y_{rj} $(r = 1, 2, \ldots, s)$ are input and output of DMU_j $(j = 1, 2, \ldots, n)$. A standard DEA model for assessing DMU_p which is known as the CCR model [1], is formulated in Model (1).

$$\begin{aligned} \max \quad & z_p = \sum_{r=1}^{s} u_r y_{rp} \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{ip} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0 \\ & u_r,\ v_i \geqslant 0 \quad \forall r, i \end{aligned} \tag{1}$$

In Model (1), the optimal value (z_p^*) demonstrates the relative efficiency score associated with DMU_p which is under evaluation. v_i and u_r are the associated input and output weights. In this model DMU_p is efficient if $z_p^* = 1$.

2.2. Cross-efficiency evaluation

In the cross-efficiency method, the elements of the cross-efficiency matrix is the efficiency score of each DMU which is calculated using the weights obtained from the optimal solution of the DMU that is under evaluation. The results of all the DEA cross-efficiency scores, calculated by solving n linear programming problems corresponding to n DMUs under evaluation, can be described as follows:

$$z_{jp}^* = \frac{\sum_{r=1}^{s} u_{rp}^* y_{rj}}{\sum_{i=1}^{m} v_{ip}^* x_{ij}} \quad p,\ j = 1, 2, \ldots, n \tag{2}$$

where z_{jp}^* represents the efficiency score of DMU_j while DMU_p is being evaluated. u_{rp}^* and v_{ip}^* indicate the optimal weights evaluated by the linear programming problem associated with DMU_p. On the other hand, DMU_j is evaluated by the optimal weights of DMU_p.

To rank the DMUs using the cross-efficiency method, the average cross-efficiency score is calculated as:

$$\bar{z}_j = \frac{\sum_{p=1}^{n} z_{jp}^*}{n} \quad j = 1, 2, \ldots, n \tag{3}$$

There are some other scores that can be used for ranking DMUs, such as median and minimum of scores (see Doyle and Green [13]). One drawback of the above mentioned method is that the input and output weights are not unique and thus there are multiple optimal solutions for the traditional DEA models. To remedy this shortcoming, the following models which are known as the aggressive Model (4) and the benevolent Model (5) are often used:

$$\begin{aligned} \min \quad & \sum_{r=1}^{s} u_{rp} \left(\sum_{\substack{j=1 \\ j \neq p}}^{n} y_{rj} \right) \\ s.t. \quad & \sum_{i=1}^{m} v_{ip} \left(\sum_{\substack{j=1 \\ j \neq p}}^{n} x_{ij} \right) = 1 \\ & \sum_{r=1}^{s} u_{rp} y_{rp} - z_{pp}^* \sum_{i=1}^{m} v_{ip} x_{ip} = 0 \\ & \sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} \leqslant 0 \; j \neq p \\ & u_{rp},\ v_{ip} \geqslant 0 \quad \forall r, i \end{aligned} \tag{4}$$

$$\begin{aligned} \max \quad & \sum_{r=1}^{s} u_{rp} \left(\sum_{\substack{j=1 \\ j \neq p}}^{n} y_{rj} \right) \\ s.t. \quad & \sum_{i=1}^{m} v_{ip} \left(\sum_{\substack{j=1 \\ j \neq p}}^{n} x_{ij} \right) = 1 \\ & \sum_{r=1}^{s} u_{rp} y_{rp} - z_{pp}^* \sum_{i=1}^{m} v_{ip} x_{ip} = 0 \\ & \sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} \leqslant 0 \; j \neq p \\ & u_{rp},\ v_{ip} \geqslant 0 \quad \forall r, i \end{aligned} \tag{5}$$

where $z^*_{pp} = z^*_p = \sum_{r=1}^{s} u^*_{rp} y_{rp}$ is the efficiency score of CCR model related to DMU_p.

2.3. Cook and Kress method

Cook and Kress [20] proposed a method that is based on DEA to aggregate the votes from a preferential ballot. For this purpose, they used the following DEA Model (6) in which output are number of first place votes, second place votes and so on that a DMU obtained and a single input with value 1.

$$\begin{aligned} \max \quad & \beta_p = \sum_{k=1}^{n} \mu_k w_{pk} \\ s.t. \quad & \\ & \sum_{k=1}^{n} \mu_k w_{jk} \leqslant 1 \quad j = 1, 2, \ldots, m \\ & \mu_k - \mu_{k+1} \geqslant d(k, \varepsilon) \quad k = 1, 2, \ldots, n-1 \\ & \mu_n \geqslant d(n, \varepsilon) \end{aligned} \tag{6}$$

where w_{jk} is the number of rank k vote that DMU_j obtained and μ_k is the weight of rank k calculated by Model (6). It is clear that $\mu_k \geqslant \mu_{k+1}$, so the extra constraint $\mu_k - \mu_{k+1} \geqslant d(k, \varepsilon)$ indicates how much vote $k + 1$ is preferred to vote k. The notation $d(k, \varepsilon)$ is a function which is non-decreasing in ε and is referred to as a discrimination intensity function. Model (6) is solved for each candidate $j = 1, 2, \ldots, m$.

3. The proposed approach

The proposed method for ranking DMUs under the notion that pursuing the best ranking is more important than maximizing the individual efficiency score comprises six stages.

Stage 1. Determine the efficiency score of all DMUs using the CCR model, i.e. Model (1). Let z^*_{pp} be efficiency score of DMU_p.

Stage 2. Construct the cross-efficiency matrix $Z = (z^*_{jp})_{n \times n}$ using the aggressive or the benevolent model. The z^*_{pp} values that were determine in Stage 1 are used as the diagonal elements of the cross-efficiency matrix associated to DMU_p. The same z^*_{pp} values are also used in solving Model (5). Elements (j, p) of the matrix are the z^*_{jp} values calculated using formula (2).

Stage 3. Convert the cross-efficiency matrix that was constructed in Stage 2 into a cross-ranking matrix $R = (r_{jp})_{n \times n}$ where r_{jp} is the ranking order of z^*_{jp} in column p of matrix Z.

Stage 4. Construct the preference matrix $W = (w_{jk})_{n \times n}$ with reference to matrix R where w_{jk} is the number of time that DMU_j is placed in rank k.

Stage 5. Construct matrix $\Omega = (\hat{\theta}_{jk})_{n \times n}$ where $\hat{\theta}_{jk}$ is the summation of the efficiency scores in matrix Z which corresponds to DMU_j being placed in rank k.

Stage 6. Obtain a common set of weight for final ranking of DMUs using the following modified Cook and Kress [20] method:

$$\begin{aligned} \max \quad & \beta = \sum_{j=1}^{n} \frac{\sum_{k=1}^{n} \mu_k \hat{\theta}_{jk}}{\beta^*_j} \\ s.t. \quad & \\ & \sum_{k=1}^{n} \mu_k \hat{\theta}_{jk} \leqslant 1 \quad j = 1, 2, \ldots, n \\ & \mu_k - \mu_{k+1} \geqslant d(k, \varepsilon) \quad k = 1, 2, \ldots, n-1 \\ & \mu_n \geqslant d(n, \varepsilon) \end{aligned} \tag{7}$$

where $\hat{\theta}_{jk}$ is as defined in Stage 5. Note that we have used $\hat{\theta}_{jk}$ instead of w_{jk}. β^*_j is the optimal solution of Model (6) with w_{jk} being replaced by $\hat{\theta}_{jk}$. In case of discrimination, a straightforward linear programming problem presented in Model (7) is solved, since $\beta^*_j = 1$. The value of ε is obtained as indicated in Cook and Kress [20]. Finally, the DMUs are ranked based on their $z^*_j = \sum_{k=1}^{n} \mu^*_k \hat{\theta}_{jk}$ values.

4. Illustration with a numerical example

Example. A real data set taken from a previous study by Sherman and Gold [25] for comparing the efficiency of 14 bank branches is given in Table 1. The comparison is based on three input and four output as follows:

Input 1: Rent (thousands of dollars).
Input 2: Full time equivalent personnel.
Input 3: Supplies (thousands of dollars).

Table 1
Sherman and Gold data set on 14 bank branches.

DMU	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3	Output 4
1	140,000	42,900	87,500	484,000	4,139,100	59,860	2,951,430
2	48,800	17,400	37,900	384,000	1,685,500	139,780	3,336,860
3	36,600	14,200	29,800	209,000	1,058,900	65,720	3,570,050
4	47,100	9,300	26,800	157,000	879,400	27,340	2,081,350
5	32,600	4,600	19,600	46,000	370,900	18,920	1,069,100
6	50,800	8,300	18,900	272,000	667,400	34,750	2,660,040
7	40,800	7,500	20,400	53,000	465,700	20,240	1,800,250
8	31,900	9,200	21,400	250,000	642,700	43,280	2,296,740
9	36,400	76,000	21,000	407,000	647,700	32,360	1,981,930
10	25,700	7,900	19,000	72,000	402,500	19,930	2,284,910
11	44,500	8,700	21,700	105,000	482,400	49,320	2,245,160
12	42,300	8,900	25,800	94,000	511,000	26,950	2,303,000
13	40,600	5,500	19,400	84,000	287,400	34,940	1,141,750
14	76,100	11,900	32,800	199,000	694,600	67,160	3,338,390

Table 2
Cross-efficiency matrix Z.

DMU	DMU														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Ave
1	1.000	0.149	0.216	0.916	0.839	0.344	0.722	0.421	0.285	0.238	0.256	0.268	0.246	0.246	0.438
2	1.000	1.000	0.701	1.000	1.000	0.673	1.000	0.856	0.523	0.674	1.000	0.929	1.000	1.000	0.883
3	0.772	0.627	1.000	0.837	0.879	0.449	1.000	0.582	0.361	0.892	0.914	1.000	0.864	0.864	0.725
4	0.957	0.203	0.453	1.000	1.000	0.515	0.944	0.553	0.302	0.724	0.709	0.736	0.679	0.679	0.675
5	0.785	0.203	0.336	0.880	0.904	0.305	0.808	0.287	0.121	0.702	0.758	0.727	0.786	0.786	0.599
6	0.828	0.239	0.537	0.935	1.000	1.000	1.000	1.000	0.743	1.000	1.000	1.000	0.971	0.971	0.873
7	0.631	0.173	0.452	0.717	0.762	0.215	0.782	0.226	0.134	0.767	0.727	0.760	0.694	0.694	0.553
8	0.718	0.474	0.738	0.794	0.841	0.829	0.924	1.000	0.603	0.856	0.904	0.948	0.866	0.866	0.812
9	0.093	0.310	0.558	0.094	0.097	0.163	0.122	0.257	1.000	0.101	0.094	0.116	0.086	0.086	0.227
10	0.523	0.271	0.911	0.649	0.728	0.278	0.902	0.341	0.196	1.000	0.853	1.000	0.781	0.781	0.658
11	0.567	0.387	0.517	0.669	0.740	0.368	0.791	0.393	0.250	0.834	0.967	0.926	0.956	0.956	0.666
12	0.581	0.222	0.558	0.687	0.747	0.322	0.817	0.353	0.188	0.847	0.799	0.852	0.757	0.757	0.606
13	0.519	0.300	0.288	0.607	0.661	0.466	0.615	0.431	0.223	0.621	0.869	0.723	0.905	0.905	0.581
14	0.593	0.308	0.450	0.710	0.788	0.510	0.802	0.501	0.313	0.867	1.000	0.928	1.000	1.000	0.698

Table 3
Cross-ranking matrix R.

	DMU													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
DMU1	1	13	13	3	5	9	10	7	8	12	11	11	12	12
DMU2	1	1	4	1	1	3	1	2	4	10	1	3	1	1
DMU3	5	2	1	5	3	7	1	3	5	2	3	1	6	6
DMU4	2	11	8	1	1	4	2	4	7	8	10	8	11	11
DMU5	4	11	11	4	2	11	6	10	14	9	8	9	7	7
DMU6	3	9	6	2	1	1	1	1	2	1	1	1	2	2
DMU7	7	12	9	7	7	13	9	12	13	7	9	7	10	10
DMU8	6	3	3	6	4	2	3	1	3	5	4	2	4	5
DMU9	13	5	5	13	12	14	12	11	1	13	12	12	13	13
DMU10	11	8	2	11	10	12	4	9	11	1	6	1	8	8
DMU11	10	4	7	10	9	8	8	7	9	6	2	5	3	3
DMU12	9	10	5	9	8	10	5	8	12	4	7	6	9	9
DMU13	12	7	12	12	11	6	11	6	10	11	5	10	5	4
DMU14	8	6	10	8	6	5	7	5	6	3	1	4	1	1

Output 1: Loan applications, new pass-book loans, life insurance sales.

Output 2: New accounts, closed accounts.

Output 3: Travelers checks sold, bonds sold, bonds redeemed.

Output 4: Deposits, withdrawals, checks sold, treasury checks issued, B% checks, loan payments, pass-book loan payments, life insurance payments, mortgage payments.

Table 4
Preference matrix W – number of time that each DMU is placed in the different ranks.

	Rank													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
DMU1	1	0	1	0	1	0	1	1	1	1	2	3	2	0
DMU2	8	1	2	2	0	0	0	0	0	1	0	0	0	0
DMU3	3	2	3	0	3	2	1	0	0	0	0	0	0	0
DMU4	2	2	0	2	0	0	1	3	0	1	3	0	0	0
DMU5	0	1	0	2	0	1	2	1	2	1	3	0	0	1
DMU6	7	4	1	0	0	1	0	0	1	0	0	0	0	0
DMU7	0	0	0	0	0	0	5	0	3	2	0	2	2	0
DMU8	1	2	4	3	2	2	0	0	0	0	0	0	0	0
DMU9	1	0	0	0	2	0	0	0	0	0	1	4	5	1
DMU10	2	1	0	1	0	1	0	3	1	1	3	1	0	0
DMU11	0	1	2	1	1	1	2	2	2	2	0	0	0	0
DMU12	0	0	0	1	2	1	1	2	4	2	0	1	0	0
DMU13	0	0	0	1	2	2	1	0	0	2	3	3	0	0
DMU14	3	0	1	1	2	3	1	2	0	1	0	0	0	0

Table 5
Matrix Ω – summation of the efficiency scores in each rank place.

	Rank													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
DMU1	1	0.000	0.916	0.000	0.835	0.000	0.421	0.285	0.344	0.722	0.524	0.730	0.365	0.000
DMU2	8	0.856	1.602	1.224	0.000	0.000	0.000	0.000	0.000	0.674	0.000	0.000	0.000	0.000
DMU3	3	1.519	2.375	0.000	2.012	1.728	0.449	0.000	0.000	0.000	0.000	0.000	0.000	0.000
DMU4	2	1.901	0.000	1.068	0.000	0.000	0.302	1.917	0.000	0.709	1.561	0.000	0.000	0.000
DMU5	0	0.904	0.000	1.665	0.000	0.808	1.544	0.727	1.429	0.287	0.844	0.000	0.000	0.121
DMU6	7	3.522	0.827	0.000	0.000	0.537	0.000	0.000	0.239	0.000	0.000	0.000	0.000	0.000
DMU7	0	0.000	0.000	0.000	0.000	0.000	3.652	0.000	1.961	1.247	0.000	0.399	0.349	0.000
DMU8	1	1.777	2.739	1.708	1.722	1.512	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
DMU9	1	0.000	0.000	0.000	0.868	0.000	0.000	0.000	0.000	0.000	0.257	0.429	0.460	0.163
DMU10	2	0.911	0.000	0.902	0.000	0.853	0.000	1.833	0.341	0.728	1.368	0.278	0.000	0.000
DMU11	0	0.967	1.624	0.387	0.926	0.834	0.910	1.159	0.990	1.236	0.000	0.000	0.000	0.000
DMU12	0	0.000	0.000	0.874	1.375	0.799	0.799	1.100	2.782	0.544	0.000	0.188	0.000	0.000
DMU13	0	0.000	0.000	0.905	1.774	0.897	0.300	0.000	0.000	0.835	1.897	1.468	0.000	0.000
DMU14	3	0.000	0.867	0.928	1.011	1.409	0.802	1.303	0.000	0.45	0.000	0.000	0.000	0.000

Table 6
Results of the proposed model and the aggressive cross-efficiency model.

	Proposed model		Aggressive model	
	Efficiency	Rank	Efficiency	Rank
DMU1	0.221	12	0.438	13
DMU2	0.999	2	0.883	1
DMU3	0.662	3	0.725	4
DMU4	0.445	6	0.675	6
DMU5	0.245	10	0.599	10
DMU6	1.000	1	0.873	2
DMU7	0.142	14	0.553	12
DMU8	0.547	4	0.812	3
DMU9	0.145	13	0.227	14
DMU10	0.392	7	0.658	8
DMU11	0.305	8	0.668	7
DMU12	0.206	9	0.617	9
DMU13	0.176	11	0.588	11
DMU14	0.513	5	0.698	5

The use of the standard CCR model in Stage 1 resulted in 9 of the DMUs being efficient. The efficiency scores of these DMUs are the diagonal values of the cross-efficiency matrix $Z = (z^*_{jp})_{n\times n}$ in Table 2 which were constructed in Stage 2.

The cross-ranking matrix $R = (r_{jp})_{n\times n}$ that was constructed in Stage 3 by ranking the efficiency scores in each column of the cross-efficiency matrix is shown in Table 3. In the matrix, $r_{31} = 5$ means that DMU3 is ranked as number 5 when the optimal weights of DMU1 are used in determining the efficiency scores. Alternatively, the cross-ranking matrix can be viewed as a ranked voting situation where 14 decision makers voted for the 14 DMUs. Each column represents a particular decision maker's vote and $r_{31} = 5$ is now interpreted as decision maker 1 ranking DMU3 in the fifth place.

In Stage 4, a summary of the ranked voting outcomes in the cross-ranking matrix resulted in the preference matrix $W = (w_{jk})_{n \times n}$ depicted in Table 4. In the matrix, $w_{31} = 3$ means that DMU3 received three number 1 votes.

Matrix $\Omega = (\hat{\theta}_{jk})_{n \times n}$ that is constructed in Stage 5 is shown in Table 5 where $\hat{\theta}_{jk}$ is the summation of the efficiency scores in matrix Z which corresponds to DMU_j being placed in rank k. For example, $\hat{\theta}_{32} = 1.519$ means that the summation of the efficiency scores which corresponds to DMU3 being ranked as number 2 is 1.519. This value is obtained by first referring to $w_{32} = 2$ in matrix W which indicates that DMU3 received two number 2 votes. In row 3 of matrix R, rank 2 is found in columns 2 and 10. Finally, by referring to matrix Z, we obtained $z_{32} + z_{3,10} = 1.519$. The advantage of using $\hat{\theta}_{jk}$ instead of w_{jk} is that the discriminating power of the DEA model is improved.

In the final stage, Model (7) is solved and from the common set of weights that was obtained, the efficiency scores z_j^* of the DMUs and their rank are determined. These results together with the result that were obtained using the aggressive cross-efficiency model are shown in Table 6.

As expected, the result of the proposed method differs from that of the aggressive model due to the different approaches used while aggregating information from the cross-efficiency matrix. The proposed method emphasized on the ranking position of the DMUs, whereas the aggressive method relied on the efficiency score. For some pairs of DMUs, their ranking position seem to be swapped with each other, such as between DMU2 and DMU6 for the first and second ranking positions, between DMU3 and DMU8 for the third and fourth ranking positions and between DMU10 and DMU11 for the eighth and ninth ranking positions. In referring to matrix W in Table 4, DMU6 which obtained 7 first place votes and 4 second place votes is ranked better that DMU2 which obtained 8 first place votes and 1 second place vote. It seems that for DMU6, the gain of 3 second place votes out-weighed the loss of 1 first place vote. DMU3 and DMU10 clearly out-ranked DMU8 and DMU11, respectively, by getting 2 extra first place votes and the same number of second place vote.

5. Conclusion

Ranking of DMUs is a very important topic in DEA research. Many methods, each with its own strategy or logic in ranking the DMUs have been proposed. One of the popular methods is the cross-efficiency method. This paper proposed a new method of handling information from the cross-efficiency matrix. Instead of determining the average efficiency scores in each row and ranking the DMUs based on the scores, the efficiency scores in each column of the cross-efficiency are ranked and this resulted in a cross-ranking matrix. The cross-ranking matrix is then treated as a ranked voting situation and is handled as a preference aggregation problem in determining the final ranking of the DMUs. Basically, ranking of DMUs are done not only in the final step, but also in the intermediate steps. The extra emphasis that is placed on the ranking order makes this method suitable for use in certain situations such as in the assessment of R&D project proposal (see for example [26]) and material selection (see for example [27]) whereby the ranking order of DMUs are more important than individual efficiency score.

The proposed method combines the cross-efficiency method with the Cook and Kress method. Since both of these methods are known to be strong at discriminating the DMUs, this special characteristic is also retained by the proposed method.

The idea of converting ordinary data into ordinal data, as proposed in this paper, can be extended to other areas of DEA research such as in multi-period analysis, which is commonly dealt with using DEA window analysis (see for example [28]). Besides that, the idea can also be used for the evaluation of alternatives in MCDM (Multiple Criteria Decision Making).

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