$B_s - \bar{B}_s$ mixing in $Z'$ models with flavor-changing neutral currents

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Abstract

In models with an extra $U(1)'$ gauge boson family non-universal couplings to the weak eigenstates of the standard model fermions generally induce flavor-changing neutral currents. This phenomenon leads to interesting results in various $B$ meson decays, for which recent data indicate hints of new physics involving significant contributions from $b \to s$ transitions. We analyze the $B_s$ system, emphasizing the effects of a $Z'$ on the mass difference and $CP$ asymmetries.

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1. Introduction

The study of $B$ physics and the associated $CP$-violating observables has been suggested as a good means to extract information on new physics at low energy scales [1–7]. Since $B - \bar{B}$ mixing is a loop-mediated process within the standard model (SM), it offers an opportunity to see the footprints of physics beyond the SM. The currently observed $\Delta M_d = 0.489 \pm 0.008$ ps$^{-1}$ [8] and its mixing phase $\sin 2\beta = 0.736 \pm 0.049$ extracted from the $B_d \to J/\psi K_S$ mode [9] agree well with constraints obtained from other experiments [10]. However, no such information other than a lower bound $\Delta M_s > 14.5$ ps$^{-1}$ [11] is available for the $B_s$ meson yet.

Based upon SM predictions, $\Delta M_{B_s}$ is expected to be about 18 ps$^{-1}$ and its mixing phase $\phi_s$ only a couple of degrees. In contrast to the $B_d$ system, the more than 25 times larger oscillation frequency and a factor of four lower hadronization rate from $b$ quarks pose the primary challenges in the study of $B_s$ oscillation and $CP$ asymmetries. Since the $B_s \to J/\psi \phi$ decay is dominated by a Cabibbo–Kobayashi–Maskawa (CKM) favored tree-

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level process, \( b \rightarrow c \bar{c} \tau \), that does not involve a different weak phase in the SM, its asymmetry provides the most reliable information about the mixing phase \( \phi_s \).

Although new physics contributions may not compete with the SM processes in most of the \( b \rightarrow c \bar{c} s \) decays, they can play an important role in \( B_s \bar{B}_s \) mixing because of its loop nature in the SM [12]. In particular, the mixing can be significantly modified in models in which a tree-level \( b \rightarrow s \) transition is present. Thus, measurement of the properties of \( B_s \) meson mixing is of high interest in future \( B \) physics studies as a means to reveal new physics [13,14]. Since the current \( B \) factories do not run at the \( \Upsilon(5S) \) resonance to produce \( B_s \) mesons, it is one of the primary objectives of hadronic colliders to study \( B_s \) oscillation and decay in the coming years [15,16].

Flavor changing neutral currents (FCNC) coupled to an extra \( U(1)' \) gauge boson arise when the \( Z' \) couplings to physical fermion eigenstates are non-diagonal. One way for this to happen is by the introduction of exotic fermions with different \( U(1)' \) charges that mix with the SM fermions [17–21] as occurs in \( E_6 \) models. In the \( E_6 \) case, mixing of the right-handed ordinary and exotic quarks, all \( SU(2)_L \) singlets, induces FCNC mediated by a heavy \( Z' \) or by (small) \( Z–Z' \) mixing, so the quark mixing can be large. Mixing between ordinary (doublet) and exotic (singlet) left-handed quarks induces FCNC mediated by the SM \( Z \) boson [21]. We will also allow for this possibility, but in this case the quark mixing must be very small.

Another possibility involves family non-universal couplings. It is well-known that string models naturally give extra \( U(1)' \) groups, at least one of which has family non-universal couplings to the SM fermions [22–25]. Generically, the physical and gauge eigenstates do not coincide. Here, unlike the above-mentioned \( E_6 \) case, off-diagonal couplings of fermions to the \( Z' \) boson cannot be obtained without mixing with additional fermion states. In these types of models, both left-handed and right-handed fermions can have family non-diagonal couplings with the \( Z' \), while couplings to the \( Z \) are family diagonal (up to small effects from \( Z–Z' \) mixing).

The \( Z' \) contributions to \( B_s \bar{B}_s \) mixing are related to those for hadronic, semileptonic, and leptonic \( B \) decays in specific models in which the diagonal \( Z' \) couplings to \( q \bar{q}, \ell^+ \ell^-, \) etc., are known, but are independent in general.\(^1\) We have found that in specific models, \( B_s \bar{B}_s \) mixing effects can be significant while being consistent with the other constraints; these results will be presented elsewhere. In the present Letter, we will treat the mixing in a model-independent way.

Recently, we have studied the implications of a sizeable off-diagonal \( Z' \) coupling between the bottom and strange quark in the indirect CP asymmetry of \( B \rightarrow \phi K_S \) decay [28], which appears to show a significant deviation from the SM prediction [5,6,29,30]. Here we extend our analysis to \( B_s \bar{B}_s \) mixing where the \( Z' \) contributions also enter at the tree level. Applications to the \( B \rightarrow \pi K \) anomaly are under investigation [31].

The Letter is organized as follows. In Section 2, we review the basic formalism of \( B_s \bar{B}_s \) mixing. In Section 3, we evaluate \( \Delta M_s \) in the SM. In Section 4, we include the \( Z' \) contributions, allowing both left-handed and right-handed couplings in the mixing, and study their effects on observables. Our main results are summarized in Section 5.

### 2. \( B_s \bar{B}_s \) mixing

In the conventional decomposition of the heavy and light eigenstates

\[
|B_s\rangle_L = p|B_s^0\rangle + q|\bar{B}_s^0\rangle, \quad |B_s\rangle_H = p|B_s^0\rangle - q|\bar{B}_s^0\rangle,
\]

the mixing factor

\[
\left(\frac{q}{p}\right)_{\text{SM}} \approx \sqrt{\frac{M^{SM}_{12}}{M^{SM}_{12}}},
\]

\(^1\) \( B_s \bar{B}_s \) mixing in leptophobic \( E_6 \) models was considered in Ref. [21]. A model with non-universal right-handed couplings was discussed in [26]. Mechanisms for flavor change in dynamical symmetry breaking models are described in [27].
has a phase

\[ \phi_{s}^{SM} = 2 \arg(V_{tb} V_{ts}^{*}) = -2 \lambda^{2} \tilde{\eta} \simeq -2^\circ, \quad \sin 2 \phi_{s}^{SM} \simeq -0.07, \]  

(3)

where the theoretical expectation \( \Gamma_{12}^{SM} \ll M_{12}^{SM} \) is used. The approximate formula Eq. (2) receives a small correction once \( \Gamma_{12}^{SM} \) is included. Model independently, this only shifts \( \phi_{s} \) at the few percent level. With errors on \( \lambda \) and \( \tilde{\eta} \) included, we have the SM expectation that \( \sin 2 \phi_{s}^{SM} \simeq -0.07 \pm 0.01 \).

The off-diagonal element of the decay matrix, \( \Gamma_{12}^{SM} \), is evaluated by considering decay channels that are common to both \( B_{s} \) and \( \bar{B}_{s} \) mesons, and \( M_{12} \) is the off-diagonal element of the mass matrix. Due to the CKM enhancement, \( \Gamma_{12}^{SM} \) is dominated by the charm-quark contributions over the up-quark contribution in a box diagram. Unlike the Kaon system, \( \Gamma_{12}^{SM} \) is much smaller than \( M_{12}^{SM} \) for \( B \) mesons because the former is related to the \( B \) meson decays and set by the scale of its mass, whereas the latter is proportional to \( m_{s}^{2} \). We can safely assume that \( \Gamma_{12} \) is not significantly modified by new physics because \( \Gamma_{12} \) receives major contributions from CKM favored \( b \rightarrow c \bar{c} s \) decays in the SM, and the SM result \( \Gamma_{12} \ll M_{12} \) is unlikely to change.

The mass difference of the two physical states is

\[ \Delta M_{s} \equiv M_{H} - M_{L} \simeq 2|M_{12}|. \]  

(4)

The width difference is

\[ \Delta \Gamma \equiv \Gamma_{H} - \Gamma_{L} = \frac{2 \text{Re}(M_{12}^{*} \Gamma_{12})}{|M_{12}|} = 2|M_{12}| \cos \theta, \]  

(5)

where the relative phase is \( \theta = \arg(M_{12}/\Gamma_{12}) \). Since \( \Gamma_{12} \) is dominated by the contributions from CKM favored \( b \rightarrow c \bar{c} s \) decays, we have \( \theta = \arg(-V_{tb} V_{ts}^{*} / V_{cb} V_{cs}^{*}) \simeq \pi \) [32], and thus \( \Delta \Gamma \simeq -2|\Gamma_{12}| \) is negative in the SM. Although \( \Gamma_{12} \) is unlikely to be affected by new physics, the width difference always increases as long as the weak phase of \( M_{12} \) gets modified [33].

The observability of \( B_{s} - \bar{B}_{s} \) oscillations is often indicated by the parameter

\[ x_{s} \equiv \frac{\Delta M_{s}}{\Gamma_{s}}, \]  

(6)

where \( \Gamma_{s} = (4.51 \pm 0.18) \times 10^{-13} \) GeV, converted from the world average lifetime \( \tau_{s} = 1.461 \pm 0.057 \) ps [8]. The expected large value of \( x_{s} \) is a challenge for experimental searches. Currently, the result from all ALEPH [34], CDF [35], DELPHI [36], OPAL [37], and SLD [38] studies of \( \Delta M_{s} \) with a combined 95% confidence level (CL) sensitivity on \( \Delta M_{s} \) of 18.3 ps\(^{-1}\) gives [11]

\[ \Delta M_{s} > 14.5 \text{ ps}^{-1} \quad \text{and} \quad x_{s} > 20.8. \]  

(7)

It is also measured that \( m_{B_{s}} = 5369.6 \pm 2.4 \) MeV [8] and \( \Delta \Gamma_{s}/\Gamma_{s} = -0.16^{+0.15}_{-0.16} (|\Delta \Gamma_{s}|/\Gamma_{s} < 0.54) \) (with the 95% CL upper bound given in parentheses [11]) consistent with recent next-to-leading-order (NLO) QCD estimates [39]. In comparison, the \( B_{d} \) system has \( m_{B_{d}} = 5279.4 \pm 0.5 \) MeV, \( \Delta M_{d} = (0.489 \pm 0.008) \) ps\(^{-1}\), \( x_{d} = 0.755 \pm 0.015 \), and \( \tau_{B_{d}} = 1.542 \pm 0.076 \) ps [8].

3. \( \Delta M_{s} \) in the SM

The \( |\Delta B| = 2 \) and \( |\Delta S| = 2 \) operators relevant for our discussions are:

\[ O_{LL}^{L} = \left[ \hat{s} \gamma_{\mu} (1 - \gamma_{5}) b \right] \left[ \hat{s} \gamma_{\mu} (1 - \gamma_{5}) b \right], \quad O_{LR}^{L} = \left[ \hat{s} \gamma_{\mu} (1 - \gamma_{5}) b \right] \left[ \hat{s} \gamma_{\mu} (1 + \gamma_{5}) b \right], \]

\[ O_{2L}^{R} = \left[ \hat{s} (1 - \gamma_{5}) b \right] \left[ \hat{s} (1 + \gamma_{5}) b \right], \quad O_{RR}^{R} = \left[ \hat{s} \gamma_{\mu} (1 + \gamma_{5}) b \right] \left[ \hat{s} \gamma_{\mu} (1 + \gamma_{5}) b \right]. \]  

(8)
Because of the $V-A$ structure, only the operator $O^{LL}_{12}$ contributes to $B_s-\bar{B}_s$ mixing in the SM. The other three operators appear in the $Z'$ models because of the right-handed couplings and operator mixing through renormalization, as considered in the next section.

In the SM the contributions to
\[ M^{\text{SM}}_{12} \approx \frac{1}{2m_{B_s}} \langle 0^+ \rangle^\dagger \left( B_s | \Lambda_{\text{eff}} | 0^+ \right) \]
are dominated by the top quark loop. The result, accurate to NLO in QCD, is given by [40]
\[ M^{\text{SM}}_{12} = \frac{G_F^2}{12\pi^2} m_{B_s} f_{B_s}^2 (V_{tb} V_{ts}^\ast)^2 \eta_{2B} S_0(x_t) [\alpha_s(m_b)]^{-6/23} \left[ 1 + \frac{\alpha_s(m_b)}{4\pi} J_5 \right] B^{LL}(m_b), \]
where $x_t = (m_t/m_W)^2$ and
\[ S_0(x) = \frac{4x - 11x^2 + x^3}{4(1 - x)^2} - \frac{3x^3 \ln x}{2(1 - x)^3}. \]
Using $m_t(m_t) = 170 \pm 5$ GeV, we find $S_0(x_t) = 2.463$. The NLO short-distance QCD corrections are encoded in the parameters $\eta_{2B} \simeq 0.551$ and $J_5 \simeq 1.627$ [40]. The bag parameter $B^{LL}(\mu)$ is defined through the relation
\[ \langle \bar{B}_s | O^{LL} | B_s \rangle \equiv \frac{8}{3} m_{B_s}^2 f_{B_s}^2 B^{LL}(\mu). \]

In the following numerical analysis, we will use $G_F = 1.16639 \times 10^{-5}$ GeV$^{-2}$ and $M_W = 80.423 \pm 0.039$ GeV [8], and write the SM part of $\Delta M_s$ as
\[ \Delta M^2_s = 1.19 \left| \frac{V_{tb} V_{ts}^\ast}{0.04} \right|^2 \left( \frac{f_{B_s}}{23 \text{ MeV}} \right)^2 \left( \frac{B^{LL}(m_b)}{0.872} \right) \times 10^{-11} \text{ GeV}. \]

Current lattice calculations still show quite large errors on the hadronic parameters $f_{B_s} = 230 \pm 30$ MeV and $B^{LL}(m_b) = 0.872 \pm 0.005$ [41–43]. However, the ratio
\[ \xi \equiv \frac{f_{B_s}}{\sqrt{\langle \bar{B}_s | O^{LL} | B_s \rangle}} \]
can be determined with a much smaller theoretical error, where $\bar{B}_s$ is the renormalization-independent bag parameter for the $B_q$ meson ($q = d, s$). Therefore, the error on $\Delta M_s$ within the SM can be evaluated by comparing with $\Delta M_d$, i.e.,
\[ \Delta M^2_s = \Delta M^2_d \xi \frac{m_{B_s}}{m_{B_d}} \left( \frac{1 - \lambda^2}{m_{B_d}} \right)^2 \frac{\lambda^2 [1 - \tilde{\rho}^2 + \tilde{\eta}^2]}{\sqrt{\langle \bar{B}_s | O^{LL} | B_s \rangle}}. \]
Using the measured values of the Wolfenstein parameters [44] $\lambda = 0.2265 \pm 0.0024$, $\tilde{\rho} = 0.801 \pm 0.025$, $\tilde{\eta} = 0.189 \pm 0.079$, and $\bar{\lambda} = 0.358 \pm 0.044$ [10], $\xi = 1.24 \pm 0.07$ [45], and the mass parameters quoted above, we obtain the SM predictions
\[ \Delta M^2_s = (1.19 \pm 0.24) \times 10^{-11} \text{ GeV} = 18.0 \pm 3.7 \text{ ps}^{-1}, \quad x_s^\ast = 26.3 \pm 5.5. \]
As noted above, the central value of $x_s$ is slightly larger than the current sensitivity based upon the world average. Recent LHC studies show that with one year of data, $\Delta M_s$ can be explored up to 30 ps$^{-1}$ (ATLAS), 26 ps$^{-1}$ (CMS), and 48 ps$^{-1}$ (LHCb) (corresponding to $x_s$ up to 46, 42, and 75); the LHCb result is based on exclusive hadronic decay modes [16]. The sensitivity of both CDF and BTeV on $x_s$ can also reach up to 75 using the same modes [15], for a luminosity of 2 fb$^{-1}$. The sensitivity on $\sin 2\phi_s$ is correlated with the value of $x_s$, and it becomes worse as $x_s$ increases. A statistical error of a few times $10^{-2}$ can be reached at CMS and LHCb for moderate $x_s \simeq 40$ [16].
4. Z’ contributions

For simplicity, we assume that there is no mixing between the SM Z and the Z’ (small mixing effects can be easily incorporated [17]). A purely left-handed off-diagonal Z’ coupling to b and s quarks results in an effective $|\Delta B| = 2, |\Delta S| = 2$ Hamiltonian at the $M_W$ scale of

$$\mathcal{H}_{\text{eff}}^{Z'} = \frac{G_F}{\sqrt{2}} \left( \frac{g_2 M_Z}{g_1 M_{Z'}} B_{tb} \right)^2 \rho \gamma \approx 2^{\frac{3}{2}} \rho_0 \theta_{sL},$$

where $g_2$ is the $U(1)'$ gauge coupling, $g_1 = e/(\sin \theta_W \cos \theta_W)$, $M_{Z'}$ is the mass of the Z’, and $B_{tb}$ is the FCNC Z’ coupling to the bottom and strange quarks. The parameters $\rho_0$ and the weak phase $\theta_{sL}$ in the Z’ model are defined by the second equality. Generically, we expect that $g_2/g_1 \sim 1$ if both $U(1)$ groups have the same origin from some grand unified theory, and $M_{Z}/M_{Z'} \sim 0.1$ for a TeV-scale Z’. If $|B_{tb}| \sim |V_{tb}V_{tb}^*|$, then an order-of-magnitude estimate gives us $\rho_0 \sim O(10^{-3})$, which is in the ballpark of giving significant contributions to the $B_s^\pm - \bar{B}_s$ mixing. The Z’ does not contribute to $\Gamma_{Z'}$ at tree level because the intermediate Z’ cannot be on shell. After evolving from the $M_W$ scale to $m_b$, the effective Hamiltonian becomes [40]

$$\mathcal{H}_{\text{eff}}^{Z'} = \frac{G_F}{\sqrt{2}} \left[ 1 + \frac{\alpha_s(m_b) - \alpha_s(M_W)}{4\pi} J_s \right] R^{b/2} \rho_0^2 \theta_{sL}^2 \rho_0 \gamma \approx 2^{\frac{3}{2}} \rho_0 \theta_{sL} \rho_0 \gamma \approx 2^{\frac{3}{2}} \rho_0 \theta_{sL} \rho_0 \gamma,$$

where $R = \alpha_s(M_W)/\alpha_s(m_b)$. Although the above effective Hamiltonian is largely suppressed by the ratio $(g_2 M_Z)/(g_1 M_{Z'})$, it contains only one power of $G_F$ in comparison with the corresponding quadratic dependence in the SM because the Z’-mediated process occurs at tree level.

The full description of the running of the Wilson coefficient from the $M_W$ scale to $m_b$ can be found in [40]. We only repeat the directly relevant steps here. The renormalization group equation for the Wilson coefficients $\tilde{C}$,

$$\frac{d}{d \ln \mu} \tilde{C} = \gamma^T(g) \tilde{C}(\mu),$$

can be solved with the help of the $U$ matrix

$$\tilde{C}(\mu) = U(\mu, M_W) \tilde{C}(M_W),$$

in which $\gamma^T(g)$ is the transpose of the anomalous dimension matrix $\gamma(g)$. With the help of $d g/d \ln \mu = \beta(g)$, $U$ obeys the same equation as $\tilde{C}(\mu)$. We expand $\gamma(g)$ to the first two terms in the perturbative expansion,

$$\gamma(\alpha_s) = \gamma^{(0)}(\alpha_s) + \gamma^{(1)}(\alpha_s)^2. \quad \text{(21)}$$

To order this evolution matrix $U(\mu, m)$ is given by

$$U(\mu, m) = \left( 1 + \frac{\alpha_s(\mu)}{4\pi} J \right) U(0)(\mu, m) \left( 1 - \frac{\alpha_s(m)}{4\pi} J \right),$$

where $U(0)$ is the evolution matrix in leading logarithmic approximation and the matrix $J$ expresses the next-to-leading corrections. We have

$$U^{(0)}(\mu, m) = V \left( \begin{array}{cc} \alpha_s(\mu) & \tilde{\gamma}^{(0)/2} \\ \alpha_s(\mu) & \tilde{\gamma}^{(0)/2} \end{array} \right),$$

where $V$ diagonalizes $\gamma^{(0)T}$, i.e., $\gamma^{(0)} = V^{-1} \gamma^{(0)T} V$, and $\tilde{\gamma}^{(0)}$ is the vector containing the diagonal elements of the diagonal matrix $\gamma^{(0)}$. In terms of $G = V^{-1} \gamma^{(1)T} V$ and a matrix $H$ whose elements are

$$H_{ij} = \delta_{ij} \gamma^{(0)} + \frac{\beta_i}{2\beta_0} \frac{G_{ij}}{2\beta_0 + \gamma^{(0)} - \gamma^{(0)}},$$

where $\beta_i = \frac{d}{d \ln \mu} \alpha_s(\mu)$, we have

$$U^{(0)}(\mu, m) = V \left( \begin{array}{cc} \alpha_s(\mu) & \tilde{\gamma}^{(0)/2} \\ \alpha_s(\mu) & \tilde{\gamma}^{(0)/2} \end{array} \right) V^{-1}.$$
Fig. 1. Three-dimensional plot of $x_s$ (a) and $\sin 2\phi_s$ (b) versus $\rho_L$ and $\phi_L$ with a $Z'$-mediated FCNC for left-handed $b$ and $s$ quarks. The color shadings in both plots have no specific physical meaning.

the matrix $J$ is given by $J = VH V^{-1}$.

The operators $O^{LL}$ and $O^{RR}$ do not mix with others under renormalization. Their Wilson coefficients follow exactly the same RGEs, where the above-mentioned matrices are all simple numbers. The factor

$$\left[ 1 + \frac{\alpha_s(m_b) - \alpha_s(M_W)}{4\pi} J_5 \right] R^{6/23}$$

in Eq. (18) reflects the RGE running. On the other hand, $O_1^{LR}$ and $O_2^{LR}$ form a sector that is mixed under RG running. Although the $Z'$ boson only induces the operator $O_1^{LR}$ at high energy scales, $O_2^{LR}$ is generated after evolution down to low energy scales and, in particular, its Wilson coefficient $C_2^{LR}$ is strongly enhanced by the RG effects [46].

With contributions from both the SM and the $Z'$ boson with only left-handed FCNC couplings included, the $B_s$ mass difference is

$$\Delta M_s = \Delta M_s^{SM} \left( 1 + \frac{\Delta M_s^{Z'}}{\Delta M_s^{SM}} \right) = 18.0 \left| 1 + 3.858 \times 10^5 \rho_L^2 e^{2i\phi_L} \right| \text{ps}^{-1}.$$  (26)

The corresponding result for the oscillation parameter is

$$x_s = 26.3 \left| 1 + 3.858 \times 10^5 \rho_L^2 e^{2i\phi_L} \right|.$$  (27)

With couplings of only one chirality, the physical observables $\Delta M_s$, $x_s$, and $\sin 2\phi_s$ are periodic functions of the new weak phase $\phi_L$ with a period of 180°.

Fig. 1(a) shows the effects of including a $Z'$ with left-handed coupling. We see that if $\rho_L$ is small, $x_s$ is dominated by the SM contribution and has a value $\sim 26$. For $\phi_L$ around 90° and $\rho_L$ between 0.001 and 0.002, the $Z'$ contribution tends to cancel that of the SM and reduces $x_s$ to be smaller than the SM value of 26.3. In Eq. (27) and Fig. 1(a), we see that the $Z'$ has a comparable contribution to the SM if $\rho_L \gtrsim 0.002$, independent of the actual value of $\phi_L$. The planned resolution of Fermilab run II and LHCb are both about $x_s \lesssim 75$ [15,16]. Thus, a $\rho_L$ greater
than about 0.003 will result in an $x_s$ beyond the planned sensitivity. If $x_s$ is measured to fall within a range, one can read from the plot what the allowed region is for the chiral $Z'$-model parameters. The same discussion can easily be applied to a $Z'$ model with only right-handed couplings. Fig. 1(b) shows $\sin 2\phi_s$ as a function of $\rho_L$ and $\phi_L$. As $\rho_L$ increases, $\sin 2\phi_s$ goes through more oscillations when $\phi_L$ varies from 0 to $\pi$.

In Fig. 2, we show the overlayed plot of the contours of fixed $x_s$ and those of fixed $\sin 2\phi_s$. The shaded region in the center shows the experimentally excluded points in the $\phi_L$–$\rho_L$ plane that induce $x_s$ values smaller than 20.6. Contours for higher values of $x_s$ are also shown. The SM predicted $\sin 2\phi_s \simeq -0.07 \pm 0.01$ would appear as narrow bands around the $\sin 2\phi_s = -0.07$ curves. Note that even if the $x_s$ measurement turns out to be consistent with the SM expectation, it is still possible that the new physics contributions, such as the $Z'$ model considered here, can alter the $\sin 2\phi_s$ value significantly. It is therefore important to have a clean determination of both quantities simultaneously. Once $x_s$ and $\sin 2\phi_s$ are extracted from $B_s$ decays, one can determine $\rho_L$ up to a two-fold ambiguity and $\phi_L$ up to a four-fold ambiguity, except for the special case with $\sin 2\phi_s \simeq 0$.

$\Delta \Gamma_s$ can be determined with a high sensitivity by measuring the lifetime difference between decays into $CP$-specific states and into flavor-specific states. Using the $J/\psi \phi$ mode, simulations determine [16] that the LHC can measure the ratio $\Delta \Gamma_s/\Gamma_s$ with a relative error $\lesssim 10\%$ for an actual value around $0.15$. Tevatron simulations show that $\Delta \Gamma_s/\Gamma_s$ can be measured with a statistical error of $\sim 0.02$. For a sufficiently large $\rho_L$, the $\cos \theta$ factor in Eq. (5) increases from $-1$ at $\phi_L = 0^\circ$ (mod 180$^\circ$) to the maximum 1 at $\phi_L = 90^\circ$ (mod 180$^\circ$). We are left with the phase ranges $0^\circ \lesssim \phi_L \lesssim 30^\circ$, $60^\circ \lesssim \phi_L \lesssim 120^\circ$, and $150^\circ \lesssim \phi_L \lesssim 180^\circ$ (mod 180$^\circ$) where a $3\sigma$ determination of $\Delta \Gamma_s$ can be made.

Once the right-handed $Z'$ couplings are introduced, we also have to include the new $|\Delta B| = 2$ operators $O_1^{LR}$, $O_2^{LR}$, and $O_3^{RR}$ defined in Eq. (8) into the effective Hamiltonian that contributes to the $B_s$–$\bar{B}_s$ mixing. The matrix element of $O_3^{RR}$ is the same as that of $O_2^{LL}$, while those of $O_1^{LR}$ and $O_2^{LR}$ are

$$
\langle \bar{B}_s | O_1^{LR} | B_s \rangle = -4 \left( \frac{m_{B_s}(m_b) + m_{\phi}(m_b)}{m_{B_s}(m_b) + m_{\phi}(m_b)} \right) \frac{m_{B_s}^2 f_{B_1}^2 B_1^{LR}(m_b)}{m_{B_1}^2 f_{B_1}^2 B_1^{LR}(m_b)},
$$

(28)
\[ \langle \bar{B}_s | O_{1,2}^L | B_s \rangle = 2 \left( \frac{m_{B_s}}{m_{b}(m_b) + m_s(m_b)} \right)^2 m_{B_s}^2 f_{B_s}^2 B_{2}^{LR}(m_b), \]  

(29)

For the \( Z' \) coupling to right-handed currents, we define new parameters \( \rho_R \) and the associated weak phase \( \phi_R \):

\[ \rho_R e^{i\phi_R} = \frac{g_2 M_Z}{g_1 M_{Z'}} B_{1}^{LR}. \]  

(30)

At the \( M_W \) scale, we have additional contributions to the effective Hamiltonian due to the right-handed currents, similar to Eq. (17). The terms due to the left–right mixing are

\[ \mathcal{H}_{\text{eff}}^{Z'} \supset \frac{G_F}{\sqrt{2}} \rho_L \rho_R e^{i(\phi_L + \phi_R)} ( O_{1}^{LR} + O_{1}^{RL}, O_{2}^{LR} + O_{2}^{RL} ) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]  

(31)

In the RGE running, the Wilson coefficient of \( O_{1}^{LR} \) mixes with that of \( O_{2}^{LR} \); the relevant anomalous dimension matrices are [46]

\[ \gamma^{(0)} = \begin{pmatrix} \frac{2}{5} & 12 \\ 0 & -6N_c + \frac{4}{3}N_f \end{pmatrix}, \]  

(32)

\[ \gamma^{(1)} = \begin{pmatrix} \frac{17}{6} + \frac{15}{2N_f} - \frac{2N_f - 6}{12} & -\frac{102}{15}N_f - \frac{8}{3}f - 44 \frac{1}{3}f \\ \frac{71}{4} + \frac{9}{N_c} - 2f & -\frac{203}{6}N_f^2 - \frac{479}{6} + \frac{15}{N_c} + \frac{10}{N_c}f - \frac{22}{3N_f}f \end{pmatrix} \]  

(33)

where \( N_c \) is the number of colors and \( f \) is the number of active quarks. At the scale of the \( B \) meson masses, the value of \( f \) is 5.

We take \( m_{b}(m_b) = 4.4 \text{ GeV}, m_s(m_b) = 0.2 \text{ GeV}, \) and \( \Lambda_{\text{MS}}^{(5)} = 225 \text{ MeV}. \) Following Eqs. (23) and (24), we find the effective Hamiltonian terms for the operators \( O_{1,2}^{LR} \) at \( m_b \) to be

\[ \mathcal{H}_{\text{eff}}^{Z'} \supset \frac{G_F}{\sqrt{2}} \rho_L \rho_R e^{i(\phi_L + \phi_R)} ( O_{1}^{LR} + O_{1}^{RL}, O_{2}^{LR} + O_{2}^{RL} ) \begin{pmatrix} 0.930 \\ -0.711 \end{pmatrix}. \]  

(34)

Note that there is no contribution of the operator \( O_{2}^{LR} \) at the \( M_W \) scale. It is induced through the operator mixing in RGE running and actually has an important effect at the \( m_b \) scale, as one can see from its Wilson coefficient in Eq. (34).

In the numerical analysis, we use the central values of \( B_{1}^{LR}(m_b) = 1.753 \pm 0.021 \) and \( B_{2}^{LR}(m_b) = 1.162 \pm 0.007 \) given in Ref. [42] with the decay constant \( f_{B_s} \) the same as before. The predicted mass difference with all the \( Z' \) contributions included is then

\[ \Delta M_s = 18.0 \times 10^5 \times 1 \times 3.858 \times 10^5 \times ( \rho_L^2 e^{2i\phi_L} + \rho_R^2 e^{2i\phi_R} ) - 2.003 \times 10^6 \rho_L \rho_R e^{i(\phi_L + \phi_R)} | \text{ps}^{-1}. \]  

(35)

The overall contribution to \( x_s \) from the SM and \( Z' \) is

\[ x_s = 26.3 \times 1 \times 3.858 \times 10^5 \times ( \rho_L^2 e^{2i\phi_L} + \rho_R^2 e^{2i\phi_R} ) - 2.003 \times 10^6 \rho_L \rho_R e^{i(\phi_L + \phi_R)} | \text{ps}^{-1}. \]  

(36)

To illustrate the interference among different contributions, we set \( \rho_L = \rho_R = 0.001 \) and plot \( x_s \) and sin \( 2\phi_s \) versus the weak phases \( \phi_L \) and \( \phi_R \) in Fig. 3(a) and (b), respectively.

First, we note that after the RGE running the operators \( O_{1}^{LR} \) and \( O_{2}^{LR} \) interfere constructively. This can be seen from the relative minus sign between the Wilson coefficients in Eq. (34) and a corresponding relative minus sign in the hadronic matrix elements given in Eqs. (28) and (29). Because of the constructive interference and the fact that the bag parameters \( B_{1}^{LR} \) and \( B_{2}^{LR} \) are twice as large as \( B_{2}^{LL} \), the LR and RL operators together become the dominant contributions. The interference of all the terms makes \( x_s \) reach its maximum when one of the weak phases is 180° and the other is 0° (mod 360°). If \( \rho_L \) and \( \rho_R \) are both much smaller than \( 10^{-3} \), the variation in \( x_s \) in the \( \phi_L - \phi_R \) space will be indistinguishable from the predicted SM range. Compared to Fig. 1(a) for \( Z' \) with
only \( LL \) couplings, Fig. 3(a) shows that even for large values of \( \rho_L \) and \( \rho_R \), \( x_s \) can be smaller than 20.6 due to the interference among all the terms in Eq. (36). The current \( x_s \geq 20.6 \) bound excludes the regions with \( \phi_L + \phi_R \simeq 0^\circ \) (mod 360\(^\circ\)). Because of the assumed equal values of \( \rho_L \) and \( \rho_R \), the parameter space points with the same sin 2\( \phi \) output lie along directions that are roughly parallel to the \( \phi_L + \phi_R = 360^\circ \) line. For the more general cases of different \( \rho_L \) and \( \rho_R \) values, the crests and troughs in Fig. 3(b) are no longer parallel to the \( \phi_L + \phi_R = 360^\circ \) line.

5. Conclusions

In this Letter we discuss the effects of a \( Z' \) gauge boson with FCNC couplings to quarks on the \( B_s - \bar{B}_s \) mixing. We show how the mass difference and CP asymmetry are modified by the left-handed and right-handed \( b - s - Z' \) couplings that may involve new weak phases \( \phi_L \) and \( \phi_R \). In the particular case of a left-chiral (right-chiral) \( Z' \) model, one can combine the measurements of \( \Delta M_s \) (or \( x_s \)) and \( \sin 2\phi_s \) to determine the coupling strength \( \rho_L \) (\( \rho_R \)) and the weak phase \( \phi_L \) (\( \phi_R \)) up to discrete ambiguities. Once comparable left- and right-chiral couplings are considered at the same time, we find the left–right interference terms dominate over the purely left- or right-handed terms, partly because of the renormalization running effects and partly because of the larger bag parameters.

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References

[12] For recent new physics studies, see, for example:


