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Bihomogeneity and Menger manifolds ☆

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Abstract

It is shown that for every triple of integers (α, β, γ) such that $\alpha \geqslant 1$, $\beta \geqslant 1$, and $\gamma \geqslant 2$, there is a homogeneous, non-bihomogeneous continuum whose every point has a neighborhood homeomorphic to the Cartesian product of Menger compacta $\mu^\alpha \times \mu^\beta \times \mu^\gamma$. In particular, there is a homogeneous, non-bihomogeneous Peano continuum of covering dimension four. © 1998 Elsevier Science B.V.

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Introduction

A space is *n-homogeneous* if, for any pair of n-point sets, the space admits a homeomorphism sending one of the sets onto the other. A *homogeneous* space is a 1-homogeneous space. A space X is *bihomogeneous* if for any pair of points p and q in X, there is a homeomorphism $h : X \to X$ such that $h(p) = q$ and $h(q) = p$. A space X is *strongly locally homogeneous* if, for every $p \in X$ and every neighborhood U_p, there is a neighborhood V_p such that for every $q \in V_p$ there is a homeomorphism $h : X \to X$ with $h(p) = q$ and $h(x) = x$ for $x \notin U_p$. A continuum is a compact, connected, metric space containing more than one point.

Around 1921, Knaster asked whether homogeneity implies bihomogeneity, and Kuratowski (Kazimierz Kuratowski) [18] gave an example of a 1-dimensional, non-locally

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compact, homogeneous, non-bihomogeneous subset of the plane. An example similar to that of Kuratowski can be easily described as follows: Let p and q be distinct points in the same composant of a nontrivial solenoid Σ. The composant of $\Sigma - \{p\}$ containing q is homogeneous but it is not bihomogeneous; one end of the composant is dense in it whereas the other end is not, making swapping points impossible. It is also easy to obtain nonmetric examples. In 1986, Cook [8] described a locally compact, 2-dimensional, homogeneous, non-bihomogeneous metric space. It is still not known if there is a 1-dimensional [locally] compact metric example.

In 1930, van Dantzig [9] restated Knaster's question for continua, which was answered in [15] by an example of a locally connected, homogeneous, non-bihomogeneous continuum. The construction of the example starts with a space that is bihomogeneous but has a property in some sense contrary to bihomogeneity. This space consists of compatibly oriented circular fibers such that any homeomorphism maps a fiber onto a fiber, and to swap certain fibers the homeomorphism must reverse orientation of the fibers. The next step is to replace each fiber with a homogeneous space containing it as a retract and not admitting a homeomorphism reversing orientation of the original fiber. The fibers are held together by a rigid grid that is locally homeomorphic to the Cartesian product of two Menger universal curves. Each circle is then replaced by a larger fiber, a manifold, which contains S^1 as its retract, but admits no homeomorphism changing the sign of the generator of the first homology group represented by this S^1. The dimension of the example equals the dimension of the manifold plus two, giving a 7-dimensional continuum. The seemingly unrelated property of local connectedness is important for the notion of 2-homogeneity: Ungar [21] proved that 2-homogeneous continua are locally connected. However, homogeneity does not imply 2-homogeneity for Peano continua [16,12,13,17,10] and, as the above example shows, it does not imply bihomogeneity.

A substantially simpler, although not locally connected, example of a homogeneous, non-bihomogeneous continuum was given by Minc [20]. The "model" space of Minc's example is a solenoid, whose each arc component is replaced by a sequence of "glued together" mapping cylinders of a degree $m \geqslant 2$ map of S^1 onto S^1. For most pairs of composants of a solenoid, a homeomorphism swapping the composants must be orientation reversing (see [20]). Hence in the above continuum, not every two arc components can be swapped. To achieve homogeneity, Minc takes the Cartesian product of this continuum and the Hilbert cube. To get a finite dimensional example, a manifold of the same homotopy type as the above mapping cylinder can be used to replace the solenoid composants. Recently, Kawamura [11] noticed that using Menger manifolds and "n-homotopy mapping cylinders" (see [6,7]), the dimension of Minc's example can be lowered to 2.

This paper shows that by applying Kawamura's idea to the construction of [15], a locally connected, homogeneous, non-bihomogeneous continuum of dimension 4 can be obtained. The factorwise rigidity of the Cartesian products of Menger compacta, immediately gives such examples in all dimensions greater than or equal to four.

1. Factorwise rigidity

Menger [19] defined n-dimensional universal compacta in terms of the intersection of a sequence of polyhedra in $\mathbb{R}^{2n+1}$. Anderson [1,2] proved that the 1-dimensional universal compactum, the Menger universal curve μ^1, is homogeneous and strongly locally homogeneous. Not much was known about the higher-dimensional Menger universal compacta until Bestvina [5] characterized the Menger universal compactum μ^n as a space that is topologically defined as follows:

(1) μ^n is a compact n-dimensional metric space,
(2) μ^n is LC^{n-1},
(3) μ^n is C^{n-1},
(4) μ^n satisfies the Disjoint n-Disk Property, $\mathrm{DD}^n\mathrm{P}$.

By [5], the compacta μ^n are homogeneous and strongly locally homogeneous. An *n-dimensional Menger manifold*, i.e., *μ^n-manifold*, is a metric space whose every point has a neighborhood homeomorphic to μ^n.

Definition. The Cartesian product $X = \prod_{\lambda\in\Lambda} X_\lambda$ is *factorwise rigid* if every homeomorphism $h: X \to X$ preserves the Cartesian factors: specifically, there is a permutation $\tau: \Lambda \to \Lambda$ and homeomorphisms $h_\lambda : X_{\tau(\lambda)} \to X_\lambda$ such that if $h(\langle x_\lambda\rangle) = \langle y_\lambda\rangle$, then $y_\lambda = h_\lambda(x_{\tau(\lambda)})$.

Definition. The *i-fiber* of $X = \prod_{\lambda\in\Lambda} X_\lambda$ is a subset of X of the form $\prod_{\lambda\in\Lambda} A_\lambda$, where $A_i = X_i$ and the remaining factors are single points. An *i-cofiber* of $X = \prod_{\lambda\in\Lambda} X_\lambda$ is a subset of X of the form $\prod_{\lambda\in\Lambda} A_\lambda$, where A_i is a single point and $A_j = X_j$ for $j \neq i$.

Definition. Points $x, y \in X$ are *homologically separated in dimension n*, if they have respective neighborhoods U_x and U_y such that

$$i_*\big(\check{H}_n(U_x)\big) \cap j_*\big(\check{H}_n(U_y)\big) = 0,$$

where $i: U_x \hookrightarrow X$, $j: U_y \hookrightarrow X$ are the inclusions, and $\check{H}$ is the nth Čech homology group.

The factorwise rigidity of the Cartesian product of two Menger universal curves was first determined in [16] to show that the product is not 2-homogeneous. The factorwise rigidity of the Cartesian products of pseudo-arcs was shown in [3,4]. Kennedy Phelps [12] proved that the Cartesian product of arbitrarily many copies of μ^1 is factorwise rigid, and by an unpublished result of Yagasaki, Kennedy's theorem extends to the Cartesian products of copies of μ^n (see [7, Section 3]). Garity [10] used the Künneth and Eilenberg–Zilber formulas to show that finite products of at least two Menger universal compacta (of equal or different dimensions, but excluding the product with all factors μ^0) are not 2-homogeneous. His proof is very close to imply factorwise rigidity. The notion of homology separation was introduced in [17].

For dimensional reasons, any two points in μ^n are homologically separated in dimension n. At every $x \in \mu^n$, there are arbitrarily small spheres S^n embedded in μ^n as retracts. Let $X = \mu^{n_1} \times \cdots \times \mu^{n_k}$. The m-cycles, where $m = n_1 + \cdots + n_k = \dim X$, carried by two disjoint tori of form $S^{n_1} \times \cdots \times S^{n_k}$ are not homologous. Therefore, if $h_1, h_2 : X \to X$ are isotopic homeomorphisms, then $h_1 = h_2$. The lemma below has analogs in the above-mentioned papers, but treats factorwise rigidity as a local property.

Lemma 1. *Let $X = X_1 \times \cdots \times X_k$, where X_i is homeomorphic to μ^{α_i}, $1 \leqslant \alpha_1 \leqslant \cdots \leqslant \alpha_k$. Let $U = U_1 \times \cdots \times U_k$, $U_i \subset X_i$, be an open connected subset of X, and let $h : U \to X$ be an open embedding. Then $h(x_1, \ldots, x_k) = (h_1(x_{\tau(1)}), \ldots, h_k(x_{\tau(k)}))$, where τ is a permutation and $h_i : U_{\tau(i)} \to X_i$ is an embedding.*

Proof. Let $x = (x_1, \ldots, x_k)$ and $\bar{x} = (x_1, \bar{x}_2 \ldots, \bar{x}_k)$ be two points in the same 1-cofiber of U. There is a sequence of α_1-dimensional spheres C_n in U_1 containing x_1 with diam$(C_n) \to 0$. Let

$$K_n = \{(y^n, x_2, \ldots, x_k) \mid y^n \in C_n\} \quad \text{and} \quad \overline{K}_n = \{(y^n, \bar{x}_2, \ldots, \bar{x}_k) \mid y^n \in C_n\}.$$

The spheres K_n are retracts of X, and the spheres $h(K_n)$ are retracts of $h(U)$. Every point of $h(U)$ has small neighborhoods $V_1 \times \cdots \times V_k$ in $h(U)$, which are retracts of X. Hence sufficiently small spheres $h(K_n)$ are retracts of X. For some i and infinitely many n's, $\pi_i \circ h : K_n \to X$ is essential. Note that $\alpha_1 = \alpha_i$, so by the classical Hurewicz theorem, an essential map $S^{\alpha_1} \to \mu^{\alpha_i}$ is homologically nontrivial. If the i-coordinates of $h(x)$ and $h(\bar{x})$ are different, then for sufficiently large n, the images $\pi_i(h(K_n))$ and $\pi_i(h(\overline{K}_n))$, are disjoint. Since the 1-cofibers of U are arcwise connected, there is a tube $S^{\alpha_1} \times [0, 1]$ in U joining K_n and $\overline{K}_n$, which implies that the nontrivial cycles represented by $\pi_i(h(K_n))$ and $\pi_i(h(\overline{K}_n))$ are homologous, contradicting the fact that distinct points in μ^{α_i} are homologically separated in dimension α_i. Therefore the points $h(x)$ and $h(\bar{x})$ are in the same i-cofiber of X. Hence every 1-cofiber in U is mapped into a cofiber in X. By continuity, 1-cofibers close to a 1-cofiber mapped into an i-cofiber are mapped into i-cofibers for the same i. The compositions of the natural inclusions of $U_2 \times \cdots \times U_k$ into 1-cofibers, h restricted to these 1-cofibers, and the projection π_i are isotopic, so they are identical. The proof is completed by induction. □

Corollary (see [17, Problem 2]). *Finite Cartesian products of Menger universal compacta (also Menger manifolds) are factorwise rigid.*

2. Grids

A homeomorphism $h : X \to X$ is *periodic* with *period* $k \geqslant 1$ if $h^k(p) = p$ for every $p \in X$, but for every $1 \leqslant i < k$ and $p \in X$, $h^i(p) \neq p$. A closed subset A of a compact metric space X is a *Z-set* if for every $\varepsilon > 0$, there is a map $f : X \to X$ ε-close to the identity with $f(X) \cap A = \emptyset$.

For positive integers α, β, and $k \geqslant 2$, choose M, f_M, N, A, B, f_N, F, and Q as follows:

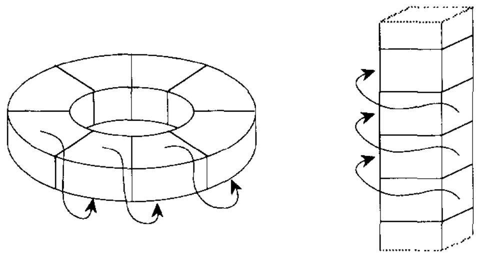

Fig. 1. The symmetry of Q.

(1) $f_M : M \to M$, where M is a μ^α-manifold, is a periodic homeomorphism of period k,
(2) $N = \mu^\beta$,
(3) A and B are disjoint, nonempty, homeomorphic Z-sets in N, and $f_N : A \to B$ is a homeomorphism, such that the quotient space of N obtained by identifying each point $n \in A$ with its image $f_N(n) \in B$, is a μ^β-manifold,
(4) $F : M \times A \to M \times B$ is given by $F(m,n) = (f_M(m), f_N(n))$,
(5) $Q = (M \times N)/F$ is the quotient space obtained by identifying each point (m,n) with the point $F(m,n)$.

We refer to the continuum Q as the *k-grid*. By a slight abuse of notation, points in the quotient space are denoted in the same way as the corresponding points in $M \times (N - A)$.

For $p = (m_p, n_p) \in Q$, let

$$\begin{aligned}
M_p &= \{(m,n) \in Q \mid n = n_p\},\\
N_p &= \{(m,n) \in Q \mid m = f_M^i(m_p),\ i = 0, \ldots, k-1\},\\
O_p &= M_p \cap N_p.
\end{aligned}$$

Remark. If $\alpha = \beta$, the position of the sets M_p and N_p in Q is symmetrical, see Fig. 1.

Call the sets M_p and N_p *horizontal* and *vertical fibers* respectively. The intersection of a horizontal fiber and a vertical fiber is a *necklace* and its elements are *beads*. Note that the number of beads on each necklace O_p is k.

Lemma 2. *A homeomorphism $h : Q \to Q$ takes each horizontal and vertical fiber onto a horizontal or vertical fiber, and a necklace onto a necklace.*

Proof. Using Lemma 1, the proof is identical to the proof of Lemmas 5 and 6 in [15], where this is shown for the case $\alpha = \beta = 1$ and a specific k. $\square$

There is a cyclic order of a given set of beads on a necklace, which cannot be arbitrarily disturbed by a homeomorphism of Q. Let $\phi : Q \to Q$ be the homeomorphism given by $\phi(m,n) = (f_M(m), n)$. For a point p_0 in Q, denote by p_i the point $\phi^i(p_0)$. Thus the necklace O_{p_0} is the set $\{p_0, \ldots, p_{k-1}\}$.

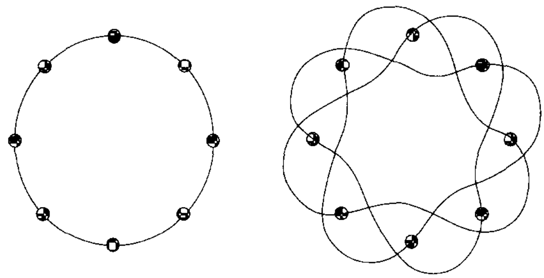

Fig. 2. Beads on a necklace.

Lemma 3 (Compare with Lemma 7 in [15]). *Suppose that $h:Q \to Q$ is a homeomorphism and $h(p_0) = p_0$. Then there is an s such that $h(p_i) = p_{(si) \bmod k}$ for $i = 0, \ldots, k-1$.*

Proof. There is an arc L_0 joining p_0 and p_1 such that distinct $\phi^i(L_0)$ and $\phi^j(L_0)$ do not intersect except for a possible common end point. Denote $\phi^i(L_0)$ by L_i, and by K the simple closed curve $\bigcup_{i=0}^{k-1} L_i$. Note that K is the union of necklaces. Hence $h(K)$ is the union of necklaces; if $p \in h(L_0)$ then $O_p \subset h(K)$. We have (see Fig. 2)

$$h(K) = \bigcup_{i=0}^{k-1} h(L_i) = \bigcup_{i=0}^{k-1} \phi^i h(L_0).$$

The points $h(p_i)$ are ordered on the simple closed curve $h(K)$ in such a way that the difference modulo k in the indices between $h(p_i)$ and $h(p_{i+1})$ is a constant. Therefore if $\phi(p_1) = p_s$, then $h(p_i) = p_{(si) \bmod k}$. □

Lemma 4. *If $h:Q \to Q$ is a homeomorphism and $h(p_0) \in O_{p_0}$, then there are integers r and s such that $h(p_i) = p_{(r+si) \bmod k}$ for $i = 0, \ldots, k-1$.*

Proof. Take r and s such that $h(p_0) = p_r$ and $\phi^{-r} \circ h(p_1) = p_s$. □

3. Circular fibers and fiber replacing

The next step is to construct a continuum built on the k-grid Q (see Fig. 3) obtained from $Q \times I$ by the identification $(p, 1) = (\phi^b(p), 0)$, where k is the product of two positive integers a and b. Each of the sets $O_p \times I$ transforms into b circles called *circular fibers*; X decomposes into pairwise disjoint copies of S^1. The number of beads of $O_p \times \{0\}$ on each circle C is a. The order of the beads determines the orientation of C. By [17, Lemma 3.1], we have:

Lemma 5. *A homeomorphism $h:X \to X$ takes each circular fiber onto a circular fiber.*

Every point of X has a neighborhood homeomorphic to the Cartesian product $M \times N \times I$. Orient the I-fiber of $Q \times I$ and transfer the orientation to the circular fibers of X.

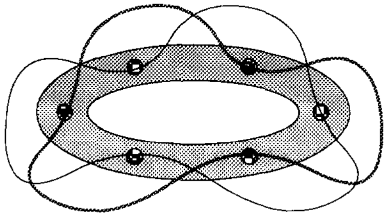

Fig. 3. Circular fibers of X going through the beads of a necklace in $Q \times \{0\}$.

Similarly as in [15, Lemma 11], a homeomorphism of X onto itself either preserves orientation on all circular fibers (it is then *orientation preserving*), or reverses orientation on all circular fibers (it is then *orientation reversing*).

As in the previous section let $O_{p_0} = \{p_0, \ldots, p_{k-1}\}$ be a necklace in Q. Denote the point $(p_i, 0) \in X$ by q_i.

Lemma 6. *If $a \geqslant 3$, then every homeomorphism $h: X \to X$ such that $h(q_0) = q_1$, $h(q_1) = q_0$, and $h(Q \times \{0\}) = Q \times \{0\}$ is orientation reversing.*

Proof. By Lemma 4, $h(q_i) = q_{(1-i) \bmod ab}$. Suppose that h is orientation preserving. Then $h(\{q_0, q_b, q_{2b}, \ldots\}) = \{q_1, q_{1+b}, q_{1+2b}, \ldots\}$ preserving order. Hence $h(q_b) = q_{1+b}$ and $1 + b = (1 - b) \bmod ab$. So $2 = 0 \bmod a$, which is a contradiction. □

If one were to follow the procedure described in [15], each circular fiber C of X would be replaced by a manifold E which contains C as its retract, and such that every autohomeomorphism of E takes the element of the first homology group represented by C onto itself; in particular it does not change the sign of this element. (Note that in [15], C, E and X are denoted by different symbols.) The resulting continuum D is the union of pairwise disjoint copies of the same manifold E, called manifold fibers. Since in [15] $a = b = 3$; each circular fiber in X consists of three segments; each manifold fiber of D consists of three identical pieces, as shown in Fig. 4. D contains a copy of X as its retract. Although X needs not be invariant under a homeomorphism $h: D \to D$, h

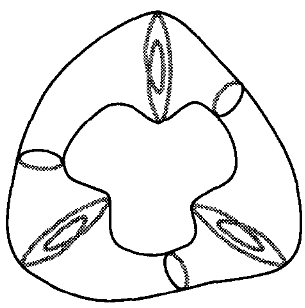

Fig. 4. Replacing circular fibers.

induces a homeomorphism of X preserving the correspondence of the circular fibers to the manifold fibers given by the inclusion $X \hookrightarrow D$. It is shown that:

(1) every homeomorphism $h: D \to D$ maps a manifold fiber onto a manifold fiber,

(2) some manifold fibers cannot be swapped by a homeomorphism of D.

Kawamura's idea [11] to modify Minc's example [20] can be also applied to modify D. Instead of the manifold E take a Menger manifold Ω consisting of a identical pieces homeomorphic to a μ^γ-manifold P, where $\gamma \geqslant 2$. P corresponds to the mapping cylinder of a degree two map of $S^1 \to S^1$. The Menger manifold Ω is similar to the Menger manifold L_n in [11, Section 3], and has the following properties:

(1) $\Omega = \bigcup_{i=0}^{a-1} \Omega_i$, and there are homeomorphisms $\tau_i : P \to \Omega_i$.

(2) $\Omega_i \cap \Omega_j = \emptyset$ if $|(i-j) \bmod a| \neq 1$.

(3) There are two disjoint homeomorphic Z-sets in P, P_0 and P_1, such that

$$\Omega_i \cap \Omega_j = \begin{cases} \tau_i(P_0) = \tau_j(P_1) & \text{if } (i-j) \bmod a = 1, \\ \tau_i(P_1) = \tau_j(P_0) & \text{if } (j-i) \bmod a = 1. \end{cases}$$

(4) There is a simple closed curve $K \subset \Omega$ intersecting each Ω_i in an arc such that K is a retract of Ω, every homeomorphism $\Omega \to \Omega$ takes the element of the first homology group represented by (oriented) K onto itself without changing its sign.

The Menger manifold P can be used to replace the circular fiber of the continuum X to obtain a non-bihomogeneous continuum Y. Namely, Y is the quotient space obtained from $Q \times P$ by identifying each point (p, x) with $(\phi^b(p), \tau^{-1}_{(i+1) \bmod a} \circ \tau_i(x))$, where Q is the k-grid considered in the beginning of this section, $p \in Q$, and $x \in P_1$. Note that $\tau^{-1}_{(i+1) \bmod a} \circ \tau_i : P_1 \to P_0$.

Lemma 7. *The continuum Y is homogeneous.*

Proof. The proof uses strong local homogeneity of the Menger compacta and is similar to the proofs of Lemmas 3 and 4 in [15]. □

Lemma 8. *If $a \geqslant 3$ and $b \geqslant 2$, then Y is not bihomogeneous.*

Proof. Since Y is locally homeomorphic to $\mu^\alpha \times \mu^\beta \times \mu^\gamma$, the local factorwise rigidity holds. Using the local Cartesian product structure, we may define the μ^α-, μ^β-, and μ^γ-fibers, as well as the μ^α-, μ^β-, and μ^γ-cofibers. The μ^γ-fibers are homeomorphic to P, and the μ^γ-cofibers are homeomorphic to Q. The necklaces in the μ^γ-cofibers have ab beads, whereas the necklaces in the μ^α- and μ^β-cofibers contain only a beads. Therefore every homeomorphism $Y \to Y$ maps the μ^γ-fibers onto μ^γ-fibers and μ^γ-cofibers onto μ^γ-cofibers even if $\alpha = \beta = \gamma$. Since $b \geqslant 2$, there are at least two distinct μ^γ-fibers passing through the same necklace of $Q \times \{x\}$, where x is a point in $P - P_1$. By Lemma 6 and by the above consideration, there are two such fibers that cannot be swapped. □

The above lemma could be compared to Lemmas 13–16 in [15]. However, factorwise rigidity involving all three factors, μ^α, μ^β, and μ^γ, makes the proof much simpler.

Theorem. *For every triple of integers (α, β, γ) such that $\alpha \geqslant 1$, $\beta \geqslant 1$, and $\gamma \geqslant 2$, there is a homogeneous, non-bihomogeneous continuum whose every point has a neighborhood homeomorphic to $\mu^\alpha \times \mu^\beta \times \mu^\gamma$.*

Remark. Y is the quotient space of $\mu^\alpha \times \mu^\beta \times \mu^\gamma$ with some identifications made along Z-sets. It is not known whether the same effect can be achieved on the product $\mu^1 \times \mu^1 \times \mu^1$.

Question 1. Does there exist a homogeneous, non-bihomogeneous Peano continuum of dimension lower than 4?

Question 2. Does there exist a homogeneous, non-bihomogeneous Peano continuum whose every point has a neighborhood homeomorphic to an open set in $\mu^1 \times \mu^1 \times \mu^1$?

While the second question remains open, since the submission of this paper, the first question has been answered by G. Kuperberg [14]. For any pair of integers α, β such that $\alpha \geqslant 1$ and $\beta \geqslant 2$, he constructs a homogeneous, non-bihomogeneous Peano continuum with the local structure of $\mu^\alpha \times \mu^\beta$.

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