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Sharing the cost of multicast transmissions in wireless networks☆

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Abstract

A crucial issue in non-cooperative wireless networks is that of sharing the cost of multicast transmissions to different users residing at the stations of the network. Each station acts as a *selfish* agent that may misreport its *utility* (i.e., the maximum cost it is willing to incur to receive the service, in terms of power consumption) in order to maximize its individual welfare, defined as the difference between its true utility and its charged cost. A provider can discourage such deceptions by using a strategyproof *cost sharing mechanism*, that is a particular public algorithm that, by forcing the agents to truthfully reveal their utility, starting from the reported utilities, decides who gets the service (the receivers) and at what price. A mechanism is said *budget balanced* (BB) if the receivers pay exactly the (possibly minimum) cost of the transmission, and *β-approximate budget balanced* (β-BB) if the total cost charged to the receivers covers the overall cost and is at most β times the optimal one, while it is *efficient* if it maximizes the sum of the receivers' utilities minus the total cost over all receivers' sets. In this paper, we first investigate cost sharing strategyproof mechanisms for symmetric wireless networks, in which the powers necessary for exchanging messages between stations may be arbitrary and we provide mechanisms that are either efficient or BB when the power assignments are induced by a fixed universal spanning tree, or $(3\ln(k+1))$-BB (k is the number of receivers), otherwise. Then we consider the case in which the stations lay in a d-dimensional Euclidean space and the powers fall as $1/d^{\alpha}$, and provide strategyproof mechanisms that are either 1-BB or efficient for $\alpha = 1$ or $d = 1$. Finally, we show the existence of $2(3^d - 1)$-BB strategyproof mechanisms in any d-dimensional space for every $\alpha \geqslant d$. For the special case of $d = 2$ such a result can be improved to achieve 12-BB mechanisms.

Keywords: Wireless networks; Power management; Algorithmic mechanism design; Cost sharing mechanisms; Approximate budget balanceness; Efficiency

1. Introduction

Wireless networks have received significant attention during the recent years. In particular, ad hoc wireless networks can be deployed for applications such as emergency disaster relief, automated battlefield applications, etc. [45,50]. Unlike traditional wired networks or cellular networks, they do not require the installation of any wired backbone

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infrastructure. Each station of the network has a fixed position and is equipped with an omnidirectional antenna which is responsible for sending and receiving signals.

Wireless network model: a wireless network is usually modeled as a complete graph (S, c), called the *cost graph*, in which $S = \{x_1, \ldots, x_n\}$ is a set of radio stations and c is a *transmission cost function* that associates to each ordered pair of stations x_i, x_j the power required to establish a connection between them. Throughout the paper we will deal with symmetric transmission cost functions which is the most adopted case in the literature. Asymmetric functions are usually used to model medium abnormalities or batteries with different energy levels [35]. A *power assignment* $\omega : S \mapsto \mathbb{R}^+$ to all stations implements a connection from x_i to x_j if $\omega(x_i) \geqslant c(x_i, x_j)$. Therefore, once a power assignment ω is fixed, a weighted *transmission digraph* $G_\omega = (S, E_\omega)$ can be abstracted, where $E_\omega = \{\langle x_i, x_j\rangle \mid \omega(x_i) \geqslant c(x_i, x_j)\}$ is the set of the connections implemented by ω. A communication session from x_i to x_j can be achieved directly (i.e., through a single-hop transmission) if $(x_i, x_j) \in E_\omega$, or through relaying by intermediate stations, otherwise. The *cost* of a power assignment ω is the overall power consumption yielded by ω, i.e., $cost(\omega) = \sum_{x_i \in S} \omega(x_i)$.

The most common power attenuation model [45] assumes *Euclidean wireless networks* where the stations are located in the d-dimensional Euclidean space ($d \geqslant 1$). In such a special case, the signal power of a station x_i falls as $1/d^\alpha$, where d is the distance from x_i and $\alpha \geqslant 1$ is a constant called the *distance-power gradient* (or attenuation parameter) depending on the environmental conditions, whose typical values are between 1 and 6. All receivers have the same *transmission-quality* threshold γ for signal detection (typically normalized to be 1). As a consequence, the power required to establish a connection between two stations x_i and x_j at distance $dist(x_i, x_j)$ is $c^{\alpha,\gamma}(x_i, x_j) = \gamma \cdot dist(x_i, x_j)^\alpha$.

A crucial issue in wireless networks is that of supporting communication patterns that are typical of traditional networks, such as multicasting (one-to-many communication), in which a message or service of a given source station must be forwarded to users residing at a subset of receiving stations.

The problem of determining an optimal power assignment implementing a multicast tree rooted at a source s and spanning a given set of receivers R, called the minimum energy multicast tree (MEMT) problem, is an interesting algorithmic issue. In fact, such a tree guarantees the multicast communication from the source s to the users in R with a minimum overall power consumption. However, it has been proved to be inapproximable within $(1 - \varepsilon) \ln n$ unless $\mathrm{NP} \subseteq \mathrm{DTIME}(n^{\mathrm{O}(\log \log n)})$ [11,49]. Caragiannis et al. [9] used a reduction to the well-known minimum node weighted steiner tree (NWST) problem to prove that a ρ-approximate solution obtained by using the reduction to NWST can be used to obtain a 2ρ approximate solution to MEMT. The two best known approximation algorithms for NWST, presented by Guha and Khuller in [28] and achieving performance guarantees of $1.35 \ln k$ and $1.5 \ln k$ (k is the number of receivers), imply the existence of a $2.7 \ln(|R| + 1)$ and a simpler $3 \ln(|R| + 1)$ approximation algorithms for MEMT.

In Euclidean wireless networks, it has been proved in [11] that the problem of broadcasting (and hence multicasting) with minimum energy consumption, called the minimum energy broadcast tree (MEBT) problem is NP-hard for $\alpha > 1$ and $d > 1$ but solvable in polynomial-time if $\alpha = 1$. Polynomial-time solvability of the case $d = 1$, already conjectured in [11], was proved independently in [8,12]. The best known approximation algorithm, called MST, has been presented with other heuristics in [50] and is based on the idea of tuning powers so as to include a minimum spanning tree of the cost graph. The performance of MST has been investigated by several authors [11,49,21] and the evaluation of its approximation ratio progressively reduced till $3^d - 1$ for every $\alpha \geqslant d$ [21]. For the case $\alpha \geqslant d = 2$, this bound has been recently improved to 6.33 in [39] and finally to 6 in [1].

Other results, also related to different communication patterns and connectivity requirements have been considered also in [4,6,7,14,32] (see [13] for a survey).

Multicasting data to a large population is likely to incur significant costs, expressed as the overall power consumption, that is natural to be shared in some manner among the receivers.

We consider the problem of sharing the cost of multicast transmissions in symmetric wireless networks in a *non-cooperative scenario*, where each station x_i acts as a *selfish agent*. In such a scenario, each agent is potentially interested in receiving the service and valuates the transmission an amount u_i, called the *utility*, interpretable as the maximum cost (in terms of power consumption) that a station is willing to incur in order to receive the service from the source. Each agent is only interested in maximizing its *individual welfare* $\omega_i = u_i - c_i$, that is the difference between the utility and the cost share, and will agree to pay for the transmission if and only if it is charged a cost not exceeding the utility. As the utility u_i is not a property of the network (it is a "private" value of x_i), each agent x_i may report a different value $v_i \neq u_i$ trying to receive the transmission at a lower cost share, in order to increase its individual welfare. The network can discourage such deceptions by using the so-called *strategyproof cost sharing mechanisms*.

A *cost sharing mechanism* is a (public) algorithm which, given any reported utility profile u, decides the subset $R(u)$ of the agents receiving the service and shares the possibly minimum global cost $C(R(u))$ incurred by the source among them, according to some method.

A cost sharing mechanism is *strategyproof* if truthfully reporting the utilities is always a dominant strategy for each agent; that is, no agent has an incentive in reporting $v_i \neq u_i$, no matter how the other agents behave, as u_i maximizes its individual welfare $w_i(u) = v_i - c_i(u)$, where $c_i(u)$ is its cost share. More precisely, for a strategyproof mechanism, denoted $v^{-i} = (v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n)$ and $(v^{-i}, a_i) = (v_1, \ldots, v_{i-1}, a_i, v_{i+1}, \ldots, v_n)$:

$$\forall x_i\ \forall v^{-i}\ \forall v_i \neq u_i :\ w_i(v^{-i}, u_i) \geqslant w_i(v^{-i}, v_i).$$

Group strategyproofness is a stronger form of strategyproofness, such that no coalition Q of agents has an incentive to jointly misreport its utility values: for any Q and for any utility profile v with $v_i = u_i$ for every $x_i \notin Q$, the group strategyproofness requires that if the inequality $w_i(v) \geqslant w_i(u)$ holds for all $x_i \in Q$, then it must hold with equality for all $x_i \in Q$. In other words, if no member of Q is made worse off by misreporting their utility values, then no member is made better off either.

In this work we are interested in defining suitable (group) strategyproof mechanisms for sharing the costs of multicast transmission services in wireless networks. More precisely, given any reported utility profile u and a source s, such a mechanism has to determine:

- the subset $R(u) \subseteq S\backslash\{s\}$ of the receivers;
- a power assignment $\omega_{R(u)}$ multicasting to $R(u)$; i.e., $\omega_{R(u)}$ has to induce a transmission digraph $G_{\omega_{R(u)}} = (S, E_{\omega_{R(u)}})$ that contains a multicast tree $T(R(u))$ rooted at s and spanning $R(u)$ as a subgraph;
- in order to share the (possibly minimum) overall cost $C(R(u)) = cost(\omega_{R(u)})$ of the multicast service among the receivers, a cost share $c_i(u)$, $\forall x_i \in R(u)$, representing the amount of power consumption attributed to each of them.

In order to meet a few natural economical requirements, a cost sharing mechanism should also satisfy these requirements, for every utility profile u and for every agent x_i:

- *No positive transfers* (NPT): $c_i(u) \geqslant 0$.
- *Voluntary participation* (VP): $w_i(u) \geqslant 0$, i.e., agents may decide not to receive the service and in such a case they are not charged.
- *Consumer sovereignty* (CS): $\exists v_i : x_i \in R(u^{-i}, v_i)$, i.e., every agent is guaranteed to get the service if it reports a high enough utility value.
- *Budget balance* (BB): the selected agents in $R(u)$ pay exactly the total cost of the service. Namely, it consists in satisfying both the two following requirements: (i) $\sum_{x_i \in R(u)} c_i(u) \geqslant C(R(u))$ (*cost recovery*), and (ii) $\sum_{x_i \in R(u)} c_i(u) \leqslant C(R(u))$ (*competitiveness*).
- *Efficiency*: Let us say a subset R is *efficient* if it maximizes the *overall welfare* $W(R) = u_R - C(R)$, where $u_R = \sum_{x_i \in R} u_i$; then the efficiency requirement consists in maximizing the *net worth* $NW(u) = W(R(u))$. Thus, $R(u)$ is an efficient set.

Since in wireless networks energy is a scarce resource and establishing a communication pattern strongly depends on the energy consumption, a natural question to be solved is to guarantee a communication with a minimum total energy consumption.

We say that a mechanism satisfies the *cost optimality* (CO) requirement if it guarantees the overall cost $C(R(u))$ is minimized. For NP-hard problems the CO requirement is not computationally feasible (unless P = NP) so, by denoting with $C^*(R)$ the minimum cost of serving R, in [29] the standard CO, BB and efficiency requirements are naturally relaxed to:

- *β-approximate CO* (β-CO): $C(R(u)) \leqslant \beta \cdot C^*(R(u))$;
- *β-approximate BB* (β-BB): It consists in satisfying both the *cost recovery* requirement and $\sum_{x_i \in R(u)} c_i(u) \leqslant \beta \cdot C^*(R(u))$ (*β-approximate competitiveness*), i.e., the receivers pay at most β times the minimum cost. Clearly, β-CO and BB imply β-BB. A 1-BB mechanism is *optimally* BB;
- *β-approximate efficiency* (β-efficiency): $\beta \cdot NW(u) \geqslant \max_{R \subseteq S\backslash\{s\}} W(R)$.

There is a vast literature on the design of cost sharing mechanisms for several non-cooperative games [5,23,24,31,30]. One of the main approaches that have been considered in the literature in order to cope with agents' selfishness is that of mechanism design [40–42], that dates back to the seminal papers by Vickrey [48], Clarke [10] and Groves [27]. Their celebrated VCG mechanism is still a fundamental technique to derive *strategyproof mechanisms* for many problems.

In Section 1.1, we review the basics of algorithmic mechanism design and address related works on cost sharing mechanisms for multicast transmissions on "classic" wired networks [2,17–19,29]. For a more extensive discussion of applications of game theoretic tools and microeconomics to the Internet we refer the reader to [3,19,20,40–42]. Readers already familiar with this area should skip Section 1.1.

1.1. Cost sharing mechanisms for non-cooperative games

The literature [25,46] on strategyproof cost sharing mechanisms emphasizes the tradeoff between efficiency and budget balance: a strategyproof mechanism which satisfies NPT, VP and CS cannot be *both* efficient and budget balanced (BB). In view of this limitation, there are two options: sacrifice either budget balance or efficiency.

The marginal cost (MC) mechanism: When the cost function C is *non-decreasing* and *submodular*, i.e., $C(\emptyset) = 0$ and $\forall R, Q \subseteq S\backslash\{s\}$:

$$Q \subseteq R \Rightarrow C(Q) \leqslant C(R), \tag{1}$$

$$C(Q \cup R) \leqslant C(Q) + C(R) - C(Q \cap R), \tag{2}$$

then the only strategyproof mechanism which is efficient is the MC mechanism [38], which is a special case of the general class of the VCG strategyproof mechanisms [10,26,27,48]. The MC mechanism is strategyproof and meets NPT, VP, CS; conversely, any strategyproof mechanism M selecting an efficient set (not necessarily the largest) at all utility profiles u, meeting NPT and VP is welfare equivalent to MC. However, MC it is not group strategyproof. Moreover, it never creates a budget surplus, but can run a deficit and in many cases it raises no revenue at all.

More precisely, as a consequence of the submodularity of C, since the union of two efficient sets is also an efficient set, the largest one $R^*(u)$ is well defined. This allows to define the MC mechanism as follows: first, select the largest efficient set $R^*(u)$ of agents; then, $\forall x_i \in R^*(u)$, assign a MC

$$c_i(u) = C(R^*(u)) - C((R\backslash\{x_i\})^*(u)). \tag{3}$$

The Shapley value mechanism: A cost sharing method for a cost function C is a function ξ which distributes $C(R)$ among the receivers in such a way that $\forall x_i \notin R,\ \xi(R, x_i) = 0$ and $\sum_{x_i \in R} \xi(R, x_i) = C(R)$. A method ξ is *cross-monotonic* if $\forall Q, R \subseteq S\backslash\{s\}$ such that $Q \subseteq R$, $\xi(Q, x_i) \geqslant \xi(R, x_i)$ for every $x_i \in Q$, whereas it is *weakly cross-monotonic* if under the same assumption, $\sum_{x_i \in Q} \xi(Q, x_i) \geqslant \sum_{x_i \in Q} \xi(R, x_i)$.

When sacrificing the efficiency requirement, as shown in [37,38], if a cross-monotonic method ξ exists for C, then the following mechanism $M(\xi)$ is BB, meets NPT, VP, CS and is group strategyproof (the converse also holds if the cost function C is submodular):

- initialize $R(u)$ to $S\backslash\{s\}$;
- while $\exists x_i \in R(u) : u_i < \xi(R(u), x_i)$ drop x_i from $R(u)$;
- set $c_i(u) = \xi(R(u), x_i), \forall x_i \in R(u)$, and then build a feasible solution spanning $R(u)$ of cost $C(R(u)) = \sum_{x_i \in R(u)} c_i(u)$.

Non-decreasing submodular cost functions support an entire class of cross-monotonic cost sharing methods, of which the *Shapley value* [47] is one of the most relevant representatives, due to its fair distribution of costs and especially because it achieves the lowest worst case efficiency loss over all the utility profiles [38]. The *Shapley value* [47] method sets

$$\xi(R, x_i) = \sum_{Q \subseteq R\backslash\{x_i\}} \frac{|Q|!(|R| - |Q| - 1)!}{|R|!} C(T(Q \cup \{x_i\})) - C(T(Q)). \tag{4}$$

When specialized to the multicast problem in wireless networks, such a formula, a bit counterintuitive, has a quite simple explanation (see Section 2.1).

For NP-hard problems, Jain and Vazirani in [29] have introduced the notion of approximate BB cost sharing method: a method ξ is β-approximate budget balanced (β-BB), for some $\beta \geqslant 1$, if $\forall R \subseteq S\backslash\{s\}, C(R) \leqslant \sum_{x_i \in R} \xi(R, x_i) \leqslant \beta \cdot C^*(R)$. The standard notions of cross-monotonic and weakly cross-monotonic cost sharing methods extend directly to β-BB cost sharing methods and ξ is said to be efficiently computable if $\forall R \subseteq S\backslash\{s\}$ both ξ and $C(R)$ can be computed in polynomial time. Extending the results in [37,38], in [29] it is proved that for any β-BB cross-monotonic cost sharing method ξ, the mechanism $M(\xi)$ is β-BB, meets NPT, VP, CS and is group strategyproof, and $M(\xi)$ is efficiently computable if ξ is.

The existence of cross-monotonic cost sharing methods for a cost function C can be related to the notion of *core* of C, denoted $core(C)$, defined as the set of all the allocation functions f such that $\sum_{x_i \in R} f(x_i) \leqslant C(R)$, $\forall R \subseteq S\backslash\{s\}$ (i.e., no subset of agents has an incentive to secede). We recall that a *cost allocation function* f is a non-negative function which distributes the cost of serving all the agents in $S\backslash\{s\}$ in such a way that $\forall x_i \in S\backslash\{s\}$, $f(x_i) \geqslant 0$, and $\sum_{x_i \in S\backslash\{s\}} f(x_i) = C(S\backslash\{s\})$. Since for every R a weakly cross-monotonic method defines an allocation function belonging to the core for R, it follows that if the core of C is empty then no weakly cross-monotonic and thus cross-monotonic method exists. As a further implication, since if C was submodular then the Shapley value would be a cross-monotonic method, also C is not submodular.

The extended Moulin–Shenker approach: The emptiness of the core prevents from using cross-monotonic cost sharing methods to achieve BB group strategyproof mechanisms. Penna and Ventre in [44] recently extended the approach of Moulin and Shenker and proposed a new technique to obtain group strategyproof BB mechanisms which also satisfy NPT, VP and CS requirements by introducing the notion of *self-cross-monotonic* cost sharing and then showed that group strategyproof mechanisms do not need to provide solutions in the core.

1.1.1. Cost sharing mechanisms for wired networks

The problem of sharing the cost of multicast transmissions in standard networks, where the cost is the total cost of the links of a multicast subtree connecting the source s to the receivers $R(u)$, has been considered by several authors [17–19,29]. In [19] a method has been proposed in which a universal spanning tree is fixed, and each multicast subtree is given by the union of all the paths in the universal tree connecting s to the receivers. The deriving cost function is non-decreasing and submodular, and thus Shapley value gives rise to a BB mechanism and the MC to an efficient one, both efficiently computable. Moreover, it is shown that the overall welfare value (sum of the utilities minus cost) of an optimal subtree is NP-hard to approximate within any constant factor.

Nevertheless, the method of [19] has the drawback that the multicast trees may be arbitrarily more expensive than the optimal ones, that is the minimum Steiner trees connecting s to the receivers. Of course, one of the problems is that such a tree is NP-hard to compute. Moreover, even if polynomial solvability is not a concern, the results of Megiddo [36] imply that there do not exist cross-monotonic cost sharing methods for minimum Steiner trees. Thus in [29], starting from the classical approximation algorithm for the minimum Steiner tree problem based on the determination of a minimum spanning tree (MST) of the subset of the receivers [34], using the primal-dual algorithm of Edmonds [16] a class of 2-BB cross-monotonic methods has been constructed parameterized by n mappings $f_i : \mathbb{R}^+ \mapsto \mathbb{R}^+$, one per user. Any of these methods is clearly efficiently computable. These results extends previous ones in [30] concerning the MST game.

1.2. Our contribution

We investigate the problem of sharing the cost of multicasting in wireless networks in both the general symmetric case and the special *Euclidean* one, where stations are located in the d-dimensional Euclidean space.

In the general symmetric case, according to the model for multicasting in wired networks adopted in [19], we first consider the case in which the power assignments are induced by a fixed universal spanning tree and we show that the deriving cost function is non-decreasing and submodular, and thus the Shapley value yields a BB mechanism, while the MC yields an efficient one. Successively, as a universal spanning tree can induce a power assignment arbitrarily more expensive than the optimal ones, in order to answer the natural question of guaranteeing communication with a minimum total energy consumption, we focus on the determination of a mechanism that distributes the cost of an optimal power assignment. Motivated by the NP-hardness of the problem and by the impossibility of having polynomial-time mechanisms satisfying β-efficiency [43], by exploiting the notion of β-BB condition [29] we provide a $(3\ln(k+1))$-BB strategyproof cost sharing mechanism for general wireless networks, where k is the number of receivers.

When restricted to Euclidean wireless networks, we show that if $\alpha = 1$ or $d = 1$ the optimal multicast cost yields a non-decreasing submodular function. Therefore, the Shapley value gives rise to a 1-BB mechanism and the MC to an efficient one, again both efficiently computable. For $\alpha > 1$ and $d > 1$ we show that optimal multicast power assignments, that are NP-hard to compute [11], in general do not admit cross-monotonic cost sharing methods.

Therefore, by extending the results in [21] concerning the approximation of the MST algorithm for broadcasting, we prove that for $\alpha \geqslant d \geqslant 2$ (i) the cost of a minimum Steiner tree connecting a source s to given set of receivers R is at most $3^d - 1$ times the optimal one for multicasting to R and (ii) any Steiner tree connecting s to R induces a power assignment that does not exceed its cost. Therefore, by exploiting the results in [29], we provide a class of $2(3^d - 1)$-BB cross-monotonic cost sharing methods that can be determined in a polynomial running time and consequently efficiently computable $2(3^d - 1)$-BB group strategyproof mechanisms for any $\alpha \geqslant d \geqslant 2$. Moreover, for the case $d = 2$ the result in [1] implies that our mechanisms are 12-BB.

The paper is organized as follows. In the next section are presented the results obtained for symmetric wireless networks, and in Section 3 the ones obtained for Euclidean wireless networks. Finally, in Section 4 we address conclusions and some open problems.

2. Cost sharing mechanisms for symmetric wireless networks

2.1. Mechanisms based on universal trees

Similar to the result for wired networks illustrated in Section 1.1.1 [19], we first consider the power assignments induced by *universal broadcast trees* in the cost graph $G = (S, c)$, that is directed trees $T(S \backslash \{s\})$ spanning all the stations in $S \backslash \{s\}$ which allow broadcast communications from s. For any subset of receivers $R \subseteq S \backslash \{s\}$, we obtain a directed tree $T(R)$ for multicasting to R by the union of all the directed paths in $T(S \backslash \{s\})$ connecting s to the agents in R. The power assignment ω_R that implements the multicast tree $T(R)$ is naturally defined as $\omega_R(x_i) = \max_{(x_i, x_j) \in T(R)} c(x_i, x_j)$.

Lemma 2.1. *The cost function C yielded by any universal tree $T(S \backslash \{s\})$ is non-decreasing and submodular.*

Proof. Clearly, $C(\emptyset) = 0$ and for every $Q \subseteq R$, as $T(Q)$ is contained in $T(R)$, it must be $C(Q) \leqslant C(R)$. In order to show that for any two subsets of receivers Q and R, $C(Q \cup R) \leqslant C(Q) + C(R) - C(Q \cap R)$ it suffices to observe that $\forall Q, R \subseteq S \backslash \{s\}$ and $\forall x_i \in S$, both these conditions are met: (i) $\omega_{Q \cup R}(x_i) = \max\{\omega_Q(x_i), \omega_R(x_i)\}$, and (ii) $\omega_{Q \cap R}(x_i) \leqslant \min\{\omega_Q(x_i), \omega_R(x_i)\}$. Thus, as $\omega_{Q \cup R}(x_i) + \omega_{Q \cap R}(x_i) \leqslant \max\{\omega_Q(x_i), \omega_R(x_i)\} + \min\{\omega_Q(x_i), \omega_R(x_i)\} = \omega_Q(x_i) + \omega_R(x_i)$, by summing over all the $x_i \in S$, it follows that C is also submodular. □

As illustrated in Section 1.1, by Lemma 2.1 the MC mechanism is efficient, strategyproof (but not group strategyproof) and meets NPT, VP and CS [38]. Moreover, according to the results in [37,38], the Shapley value mechanism is group strategyproof and satisfies BB, NPT, VP and CS.

It is worth while noticing that, as already remarked in Section 1.1, the *Shapley value formula* (4) in this case has a quite simple and intuitive explanation: the transmission power of a station is distributed among the agents using it in the multi-hop communication from the source in such a way that the portion of power used to reach the next-hop stations of some receiving agents is equally distributed among such agents. That is, given a station x in $T(R)$, if $y_1, \ldots, y_k$ are the children of x in $T(R)$ listed in non-decreasing order of cost from x (i.e., $c(x, y_i) \leqslant c(x, y_{i+1})$ for $i = 1, \ldots, k-1$), then the power emission of x, that is $\omega_R(x)$, is divided as follows: the portion of power $c(x, y_i) - c(x, y_{i-1})$ (where $c(x, y_0) = 0$), for $1 \leqslant i \leqslant k$, is equally divided among the agents whose next hop station from x is one of the stations from y_i to y_k.

The main drawback of the mechanisms just described is that a universal spanning tree can induce a power assignment arbitrarily more expensive than the optimal one.

The problem of designing strategyproof mechanisms that meet efficiency, NPT, VP, CS and CO has been recently also investigated by Penna and Ventre in [43]. By adapting the known VCG technique to this problem they showed that it admits a polynomial-time strategyproof mechanism satisfying efficiency, NPT, VP, CS and CO based on polynomial-time exact algorithms for maximizing the net worth. Motivated by the NP-hardness of the problem, they first studied the problem restricted to the special case of trees and provided a distributed algorithm which computes in polynomial time the optimal net worth. Successively, they showed that a pre-computed shortest path tree in the cost graph can be used as a (universal) tree in order to obtain a polynomial-time strategyproof mechanism which meets NPT, VP, CS and O(n)-CO for general transmission graphs, but not O(n)-efficiency. Importantly, they prove that for any $\beta > 0$ no polynomial-time algorithm can guarantee β-efficiency, unless P = NP.

2.2. An approximate BB mechanism for symmetric wireless networks

Ruled out the possibility of having polynomial-time mechanisms satisfying β-efficiency, it remains to be explored the possibility of defining a β-BB strategyproof mechanism for multicast transmissions in symmetric wireless networks. In this work, we provide a positive result to this question by providing a $3\ln(k+1)$-BB strategyproof cost sharing mechanism for multicast transmissions in such networks, where k is the number of receivers, which exploits the reduction presented in [9] from MEMT to the NWST problem. Indeed, we first prove a $1.5\ln k$-BB strategyproof cost sharing mechanism for the non-cooperative NWST, where k is the number of "terminals". Then, as such a mechanism only shares the cost of a weakly connected multicast tree, we show a second mechanism sharing the cost of the edges added in order to make it a directed spanning tree rooted at s. Informally speaking, our mechanism consists of these two mechanisms combined together.

We recall that NWST is defined as follows: given an undirected node weighted graph $H = (V, E)$ with a non-negative *node weight function* $\delta : V \to \mathbb{R}^+$ and a set of k required *terminals* $V' \subseteq V$, [1] we wish to compute a minimum weight Steiner tree spanning V'. In Section 2.2.1, we recall the approximability results of MEMT which exploit the reduction to NWST [9,28].

For a station x_i, let n_i denote the number of different transmission costs $c(x_i, x_j)$ (for $j \neq i$) in the edges incident to x_i and, for $m = 1, \ldots, n_i$, let C_i^m be the mth smallest transmission cost. Finally, for the sake of simplicity, let us assume that $s = x_n$.

2.2.1. Approximating MEMT by exploiting a reduction to NWST

Let $I_{\text{MEMT}} = \{G = (S, c), s, R\}$ be an instance for MEMT defined, as we know, by a cost graph $G = (S, c)$, a distinguished source station $s \in S$ and a subset $R \subseteq S\backslash\{s\}$ of receivers. Then, the reduction to an instance $I_{\text{NWST}} = \{H = (V, E), \delta, V'\}$ of NWST described in [9] works as follows:

- The set of nodes V is the union of $|S| = n$ disjoint sets of nodes Z_i called *supernodes*. Each supernode Z_i is composed of an *input node* Z_i^0 and n_i *output nodes* Z_i^j, for $j = 1, \ldots, n_i$.
- The set of edges E contains an edge (Z_i^m, Z_j^0) if $C_i^m \geqslant c(x_i, x_j)$. Also, for each station x_i, E contains an edge between Z_i^0 and each output node Z_i^j for $j = 1, \ldots, n_i$.
- The node weight function δ is defined as $\delta(Z_i^0) = 0$ for $i = 1, \ldots, n$ and $\delta(Z_i^m) = C_i^m$ for $m = 1, \ldots, n_i$.
- The set of terminals is defined as $V' = \{Z_i^0 \mid x_i \in R \cup \{s\}\}$.

Given a solution $T[I_{\text{NWST}}]$ for I_{NWST}, we compute a breadth first search (BFS) which starts from Z_n^0 (i.e., the input node corresponding to the source s) and number the nodes of the tree $T[I_{\text{NWST}}]$ spanning V'. Let $n(Z_i^m)$ be the BFS number of Z_i^m. Then, we construct a directed spanning tree $T[I_{\text{MEMT}}]$ for I_{MEMT} rooted at $s = x_n$ and spanning R in this way: for each edge $(Z_i^m, Z_j^{m'})$ such that $n(Z_i^m) < n(Z_j^{m'})$, the tree $T[I_{\text{MEMT}}]$ contains a directed edge $\langle x_i, x_j \rangle$. Finally, the output for I_{MEMT} is the power assignment ω defined as $\omega(x_i) = \max_{\langle x_i, x_j\rangle \in T[I_{\text{MEMT}}]} c(x_i, x_j)$, if x_i has at least one outgoing edge in $T[I_{\text{MEMT}}]$, and $\omega(x_i) = 0$, otherwise.

Caragiannis et al. [9] proved that if $T[I_{\text{NWST}}]$ is a ρ approximate solution to I_{NWST}, then ω is a 2ρ approximate solution to I_{MEMT}. In other words, a reduction to NWST is used in order to compute a weakly connected [2] multicast tree rooted at s. The solution is then made feasible by adding new edges to the multicast tree. The total cost of the added edges is at most equal to the cost of the weakly connected tree.

The first approximation algorithm for NWST, achieving an approximation ratio equal to $2\ln|k|$, where k is the number of terminals, is due to Klein and Ravi [33]. Their approach has been improved by Guha and Khuller in [28] by designing a $1.35\ln|k|$ and a simpler $1.5\ln|k|$ approximation algorithm. Consequently, there exists a $2.7\ln(|R|+1)$ and a $3\ln(|R|+1)$ approximation algorithm for MEMT. For technical reasons, that will be clear as soon as we will have defined our mechanism for NWST, the $1.35\ln|k|$-approximation algorithm cannot be used as a substrate to define our strategyproof cost sharing mechanism for the non-cooperative NWST, but we have to resort to the simpler $1.5\ln k$-approximation.

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[1] Without loss of generality, we assume that terminals have degree one, since for each terminal t, we can create a new node t' with zero weight, add the edge (t, t') and consider t' as the terminal.

[2] We recall that a direct graph is weakly connected if the undirected graph obtained by making each directed edge undirected is connected.

The $1.5 \ln k$*-approximation algorithm* $\mathcal{A}_{ST}$ *for NWST*: The $1.5 \ln |k|$-approximation algorithm is based on a decomposition theorem showing that every solution can be decomposed into complex objects called *branch-spiders*. Before recalling the algorithm, we introduce a few preliminary definitions [28]:

- A *spider* is a tree having at most one node of degree more than two. Such a node (if one exists) is called the center of the spider. In particular, an m spider ($m > 2$) is a spider with a center of degree m and an $m+$ spider is one with one node of degree at least m.
- A *branch* is a tree with at most three leaves. One of the leaves is called root.
- A *branch-spider* is constructed by merging the roots of a collection of branches into a single vertex, called the center. A 3+ branch-spider is one with at least three terminals.

Given a spider Sp, the cost $cost(Sp)$ of a spider is defined as the sum of the weights of the nodes in the spider. Denoted with T_{Sp} the set of terminals belonging to Sp, we define $ratio(Sp) = cost(Sp)/|T_{Sp}|$. Clearly, the leaves of a minimum ratio spider are terminals.

Given an instance $I_{\mathrm{NWST}} = \{H = (V, E), \delta, V'\}$ of NWST, the algorithm $\mathcal{A}_{ST}$ works iteratively, by repeatedly "shrinking" the 3+ branch-spider with minimum ratio [3] until there are at most two terminals left, and then connect them optimally. *Shrinking* a spider is the procedure of contracting all nodes of the spider and creating a new terminal to denote the contracted spider.

2.2.2. *A* $1.5 \ln k$*-BB strategyproof mechanism for NWST*

We assume a non-cooperative scenario, where each node x_i is a selfish agent (a potential terminal) which has a utility u_i and may report a different utility value v_i. In this section we propose a $1.5 \ln k$-BB strategyproof cost sharing mechanism for non-cooperative NWST. Our mechanism uses the approximation algorithm for NWST described above as a substrate to decide which terminals will be spanned in the computed solution and the cost share c_i for each of them.

With reference to the $1.5 \ln k$-approximation algorithm for NWST, we denote with $N_{Sp} = T_{Sp} \cap V$ the set of nodes of the instance graph $H = (V, E)$ which are also terminals belonging to a spider Sp, while given a terminal t we denote with N_t^+ the set of terminals belonging to t, i.e., the set of terminals which have been shrunk to form t.

A key point is the following: given any initial reported utility profile v, we need to iteratively associate a reported utility v_t to each new terminal t obtained by shrinking a spider Sp. To this aim, each such t is associated with a reported utility v_t equal to the minimum difference between a terminal's reported utility and its cost share over all the terminals belonging to the spider, multiplied by the number of terminals it contains, i.e., if $c_{t'} = \sum_{x_i \in N_{t'}^+} c_i$:

$$v_t = |T_{Sp}| \cdot \min_{t' \in T_{Sp}} \{v_{t'} - c_{t'}\}. \tag{5}$$

The NWST mechanism:

INPUT: an instance $I_{\mathrm{NWST}} = \{H = (V, E), \delta, V'\}$ of NWST and a reported utility profile v.

OUTPUT: a set of terminals (called the receivers) $R(v) \subseteq V'$, a Steiner tree $T(v)$ spanning $R(v)$ and the shared costs $c_i(v)$ for all $x_i \in R(v)$.

Initialization: $R(v) = \emptyset$; $T(v) = \emptyset$; $c_i(v) = 0, \forall x_i \in V'$;
while $R(v) \neq V'$ do
repeat the following steps until there are at most two terminals left and then connect them optimally
- find a 3+ branch-spider Sp in H with minimum ratio;
- if $ratio(Sp) \leqslant v_t$ for every $t \in T_{Sp}$, then
  $R(v) = R(v) \cup N_{Sp}$;
  $c_i(v) = ratio(Sp), \forall x_i \in N_{Sp}$;
  $c_i(v) = c_i(v) + \frac{ratio(Sp)}{|N_t^+|}, \forall x_i \in N_t^+$ and $\forall t \in T_{Sp} \setminus N_{Sp}$;
  $T(v) = T(v) \cup Sp$;
  shrink Sp, update H and let t be the obtained terminal;
  $V' = V' \cup \{t\}$;
  set v_t according to Eq. (5);

[3] See [28] for the procedure for finding a minimum weight 3+ branch-spider.

else for every $t \in T_{Sp}$ such that $ratio(Sp) > v_t$ do
let $X \subseteq V'$ be the set of terminals $x_i \in N_t^+$ s.t. $v_i - c_i(v) < \frac{v_t}{|N_t^+|}$;
$V' = V' \backslash X$;
go to the Initialization step.

Let us denote by $C(R(v))$ the weight of the Steiner tree $T(v)$ output by the mechanism (i.e., the global cost of the solution) and by $C^*(R(v))$ the cost of the optimal Steiner tree $T^*(v)$ spanning $R(v)$.

Theorem 2.2. *The mechanism returns a Steiner tree* $T(v)$ *such that* $C(R(v)) \leqslant (1.5 \ln |R(v)|) \cdot C^*(R(v))$.

Proof. The claim follows directly by showing that the solution $T_{\mathcal{A}}(R(v))$ returned by the approximation algorithm for the instance $I_{\text{NWST}} = \{H = (V, E), \delta, R(v)\}$ coincides with $T(v)$. To this aim suppose by contradiction that this is not the case. Consider the first spider Sp picked by the approximation algorithm. If the mechanism picks a different spider Sp', since the set of terminals is the same for both the algorithm and the mechanism, it must be that $ratio(Sp') < ratio(Sp)$, a contradiction. Using an inductive argument, this implies that the set of terminals remains the same at each step and that all the spiders picked by the algorithm are exactly the ones picked by the mechanism. Hence, they return identical solutions. □

As the mechanism clearly meets cost recovery, previous theorem shows that the mechanism is $1.5 \ln |R(v)|$-BB. It is quite easy to see that it runs in polynomial time and satisfies *VP*, *NPT* and *CS*.

In order to prove strategyproofness, it suffices to prove that the cost charged to any receiver is independent from its reported utility. To this aim, it is important to note that as far as the mechanism proceeds in its iterations, the cost shares of the potential receivers are always increasing. Thus, if some node is discarded from the solution at some point, it would have been essentially the same if it had been discarded from the beginning.

Theorem 2.3. *The mechanism described above is a strategyproof.*

Proof. First of all we stress that the mechanism does only comparisons on x_i's reported utility v_i, thus the amount v_i determines only if x_i belongs to $R(v)$ or not, but not the structure of $T(v)$ and thus the shared cost $c_i(v)$.

Consider the case in which $v_i < u_i$. If $x_i \notin R(v)$ then $w_i = 0$ and so x_i's welfare cannot decrease if x_i reports the true utility u_i. If $x_i \in R(v)$ then, since the cost share of an agent is independent from its reported utility, again x_i's welfare cannot decrease if x_i reports the true utility u_i.

Now consider the case in which $v_i > u_i$. If $x_i \notin R(v)$ then $w_i = 0$ and so x_i's welfare cannot decrease if x_i reports the true utility u_i. If $x_i \in R(v)$ we have that $c_i(v) \leqslant u_i$ because of the *VP* property. Again, since the cost share of an agent is independent from its reported utility, x_i still belongs to the receivers' set and its welfare cannot decrease if it reports the true utility u_i. □

Unfortunately, the mechanism is not group strategyproof as shown in the following example.

Consider the instance represented by the graph depicted in Fig. 1a and set $V' = \{1, 5, 6, 7\}$. Let us consider the outcome of the cost sharing mechanism when the nodes report their true utilities $u_1 = u_5 = u_6 = 3$, $u_7 = \frac{3}{2}$.

It is easy to check that the first spider chosen by the mechanism is one of the two minimum ratio spiders $Sp2$ and $Sp3$ depicted in Fig. 1b. Suppose that the mechanism chooses $Sp2$. Its ratio is equal to 1 and the set of spanned terminals $\{1, 5, 7\}$ can pay for the spider since $\min\{u_1, u_5, u_7\} = \frac{3}{2}$, then $c_1(u) = c_5(u) = c_7(u) = 1$. After the shrinking of $Sp2$ we are left only two terminals, the new one, say t, obtained from $Sp2$ having a reported utility $v_t = \frac{3}{2}$ and node 6, and the mechanism connects them optimally by choosing the path $1 \to 4 \to 6$ of ratio $\frac{3}{2}$. In fact, the set of connected terminals $\{t, 6\}$ can pay for the path since $\min\{u_t, u_6\} = \frac{3}{2}$, then $c_1(u) = c_5(u) = c_7(u) = \frac{3}{2}$ and $c_6(u) = \frac{3}{2}$. The individual welfare of receivers is $w_1(u) = w_5(u) = w_6(u) = \frac{3}{2}$ and $w_7(u) = 0$.

Now suppose that all the candidate receivers collude and so x_7 reports a utility $v_7 = \frac{3}{2} - \varepsilon \neq u_7$, where $\frac{3}{2} > \varepsilon > 0$. As before, the mechanism chooses $Sp2$ and contracts it thus obtaining the new terminal t, but now the set $\{t, 6\}$ cannot pay for the path $1 \to 4 \to 6$ of ratio $\frac{3}{2}$, since this time $v_t = \frac{3}{2} - 3\varepsilon$. According to the mechanism u_7 is discarded from the set of potential receivers and the computation is restarted. It follows that at the next iteration the mechanism chooses spider $Sp1$ also depicted in Fig. 1b. Its ratio is equal to $1 + \varepsilon/3$ and x_1, x_5 and x_6 can pay for it, thus

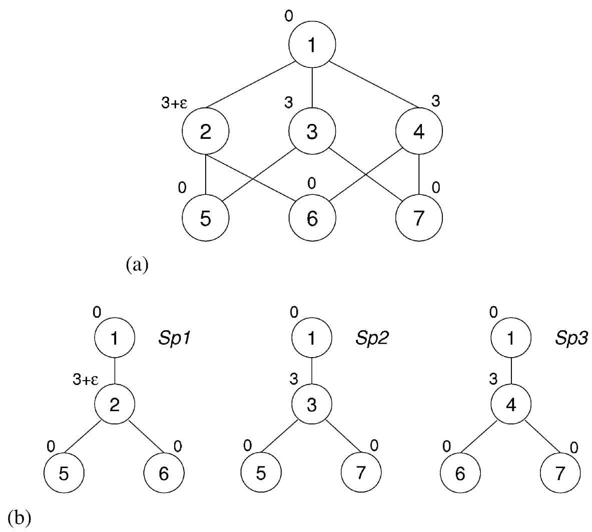

Fig. 1. (a) An instance of the NWST for which the mechanism is not group strategyproof. (b) Three spiders $Sp1$, $Sp2$ and $Sp3$.

$c_1(v) = c_5(v) = c_6(v) = 1 + \varepsilon/3$. After the shrinking of $Sp1$ all the terminals are connected, hence the mechanism terminates. In this case, the individual welfare of the receivers is $w_1(v) = w_5(v) = w_6(v) = 2 - \varepsilon/3$ and $w_7(v) = 0$. Since w_7 remains the same and w_1, w_5 and w_6 increase, the considered joined strategy dominates the one in which they report their true utilities. Hence, the mechanism is not group strategyproof.

We now explain the reason why we do not use the $(1.35 \ln k)$-approximation algorithm for NWST in defining our mechanism. This algorithm is based on a guessing of the optimal solution for the problem and for such a reason it must be run a high (but still polynomial) number of times. Since cost sharing mechanisms have to construct dynamically the set of the final spanned terminals, an exponential number of guesses needs to be performed in order to achieve the desired approximation ratio, thus making the mechanism unpractical.

We are now ready to present our cost sharing mechanism for multicast transmissions in symmetric wireless networks.

2.2.3. A $3\ln(k+1)$-BB strategyproof mechanism for multicast transmissions in wireless networks

By using the reduction introduced in [9] and illustrated in Section 2.2.1, we can transform an instance $I_{\text{MEMT}}[R, v] = \{G = (S, c), s, R, v\}$ for the MEMT in wireless networks spanning $R \subseteq S\backslash\{s\}$ with a utility profile v, to an instance $I_{\text{NWST}}[V', v'] = \{H = (V, E), \delta, V', v'\}$ of NWST with a utility profile v' such that

- $V' = \{Z_i^0 \mid x_i \in R \cup \{s\}\}$, i.e., each terminal (or receiver) is an input node Z_i^0 associated with a receiver or the source station.
- Denoted by v'_i the reported utility associated with the terminal $Z_i^0 \in V'$, the related utility profile v' is obtained from v by simply setting $v'_i = v_i$ for all $x_i \in R$ and (under the assumption that $s = x_n$) $v'_n = \infty$.

We notice that, in order for the mechanism to work, it is necessary to modify slightly the one for the NWST so that Z_n^0 is considered as a terminal to be connected, but it is not taken into account in the cost sharing process. This means that we do not have to consider Z_n^0 as a terminal when computing the ratio of a spider and consequently when sharing its cost.

The cost sharing mechanism for symmetric wireless networks:

Input: an instance $I_{\text{MEMT}}[S\backslash\{s\}, v] = \{G = (S, c), s, S\backslash\{s\}, v\}$ for the non-cooperative MEMT in wireless networks with a utility profile v.

Output: The set $R(v)$ of receiving stations, the power assignment $\omega_{R(v)}$ and the cost shares $c_i(v)$, $\forall x_i \in R(v)$.

(1) $R(v) = \emptyset$; $R' = S\backslash\{s\}$;
(2) reduce $I_{\text{MEMT}}[R', v]$ to an instance $I_{\text{NWST}}[V', v'] = \{H = (V, E), \delta, V', v'\}$ of NWST with a utility profile v' according to the method introduced above;
(3) while $R' \neq R(v)$ do

(a) let $T_{\mathrm{NWST}}[V', v']$ be the Steiner tree computed by the mechanism for NWST spanning the selected receivers $\widehat{R}(v')$ and $c_i(v')$ be the cost share for each $Z_i^0 \in \widehat{R}(v')$;
(b) let ω' be the power assignment inducing the Steiner tree $T_{\mathrm{NWST}}[V', v']$ and let ω be the solution for MEMT constructed starting from $T_{\mathrm{NWST}}[V', v']$, according to the reduction described in Section 2.2.1 which numbers the nodes of the tree $T_{\mathrm{NWST}}[V', v']$ spanning V'; by construction, ω implements a directed tree rooted at s spanning $R(v) = \{x_i \mid x_i \neq s \ \wedge \ Z_i^0 \in \widehat{R}(v')\}$;
(c) following backward the enumeration of the receivers, for each $x_i \in S$ such that $\omega(x_i) > \omega'(x_i)$, let N_i be the set of stations downstreaming from station x_i according to ω. If $\min_{x_j \in N_i} \{v_j - c_j(v)\} \geqslant \omega(x_i)/|N_i|$ then $c_j(v) \leftarrow c_j(v) + \omega(x_i)/|N_i|$, $\forall x_j \in N_i$, else let $X \subseteq S \backslash \{s\}$ be the set of stations such that $v_i - c_i(v) < \omega(x_i)/|N_i|$. Set $R(v) = R(v) \backslash X$, set $R' = R(v)$ and go to step (2).

Exploiting the same arguments used for the NWST mechanism, one can show that the above mechanism runs in polynomial time, is strategyproof, $(3 \ln(|R(v)| + 1))$-BB and meets NPT, VP and CS. Clearly, by suitably adapting the instance depicted in Fig. 1a to wireless networks, it is possible to show that the mechanism is not group strategyproof.

3. Cost sharing mechanisms for Euclidean wireless networks

In this section we consider the restriction of the problem to the special case where the stations lay in a d-dimensional Euclidean space. We first address the case in which $\alpha = 1$ or $d = 1$ in Section 3.1 and then consider the one in which $\alpha > 1$ and $d > 1$ in Section 3.2.

3.1. Optimal mechanisms for Euclidean networks

In order to show the existence of efficiently computable mechanisms that are either 1-BB or efficient for the case of Euclidean wireless networks, when $\alpha = 1$ or $d = 1$, in this section we prove that for any subset of receivers R an optimal power assignment of cost $C^*(R)$ can be efficiently determined and that C^* is a non-decreasing submodular function. Hence, the Shapley value and the MC correspond to optimal mechanisms that can be computed in polynomial time.

Lemma 3.1. *If $\alpha = 1$ or $d = 1$ then C^* is a non-decreasing submodular function and a power assignment of cost $C^*(R)$ can be determined in polynomial time for any subset $R \subseteq S \backslash \{s\}$.*

Proof. If $\alpha = 1$, the transmission cost associated with any pair of stations x_i, x_j is $c(x_i, x_j) = dist(x_i, x_j)$ and the cost paid for reaching the farthest station $x \in R$ from s is at least $dist(s, x)$, since stepping through intermediate stations cannot cost less than going to x directly by a single hop. Therefore, the power assignment obtained by setting $w(s) = dist(s, x)$ and the powers of all the other stations equal to 0 has an optimal cost and allows a multicast from s to all the stations in R, since all of them are at distance at most $dist(s, x)$ from s. Clearly such a power assignment can be determined in polynomial time. Moreover, C^* is non-decreasing and submodular since trivially $C^*(\emptyset) = 0$, $C^*(Q) \leqslant C^*(R)$ if $Q \subseteq R$ and for any two subsets of receivers R and Q the inequality $C^*(R \cup Q) \leqslant C^*(R) + C^*(Q) - C^*(R \cap Q)$ holds, because $C^*(R \cup Q) = \max\{C^*(R), C^*(Q)\}$ and $C^*(R \cap Q) \leqslant \min\{C^*(R), C^*(Q)\}$.

When $d = 1$, all the stations lay on a line. Let $x_1, \ldots, x_n$ be the stations of S ordered according to their occurrence along the line, i.e., $x_1 < x_2 < \cdots < x_n$ and let $s = x_k$ be the source. Moreover, let us denote by x_{f_R} (resp., x_{l_R}) the first (resp., last) station of any subset $R \cup \{s\} \subseteq S$ along the line, for any $R \subseteq S$, and by x_{f_ω} (resp., x_{l_ω}) the farthest station x before (resp., after) s in the line directly reachable from s (i.e., such that $\omega(s) \geqslant \gamma \cdot dist(s, x)^\alpha$), given any power assignment ω.

Without loss of generality, let us consider each subset $R \subseteq S$ as the minimal interval $[x_{f_R}, x_{l_R}]$ along the line including $R \cup \{s\}$.

Since $\alpha \geqslant 1$, it can be easily checked that for any power assignment ω multicasting to R, the power assignment ω_R to the stations such that

$$\omega_R(x_j) = \begin{cases} \omega(s) & \text{if } j = k \text{ (i.e., } x_j = s), \\ dist(x_j, x_{j+1})^\alpha & \text{if } l_\omega \leqslant j < l_R, \\ dist(x_j, x_{j-1})^\alpha & \text{if } f_R < j \leqslant f_\omega \end{cases}$$

satisfies the inequality $cost(\omega_R) \leqslant cost(\omega)$. In fact, the overall power consumption can be only reduced by maximizing the number of hops before x_{f_ω} and after x_{l_ω}.

Therefore, a power assignment ω^*_R of minimum cost $C^*(R) = cost(\omega^*_R)$ can be determined in polynomial time trying the at most n possible choices for the power emission of s and then completing as described above for reaching all the stations.

Again we prove that C^* is non-decreasing and submodular. Clearly, $C^*(\emptyset) = 0$ and $C^*(Q) \leqslant C^*(R)$ if $Q \subseteq R$. We only need to prove that $C^*(R \cup Q) \leqslant C^*(R) + C^*(Q) - C^*(R \cap Q)$ (see inequality (2)).

Let ω^*_Q and ω^*_R be the optimal power assignments for multicasting to Q and R, respectively. Let us consider: (a) the (eventually not optimal) power assignment $\omega_{R\cup Q}$ for $R \cup Q$ obtained by letting $\omega_{R\cup Q}(s) = \max\{\omega^*_R(s), \omega^*_Q(s)\}$ and then completing in the described way; (b) the power assignment $\omega_{R\cap Q}$ for $R \cap Q$ obtained by completing as above the power assignment $\omega_{R\cap Q}(s) = \min\{\omega^*_R(s), \omega^*_Q(s)\}$ to the source s.

For the sake of simplicity, we assume that $\omega^*_R(s) \leqslant \omega^*_Q(s)$. Hence, we have

$$C^*(R \cup Q) \leqslant cost(\omega_{R\cup Q}) = C^*(Q) + \Delta f(R \cup Q, Q) + \Delta l(Q, R \cup Q),$$
$$C^*(R \cap Q) \leqslant cost(\omega_{R\cap Q}) = C^*(R) - \Delta f(R, R \cap Q) - \Delta l(R \cap Q, R),$$

where for any subsets X, Y: $\Delta f(X, Y) = \sum_{f_X < j \leqslant f_Y} dist(x_j, x_{j-1})^\alpha$ and $\Delta l(X, Y) = \sum_{l_X < j \leqslant l_Y} dist(x_j, x_{j-1})^\alpha$.

Since inequality (2) is trivially satisfied if $Q \subseteq R$, we only focus on any two subsets Q and R such that neither $[x_{f_Q}, x_{l_Q}] \nsubseteq [x_{f_R}, x_{l_R}]$ nor $[x_{f_R}, x_{l_R}] \nsubseteq [x_{f_Q}, x_{l_Q}]$. Since by assumption the source s belongs to each interval, clearly the following cases suffice to conclude the proof:

- Let $x_{f_Q} < x_{f_R} \leqslant x_{l_Q} < x_{l_R}$. Since $R \cup Q = [x_{f_Q}, x_{l_R}]$ and $R \cap Q = [x_{f_R}, x_{l_Q}]$, it is $\Delta f(R \cup Q, Q) = \Delta f(R, R \cap Q) = 0$ and $\Delta l(Q, R \cup Q) = \Delta l(R \cap Q, R)$. This completes the proof of the lemma.
- Let $x_{f_R} < x_{f_Q} \leqslant x_{l_R} < x_{l_Q}$. Since $R \cup Q = [x_{f_R}, x_{l_Q}]$ and $R \cap Q = [x_{f_Q}, x_{l_R}]$, it is $\Delta f(R \cup Q, Q) = \Delta f(R, R \cap Q)$ and $\Delta l(Q, R \cup Q) = \Delta l(R \cap Q, R) = 0$. This completes the proof of the lemma. □

As illustrated in Section 1.1, by Lemma 3.1, we obtain the following theorem.

Theorem 3.2. *If $\alpha = 1$ or $d = 1$ then the Shapley value provides an efficiently computable optimally BB mechanism meeting NPT, VP, CS and the group strategyproofness. Moreover, the MC can be computed in polynomial time and provides an efficient mechanism meeting NPT, VP, CS and the group strategyproofness.*

Proof. By the above discussion it remains to show only that the MC mechanism can be computed in polynomial time both when $\alpha = 1$ and $d = 1$.

In the first case, the cost of an optimal power assignment for multicasting to a subset of receivers R is determined only by the station having maximum distance from the source. Thus, including all the intermediate stations does not increase the cost of the assignment and on the contrary can only improve the sum of the utilities of all the receiving agents. As a consequence, there are at most $n - 1$ possibilities for determining the largest efficient set, each obtained by choosing a station and including it with all the ones closer to the source in the set.

Analogously, when $d = 1$, that is all the stations belong to a line, the cost of an optimal power assignment is determined only by the first and the last receiving station along the line. Therefore, again including all the intermediate stations can only increase the efficiency and thus the largest efficient set is one of the at most n^2 subsets obtained by choosing the first and the last station and including them with all the intermediate ones in the set.

In both cases, since the optimal power assignments and thus the costs assigned to agents can be determined in polynomial time, the MC mechanism is efficiently computable. □

3.2. β-BB mechanisms for Euclidean networks

Starting from the results shown in the previous section, we now concentrate on the Euclidean case in which $\alpha > 1$ and $d > 1$.

Unfortunately, under these assumptions the problem in general is NP-hard [11], so that polynomial-time BB-mechanisms are unlikely to exist. Worse than that, even if polynomial-time solvability is not a concern, the following lemma concerning the optimal cost function C^* shows that no cross-monotonic cost sharing method in general exists for $\alpha > 1$ and $d > 1$ (and thus also the Shapley value is not cross-monotonic). Recalling the remarks on the core set

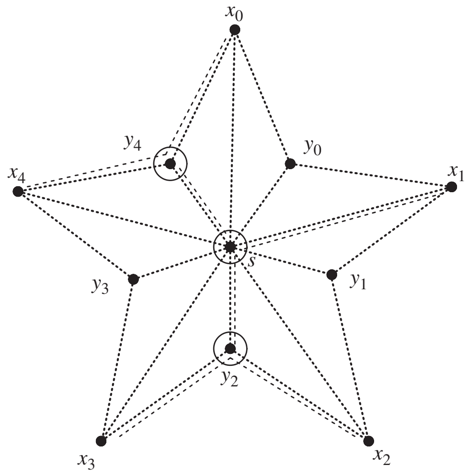

Fig. 2. An instance of the wireless multicast problem yielding an empty core.

in Section 1.1, as a further implication, C^* is not submodular. Hence, the emptiness of the core prevents from using cross-monotonic cost sharing methods to achieve BB group strategyproof mechanisms.

Lemma 3.3. *For any $\alpha > 1$ and $d > 1$ there exist instances of the wireless multicast problem for which $core(C^*)$ is empty.*

Proof. We prove the lemma by providing a suitable instance of the problem yielding an empty core. The instance is constructed as follows. For a suitably large integer m, five *external* stations $x_0, \ldots, x_4$ are placed along the border of a circle of radius m centered at the source s in correspondence of the corners of a pentagon, and another five *internal* stations $y_0, \ldots, y_4$ on the corners of a smaller pentagon of radius $m/2$, rotated in such a way that each is equidistant from the two external stations placed in the two closest adjacent corners of the external pentagon (see Fig. 2). Lines of *crossing* stations at distance one corresponding to the dotted lines in Fig. 2 connect the source to all such stations, and all the internal stations to the two closest external ones.

Since $\alpha > 1$, for m suitably large, in any optimal power assignment for multicasting to any subset of stations only the source and the five internal stations can have power assignments greater than 1. Moreover, as m increases, their contribution to the total cost becomes negligible with respect to the one yielded by the crossing stations, so that it can be ignored without affecting the correctness of the proof.

Let R be the subset of the five external stations. Then, for m suitably large, an optimal power assignment for multicasting to R corresponds to the one depicted with dashed lines in Fig. 2 in which two pairs of adjacent external stations are reached through the internal closest one, and one directly with a straight line. Clearly such a solution saves energy with respect to the one in which all the external stations are reached by straight lines, so that $C^*(\{x_j\}) > C^*(R)/5$ for every x_j. Moreover, the optimal power assignment for multicasting to any two adjacent external stations goes through the closest internal station, so that $C^*(\{x_0, x_1\}) + C^*(\{x_2, x_3\}) + C^*(\{x_4\}) = C^*(R)$ and thus $C^*(\{x_0, x_1\}) < 2C^*(R)/5$ since $C^*(\{x_0, x_1\}) = C^*(\{x_2, x_3\})$ and $C^*(\{x_4\}) > C^*(R)/5$.

Assume then by contradiction that there exists a core function f_0 for R. Then by symmetry, all the functions f_i with $1 \leqslant i \leqslant 4$ such that $f_i(x_j) = f(x_{j+i \bmod 5})$ belong to the core, and since the core set is convex, that is the convex combination of core functions belonging to the core still belongs to the core, the function f such that $f(x_j) = (f_0(x_j) + \cdots + f_4(x_j))/5$ belongs to the core and assigns the same allocation cost $C^*(R)/5$ to all the stations. Therefore, $f(x_0) + f(x_1) = 2C^*(R)/5 > C^*(\{x_0, x_1\})$: a contradiction, since f belongs to the core. □

Motivated by the above discussion, we now focus on the determination of approximate BB mechanisms. The fundamental observation leading to our mechanisms is that a Steiner tree in the cost graph $G = (S, c^{\alpha,\gamma})$ connecting s and the stations in R induces a power assignment for multicasting to R not exceeding its cost and that the cost of a minimum Steiner tree is close to the cost of an optimal power assignment for R. Thus, by applying the method of Jain and Vazirani [29], it is possible to find Steiner trees inducing a parametric family of cross-monotonic methods with a good approximation. The Steiner heuristic is very simple and consists, once determined a Steiner tree T in the cost graph connecting s and the stations in R, in orienting all the edges of T downwards and setting the powers of the internal stations so as to implement T, that is in such a way that each x_i in T has a power emission equal to the maximal cost of a descending incident edge. Clearly, denoted as $cost(T)$ the cost of the T, such a power assignment ω is such that $cost(\omega) \leqslant cost(T)$.

The following lemma is due to [21].

Lemma 3.4. *Let $Q \subseteq S$ be a subset of stations and $G(Q) = (Q, c^{\alpha,\gamma})$ be the cost graph induced by Q. Then, for any $\alpha \geqslant d$, if there exists $x_i \in Q$ such that $dist(x_i, x_j) \leqslant c$ for every other station $x_j \in Q$, a minimum spanning tree T of $G(Q)$ has cost $cost(T) \leqslant (3^d - 1)\gamma c^\alpha$.*

It is then possible to prove the following lemma.

Lemma 3.5. *Given a subset of receivers R, let T be a minimum Steiner tree in $G = (S, c^{\alpha,\gamma})$ connecting s and all the stations in R. Then, for any $\alpha \geqslant d$, $cost(T) \leqslant (3^d - 1)C^*(R)$.*

Proof. Consider an optimal power assignment ω_R^* and let $x_1, \ldots, x_k$ be the stations with strictly positive powers according to ω_R^*. Then, if Q_i is the subset of all the stations falling within the range of x_i, x_i included, i.e., $Q_i = \{x \in S \mid \omega_R^*(x_i) \geqslant \gamma \cdot dist(x, x_i)^\alpha\}$, for every i, $1 \leqslant i \leqslant k$, by applying Lemma 3.4 the cost of a minimum spanning tree T_i of $G(Q_i)$ is such that $cost(T_i) \leqslant (3^d - 1)\omega_R^*(x_i)$. The union of all the T_i's for the different values of i yields a subgraph G_R of $G = (S, c^{\alpha,\gamma})$ connecting s and all the stations in R of cost $cost(G_R) \leqslant \sum_{i=1}^{k} (3^d - 1)\omega_R^*(x_i) = (3^d - 1)C^*(R)$, and the lemma then follows by observing that the cost of a minimum Steiner tree T connecting s and the stations in R is at most $cost(G_R)$. □

Clearly, if a tree T is an ρ-approximation of a minimum Steiner tree connecting s and a subset of stations R, as a corollary of Lemma 3.5 it follows that the power assignment yielded by T corresponds to a $\rho(3^d - 1)$-approximation of an optimal power assignment for multicasting to R. Then, by exploiting the family of the 2-BB cost sharing methods for the Steiner tree problems presented in [29], it is possible to prove the following theorem.

Theorem 3.6. *For every $\alpha \geqslant d$ there exists a family of efficiently computable $2(3^d - 1)$-BB group strategyproof mechanisms for the wireless multicast problem in any d-dimensional Euclidean space.*

Proof. As shown in [29], for any weighted graph G and for any subset of nodes R in G, it is possible to find a Steiner tree T_R connecting a given source s to the nodes in R that corresponds to a 2-approximation of the minimum Steiner tree and satisfying the further property that there exists a family $\mathcal{F}$ of cross-monotonic sharing methods such that, for any $\xi \in \mathcal{F}$, $\sum_{x_i \in R} \xi(R, x_i) = cost(T_R)$, so that ξ corresponds to a 2-BB cost sharing method. Starting from $\mathcal{F}$, a $2(3^d - 1)$-BB cross-monotonic cost sharing method for the multicast problem in wireless networks can be obtained from any $\xi \in \mathcal{F}$ as follows. For any subset of receivers R, let T_R be a 2-approximation of the minimum Steiner connecting s and R in the cost graph $G = (S, c^{\alpha,\gamma})$, and let ξ be one of the corresponding cost sharing methods in $\mathcal{F}$. Then, ξ is also a $2(3^d - 1)$-BB cross-monotonic cost sharing method for the multicast problem in wireless networks. In fact, denoted as ω_R the power assignment for R determined according to the Steiner heuristic applied on T_R, it results that $\sum_{x_i \in R} \xi(R, x_i) = cost(T_R) \geqslant cost(\omega_R)$ and $\sum_{x_i \in R} \xi(R, x_i) = cost(T_R) \leqslant 2(3^d - 1)C^*(R)$. The theorem then follows directly from [29] by observing that ξ can be computed in polynomial time. □

For the special case $d = 2$ the result in Lemma 3.4 has been improved to the value 6 in [1]. Then, by applying the same arguments of Lemma 3.5 and Theorem 3.6, it is possible to prove the following theorem.

Theorem 3.7. *For every $\alpha \geqslant 2$ there exists a family of efficiently computable 12-BB group strategyproof mechanisms for the wireless multicast problem on the two-dimensional Euclidean space.*

4. Conclusions

In this paper, we provided suitably strategyproof and group strategyproof mechanisms for sharing the cost of multicast transmission services in symmetric wireless networks, both in the general and Euclidean model.

By the time the preliminary version of this paper appeared in the Proceedings of the 16th Annual ACM Symposium on Parallel Algorithms (SPAA 2004), the Euclidean mechanisms for $\alpha \geqslant d \geqslant 2$ has been improved in [43] by applying the new extended Moulin–Shenker approach based on the notion of self-cross-monotonic cost sharing method (see Section 1.1).

Interesting left open questions are the determination of cost sharing mechanisms for Euclidean wireless networks approximating the maximum efficiency for $d > 1$ and the budget balance for $\alpha < d$. To this aim, we observe that unfortunately the Steiner tree heuristic does not help, as in general it does not provide a good approximation [11].

It would be also nice to find the lowest approximation ratio that can be achieved by a BB cost sharing mechanism, even if not computable in polynomial time.

Another worth investigating issue is the determination of group strategyproof cost sharing mechanism for the general case.

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