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## Effects of phase transition induced density fluctuations on pulsar dynamics

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## ABSTRACT

We show that density fluctuations during phase transitions in pulsar cores may have non-trivial effects on pulsar timings, and may also possibly account for glitches and anti-glitches. These density fluctuations invariably lead to non-zero off-diagonal components of the moment of inertia, leading to transient wobbling of star. Thus, accurate measurements of pulsar timing and intensity modulations (from wobbling) may be used to identify the specific pattern of density fluctuations, hence the particular phase transition, occurring inside the pulsar core. Changes in quadrupole moment from rapidly evolving density fluctuations during the transition, with very short time scales, may provide a new source for gravitational waves.

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## 1. Introduction

The core of an astrophysical compact dense object, such as a neutron star, provides physical conditions where transition to exotic phases of quantum chromo-dynamics (QCD) [1] may be possible. Superfluid phases of nucleons are also believed to exist inside neutron stars with vortex de-pinning associated with glitches (though it may not provide a viable explanation for anti-glitches). In this paper we propose a technique to probe the dynamical phenomena happening inside the neutron star, which may also account for glitches and anti-glitches in a unified framework.

We consider density fluctuations which invariably arise in any phase transition. We show that even with relatively very small magnitudes, these density fluctuations may be observable with accurate measurement of pulsar timings which can detect very minute changes in the moment of inertia (MI) of the pulsar. This provides a very sensitive probe for density changes and density fluctuations (especially due to formation of topological defects) during phase transitions in the pulsar core. Non-zero off-diagonal components of moment of inertia arising from density fluctuations imply that a spinning neutron star will develop wobble leading to modulation of the peak intensity of pulses (as the direction

of the beam pointing towards earth undergoes additional modulation). This is a unique, falsifiable prediction of our model, that rapid changes in pulsar timings should, most often, be associated with modulations in changes in peak pulse intensity. It is important to note that the vortex de-pinning model of glitches is not expected to lead to additional wobble as the change in rotation caused by de-pinning of vortex clusters remains along the rotation axis. Density fluctuations will also lead to development of rapidly changing quadrupole moment which can provide a new source for gravitational wave emission due to extremely short time scales involved (despite small magnitude of this new contribution to the quadrupole moment).

The effect of phase changes on the moment of inertia has been discussed in literature. For example, moment of inertia change arising from a phase change to high density QCD phase (such as to the QGP phase) is discussed in Ref. [2]. In the scenario of Ref. [2], the transition is driven by slow decrease in the rotation speed of the pulsar, leading to increasing central density causing the transition as the central density becomes supercritical. It is assumed that as the supercritical core grows in size (slowly, over the time scale of millions of years), it continuously converts to the high density QGP phase (even when the transition is of first order). Due to very large time scale, the changes in moment of inertia are not directly observable, but observations of changes in the braking index may be possible.

Our work differs from these earlier works primarily in our focus on the *rapidly evolving density fluctuations* arising during the

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phase transitions. This has not been considered before as far as we are aware. (Though effects of density inhomogeneities in a rotating neutron star have been discussed [3].) Density fluctuations inevitably arise during phase transitions, e.g. during a first order transition in the form of nucleated bubbles, and may become very important in the critical regime during a continuous transition. The density fluctuations arising from a phase transition become especially prominent if the transition leads to formation of topological defects. Extended topological defects can lead to strong density fluctuations which can last for a relatively long time (compared to the phase transition time). It is obvious that such randomly arising density fluctuations will affect the moment of inertia of the star in important ways. Most importantly, it will lead to development of transient non-zero off-diagonal components of the moment of inertia, as well as transient quadrupole moment. Both of these will disappear after the density fluctuations decay away and the transition to a uniform new phase is complete. Net change in moment of inertia will have this transient part as well as the final value due to change to the new phase. It seems clear that this is precisely the pattern of a glitch or anti-glitch where rapid change in pulsar rotation is seen which slowly and *only partially* recovers to the original value. As we mentioned above, transient change in quadrupole moment will be important for gravitation wave emission. It is important to realize that the net change in MI (as discussed previously, e.g. in [2]) is only sensitive to the difference in the free energies of the two phases, and cannot distinguish different types of phase transitions. In contrast, density fluctuations arising during phase transitions crucially depend on the nature of phase transition, especially on the symmetry breaking pattern (e.g. via topological defects). Identification of these density fluctuations via pulsar timings (and gravitational waves) can pin down the specific transition occurring inside the pulsar core.

We consider the possibility of rapid phase transition in a large core of a neutron star. The scenario of slow transition as discussed in [2] is applicable for slowly evolving star (e.g. by accretion), with a transition which is either a weak first order, or a second order (or a crossover). If the transition is strongly first order then strong supercooling can lead to extremely suppressed nucleation rate, with transition occurring suddenly after the supercritical core becomes macroscopic in size. (This is similar to the situation of low nucleation rate leading to nucleation distance scales of the order of meters in the early universe as discussed by Witten [4].) Rapid transition in a large core of neutron star is naturally expected during the early stages of evolution of neutron star due to rapid cooling of star. It could also be driven by rapid accretion on a neutron star, coupled with a first order transition.

We will not discuss any detailed scenario of such a rapid transition in this paper, and rather just note the possibility of such transitions. Our focus will be that once such a rapid transition occurs, what are its observational implications. Clearly, change in phase will lead to net change in MI of star, affecting its rotation. This has been discussed earlier, e.g. in [2]. We will focus on additional effects associated with presence of density fluctuations which lead to qualitatively new effects, not included in the earlier investigations. These effects are a transient component of change in MI, development of the off-diagonal components of MI, and a transient, rapidly evolving quadrupole moment.

## 2. Effects of density fluctuations due to bubble nucleation

A rough estimate of change in the moment of inertia due to phase change can be taken from Ref. [2] using Newtonian approximation, and with the approximation of two density structure of the pulsar. If the density of the star changes from  $\rho_1$  to a higher

density  $\rho_2$  inside a core of radius  $R_0$ , then the fractional change in the moment of inertia is of order

$$\frac{\Delta I}{I} \simeq \frac{5}{3} \left( \frac{\rho_2}{\rho_1} - 1 \right) \frac{R_0^3}{R^3} \quad (1)$$

Here  $R$  is the radius of the star in the absence of the dense core. For a QCD phase transition, density changes can be of order one. We take the density change to be about 30% as an example. If we take the largest rapid fractional change in the moment of inertia of neutron stars, observed so far (from glitches), to be less than  $10^{-5}$ , then Eq. (1) implies that  $R_0 \leq 0.3$  km (taking  $R$  to be 10 km). For a superfluid transition, we may take change in density to be of order of superfluid condensation energy density  $\simeq 0.1$  MeV/fm<sup>3</sup> (see Ref. [5]). In such a case,  $R_0$  may be as large as 5 km. These constraints on  $R_0$  arise from observed data on glitches/anti-glitches. These estimates may also be taken as prediction of possible large fractional changes in the moment of inertia (hence pulsar spinning rate) of order few percent when a larger core undergoes rapid phase transition. For example,  $R_0$  may be of order 2–3 km for QCD transition (from estimates of high density core of neutron star [6]), or it may be only slightly smaller than  $R$  for superfluid transition.

We now discuss density fluctuations during phase transitions. First we consider fluctuations arising simply from a first order transition. Consider nucleation of relatively large number of bubbles (several thousand) inside the supercritical core. This is possible due to nonuniformities, even of purely statistical origin (e.g. from fluctuations in temperature [7]).

### 2.1. Parameters for bubble nucleation and results

We simulate random nucleation of bubbles filling up a core of size  $R_0$  ( $= 300$  meters). Effects of bubble nucleations will be characterized in terms of the following parameters.

Bubble radius  $r_0$  will be taken to vary from 5 meters to 20 meters, with bubble separation being of same order as bubble size (close packing).

Density change in bubble nucleation is taken to be about the nuclear saturation density  $\simeq 160$  MeV/fm<sup>3</sup> (e.g. for QCD scale).

We find that density fluctuations lead to fractional change in MI,  $\Delta I/I \simeq 4 \times 10^{-8}$  for  $r_0 = 20$  meters implying similar changes in pulsar timings. Change in MI remains of same order when  $r_0$  is changed from 20 meters to 5 meters. Due to random nature of bubble nucleation, off-diagonal components of the moment of inertia, as well as the quadrupole moment become non-zero and the ratio of both to the initial moment of inertia are found to be of order  $I_{xy}/I_0 \simeq Q/I_0 \simeq 10^{-11} - 10^{-10}$ .

This aspect of our model is extremely important, arising entirely due to density fluctuations generated during the transition. As these density fluctuations homogenize, finally leading to a uniform new phase of the core, both these components will dissipate away. The off-diagonal component of moment of inertia will necessarily lead to wobbling (on top of any present initially), which will get restored once the density fluctuations die away. This will lead to transient change not only in the pulse timing, but also in the pulse intensity (as the angle at which the beam points towards earth gets affected due to wobbling). We again emphasize that the conventional vortex de-pinning model of glitches is not expected to lead to additional wobble as the change in rotation caused by de-pinning of vortex clusters remains along the rotation axis. Thus, the presence of intensity modulations associated with a glitch can distinguish between our model and the vortex de-pinning model.

Generation of quadrupole moment has obvious implication for gravitational wave generation. One may think that a quadrupole

moment of order  $Q/I_0 \simeq 10^{-10}$  is too small for any significant gravity wave emission. However, note that the gravitation power depends on the (square of) third time derivative of the quadrupole moment [8]. The time scales will be extremely short here compared to the time scales considered in literature for the usual mechanisms of change in quadrupole moment of the neutron star. Here, phase transition dynamics will lead to changes in density fluctuations occurring in time scales of microseconds (or even shorter as we will see below in discussions of topological defects generated density fluctuations). This may more than compensate for the small amplitude of quadrupole moment and may lead to these density fluctuations as an important source of gravitational wave emission from neutron stars, as we will discuss below.

### 3. Density fluctuations from topological defects

Topological defects form during spontaneous symmetry breaking transitions via the so-called *Kibble mechanism* [9]. These defects can be source of large density fluctuations depending on the relevant energy scales, and their formation and evolution shows universal characteristics (e.g. scaling behavior). This may lead to reasonably model independent predictions for changes in MI, and quadrupole moment and subsequent relaxation. As bubbles, strings, domain walls, all generate different density fluctuations, with specific evolution patterns, high precision measurements of pulsar timings and intensity modulations (from wobbling) and its relaxation may be used to identify different sources of fluctuations, thereby pinning down the specific phase transition occurring. Specific phase transitions expected inside pulsars lead to different types of topological defects. Important thing is that a random network of defects will arise in any phase transition, and resulting defect distribution can be determined entirely using the symmetry breaking pattern. For example, superfluid transition leads to formation of random network of vortices. Confinement–deconfinement QCD transition can lead to formation of a network of domain walls and global strings arising from the spontaneous breaking of  $Z(3)$  center symmetry [10]. QCD transition may also give rise to only global strings, e.g. in the color flavor locked (CFL) phase with  $SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B$  symmetry (for 3 massless flavors) is broken down to the diagonal subgroup  $SU(3)_{C+L+R} \times Z_2$  [1]. As the resulting density fluctuations are mostly dominated by the nature of defect (domain walls, or strings, or both), we will model formation of these different defect networks in the following using correct energy scale and try to estimate resulting density fluctuations.

#### 3.1. Results of model simulation of defect network

We model formation of  $U(1)$  strings and  $Z_2$  domain walls by using correlation domain formation in a cubic lattice, with lattice spacing  $\xi$  representing the correlation length, as in Ref. [11]. It is not possible to carry out these simulation covering length scales of km (for star) to fm (QCD scale). Hence these simulations are necessarily restricted to very small system sizes. Each lattice site is associated with an angle  $\theta$  randomly varying between 0 and  $2\pi$  (to model  $U(1)$  global string formation), or two discrete values 0, 1 (when modeling  $Z_2$  domain wall formation). For string case, winding of  $\theta$  on each face of the cube is determined using the geodesic rule. For a non-zero winding, a string segment (of length equal to  $\xi$ ) is assumed to pass through that phase (normal to the phase). For domain wall case, any link connecting two neighboring sites differing in  $Z_2$  value is assumed to be intersected by a planar domain wall (of area  $\xi^2$ , and normal to the link). The mass density (i.e. mass per unit length) of the string was taken as 3 GeV/fm, and the domain wall tension is taken to be 7 GeV/fm<sup>2</sup>.

These values are taken as order of magnitude estimates from the numerical minimization results in Ref. [10] for the pure gauge case. (Note that logarithmic dependence of global strings on inter-string separation may lead to much larger density fluctuations than considered here.)

We consider spherical system of size  $R$  and confine defect network within a spherical core of radius  $R_c = \frac{0.3}{10}R$ . This is in view of the constraints on the supercritical core size  $R_c$  being of order 0.3 km for a neutron star with radius  $R = 10$  km. Of course, in our simulations,  $R$  is extremely small, with maximum value of 4000 fm. For  $\xi \simeq 10$  fm, we find the resulting value of  $\frac{\delta I}{I_i} \simeq 10^{-12}$ – $10^{-13}$  implying similar changes in the rotational frequency. Here  $I_i$ ,  $i = 1, 2, 3$  are the three diagonal values of the MI tensor. As we increase  $R_c$  from  $5\xi$  to about  $400\xi$ , we find that the value of  $\frac{\delta I}{I_i}$  stabilizes near  $10^{-13}$ – $10^{-14}$  as shown in Table 1. This change in  $\xi$  amounts to change in the number of string and wall segments by a factor of  $10^6$ . This gives a strong possibility that the same fractional change in the MI may also be possible when  $R$  is taken to have the realistic value of about 10 km, especially when we account for statistical fluctuations in the core. For the formation of domain walls we find fractional change in MI components (as well as quadrupole moments) to be larger by about a factor of 40. With accurate measurements, these small changes may be observable. Also, for a larger core size undergoing transition these changes can be larger. We are only presenting change in MI due to transient density fluctuations during phase transition, which as we see, can have either sign. (As we discussed above, the net change in MI will include the very large contribution of order  $10^{-5}$  due to net phase change of the core [2].) This suggests that the phase transition dynamics may be able to account for both glitch and anti-glitch events (with associated wobbling from off-diagonal components leading to pulse intensity modulation).

We now consider superfluid transition. A rapid superfluid transition could occur after transient heating of star (either due to another transition releasing latent heat, or due to accretion, etc.). (In this work we neglect any possible effect of star rotation on this mechanism of random vortex formation which is expected to lead to a much denser network than the one arising from star rotation.) We take the vortex energy per unit length to be 100 MeV/fm and correlation length for vortex formation of order 10 fm (Ref. [5]). Table 1 shows that the string induced transient fractional change in MI is of order  $10^{-10}$  (compared to net fractional change in MI of order  $10^{-5}$  as discussed in Section 2). Ratios of the quadrupole moment and off-diagonal components of MI to the net MI of the pulsar are also found to be of order  $10^{-10}$ . The transient change in the MI decays away when the string system coarsens, thereby restoring less than few percent of the original value. We note that this pattern (and numbers) are similar to that of a glitch (or anti-glitch).

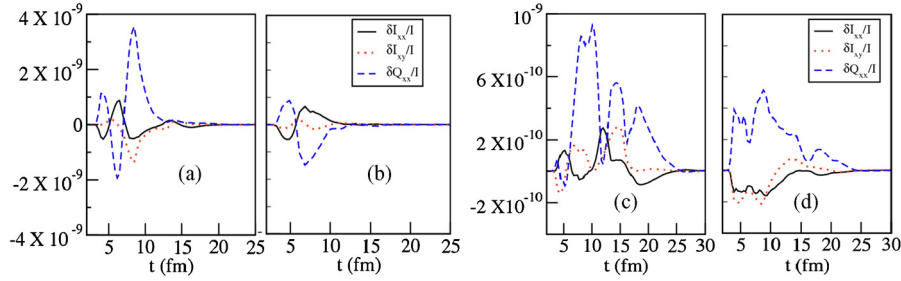
#### 3.2. Field theory simulations for QCD transition

As a further support for these estimates, we have also carried out field theory simulations of confinement–deconfinement (C–D) QCD transition using effective field theory Polyakov loop model. This leads to spontaneously broken  $Z(3)$  symmetry (for the  $SU(3)$  color group) in the QGP phase giving rise to topological domain wall defects in the QGP phase and also string defects forming at wall junctions. We carry out a field theory simulation for the C–D transition using a quench (quench is used for simplicity as only domain formation is relevant here), see Ref. [10] for details. It is not possible to carry out field theory simulation covering length scales of km (for star) to fm (QCD scale). Hence these simulations are necessarily restricted to system sizes of tens of fm only. The physical size of the lattice is taken as  $(7.5 \text{ fm})^3$  and  $(15 \text{ fm})^3$ . We

**Table 1**

Fractional change of various moments of the pulsar caused by inhomogeneities due to defects, with the correlation length  $\xi = 10$  fm. For QCD scale strings, the string tension is taken as 3 GeV/fm, while the QCD Z(3) wall tension is taken as 7 GeV/fm<sup>2</sup> (from simulations in Ref. [10]). For the superfluid vortices, the energy per unit length is taken to be 100 MeV/fm [5].

$\frac{R_c}{\xi}$	QCD strings			QCD walls			Superfluid strings		
	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{Q_{xx}}{I}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{Q_{xx}}{I}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{\delta Q_{xx}}{I}$
5	5E-10	-3E-10	-1E-10	2E-8	-1E-8	-8E-10	2E-6	-1E-6	-4E-7
50	5E-12	-2E-12	2E-12	1E-10	-8E-11	-1E-11	2E-8	-7E-9	7E-9
200	1E-13	2E-14	-7E-14	5E-12	-4E-12	-6E-12	5E-10	6E-11	-2E-10
400	-3E-15	-5E-14	-9E-14	3E-12	-2E-12	3E-14	-1E-11	-2E-10	-3E-10



**Fig. 1.** Fractional change in MI and quadrupole moment during phase transitions. (a),(b) correspond to lattice size  $(7.5 \text{ fm})^3$ , and (c),(d) correspond to lattice size  $(15 \text{ fm})^3$  respectively. Plots in (a) and (c) correspond to the C-D phase transition with Z(3) walls and strings, while plots in (b) and (d) correspond to the transition with only string formation as for the CFL phase.

use periodic boundary conditions and take a spherical region with radius  $R_c$  (representing star's core) to study change of MI, with  $R_c = 0.4 \times$  lattice size. We mention that the model of Ref. [10] does not directly apply to the case of neutron star which has large baryonic chemical potential. However, the most relevant features of the model are formation of Z(3) defects with similar energy scale as in the neutron star case. Hence we use those simulations [10] to estimate defect induced density fluctuations for the present neutron star case. We also use dissipation to relax density fluctuations but total energy is kept fixed by adding the dissipated energy. Thus we only focus on re-distribution of energy in defect network and the background. For the net change in the MI, we will use the estimates of  $\Delta I/I$  from Section 2 (Ref. [2]). QCD transition may also give rise to only global strings, e.g. in the color flavor locked (CFL) phase with  $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B$  symmetry (for 3 massless flavors) is broken down to the diagonal subgroup  $SU(3)_{c+L+R} \times Z_2$  [1]. For this case, we modify the model studied in Ref. [10] by removing terms which correspond to Z(3) structure of the vacuum manifold. This gives rise to string defects only without any domain walls, with energy scale of QCD.

Plots in Fig. 1 show the time evolution of the resulting fractional change in the MI and the quadrupole moment of the core relative to the initial total MI for these field theory simulations. Here we have taken dense core of fractional size 0.3/10 (in view of transition in core size of 300 meters for a 10 km star). We add the MI of a shell outside the core so that total system size =  $\frac{10}{0.3}$  of the core size, and the shell has the same uniform density as the core. Fractional changes for all components of MI as well as the ratio of quadrupole moment to MI are found to be of similar order. We are only presenting change in MI due to transient density fluctuations during phase transition, which as we see, can have either sign. (Net fractional change in MI will include the very large contribution of order  $10^{-5}$  due to net phase change of the core [2].) This suggests that the phase transition dynamics may be able to account for both glitch and anti-glitch events (with associated wobbling from off-diagonal components leading to pulse intensity modulation). As discussed above, rapid changes in the quadrupole moment will lead to gravitational wave emission.

#### 4. Gravitational wave generation due to density fluctuations

Despite the small values of quadrupole moments in Table 1, the power emitted in gravitational waves may not be small due to very short time scales in the present case. The defect coarsening will be governed by microphysics with time scale being of order tens of fm/c. Even with extremely dissipative motion of strings, and much larger length scales, the change in quadrupole moment due to strings can happen in an extremely short time scale (during string formation, and/or during string decay), thereby boosting the rate of quadrupole moment change. Even a very conservative value of the time scale of microseconds (e.g. for nucleation of bubbles with nucleation sites few meters apart, as discussed above) for the evolution of density fluctuations will still be 1000 times smaller than fastest pulsar rotation time of milliseconds, resulting in huge enhancement in the gravitational power, as can be seen from the following expression for the power [8]

$$\frac{dE}{dt} = -\frac{32G}{5c^5} \Delta Q^2 \omega^6 \simeq -(10^{33} \text{ J/s}) \left( \frac{\Delta Q/I_0}{10^{-6}} \right)^2 \left( \frac{10^{-3} \text{ sec.}}{\Delta t} \right)^6 \quad (2)$$

Here  $I_0$  is the MI of the pulsar, and  $\Delta Q$  is the change in the quadrupole moment occurring in time interval  $\Delta t$ . For the present case, we can take  $\Delta Q/I_0$  of order  $10^{-14}$ – $10^{-10}$  from Table 1 (which is much smaller than the value of  $10^{-6}$ , typically used for deformed neutron stars). Even though  $\Delta Q/I_0$  is very small here, the relevant time scale  $\Delta t$  is also very small. Thus, even with a conservative estimate of the time scale for the transition (e.g. bubble coalescence),  $\Delta t = 10^{-6}$ – $10^{-5}$  sec., the power in gravitational wave can be significant due to large enhancement from the  $(\frac{10^{-3} \text{ sec.}}{\Delta t})^6$  factor. A shorter time scale may make this source as potentially very prominent for gravitational waves. For an estimate of the expected strain amplitude from a pulsar at a distance  $r$ , we use the following expression [8]

$$h = \frac{4\pi^2 G \Delta Q f^2}{c^4 r} \simeq 10^{-24} \left( \frac{\Delta Q/I_0}{10^{-6}} \right) \left( \frac{10^{-3} \text{ sec.}}{\Delta t} \right)^2 \left( \frac{1 \text{ kpc}}{r} \right) \quad (3)$$

With  $\Delta Q/I_0$  of order  $10^{-10}$  and a time scale for the transition (e.g. bubble coalescence),  $\Delta t = 10^{-6}$ – $10^{-5}$  sec., we can have  $h \simeq 10^{-24}$ – $10^{-22}$  for a pulsar at 1 kpc distance. As we mentioned



above, the time scale for evolution of density fluctuations may be even shorter, leading to larger strain amplitudes (even if much smaller values of  $\Delta Q/I_0$  are taken from Table 1), and much larger power emitted in gravitational waves. Since the wave emission is only for a single burst, lasting for only duration  $\Delta t$ , net energy lost by the star remains small fraction of the star mass.

## 5. Conclusions

To summarize our results, we have shown that density fluctuations arising during a rapid phase transition lead to transient change in the MI of the star. Such density fluctuations in general lead to non-zero off-diagonal components of moment of inertia tensor which will cause the wobbling of pulsar, thereby modulating the peak intensity of the pulse. This is a distinguishing and falsifiable signature of our model. The conventional vortex de-pinning model of glitches is not expected to lead to additional wobble as the change in rotation caused by de-pinning of vortex clusters remains along the rotation axis. We find that moment of inertia can increase or decrease, which gives the possibility of accounting for the phenomenon of glitches and anti-glitches in a unified framework. Development of non-zero value of quadrupole moment (on a very short time scale) gives the possibility of gravitational radiation from the star whose core is undergoing a phase transition. Net change in MI (as discussed previously, e.g. in [2]) is only sensitive to the difference in the free energies of the two phases, and cannot distinguish different types of phase transitions. In contrast, density fluctuations arising during phase transitions crucially depend on the nature of phase transition, especially on the symmetry breaking pattern (e.g. via topological defects). Identification of these density fluctuations via pulsar timings (and gravitational waves) can pin down the specific transition occurring inside the pulsar core. This is an entirely new way of probing phase transitions occurring inside the core of a neutron star. Though our estimates suffer from the uncertainties of huge extrapolation involved from the core sizes we are able to simulate, to the realistic sizes, they strongly indicate that expected changes in moment of inertia, etc., may be well within the range of observations, and in fact may be able to even account for the phenomena of glitches and anti-glitches.

One aspect of glitches which may raise concern in our model is multiple occurrences of glitches. For vortex de-pinning model multiple glitches seem natural due to formation of vortices due to star rotation with glitches occurring with de-pinning of clusters of vortices. In our model, multiple occurrence of glitches will require multiple phase transitions. This is not very improbable, though, when the transition happens due to accretion from companion star causing heating, with subsequent cooling down. Essentially, in this case, the matter in the neutron star core will continue to remain in the vicinity of the phase boundary in the  $T-\mu$  plane (in the QCD phase diagram). Accretion will move the system across the bound-

ary by increasing  $\mu$ , causing a phase transition, and subsequent cooling will move the system back through the phase boundary with another transition. Such processes could in principle repeat for certain neutron stars leading to multiple glitches. One should also allow the possibility that glitches could occur due to multiple reasons. Some glitches/anti-glitches could occur due to the model proposed here, that is due to phase transition induced density fluctuations, while other glitches could occur due to the conventional de-pinning of vortex clusters. Main point of our work is to emphasize that if and when a transition happens in the core of a neutron star, it invariably leads to density fluctuations which manifest itself in glitch/anti-glitch like behavior, along with other implications such as wobbling of star, gravitational wave emission, etc.

It is thus important to investigate this interesting possibility further. With much larger simulations, and accounting for statistical fluctuations of temperature, chemical potential, etc., in the core, more definitive patterns of changes in moment of inertia tensor/quadrupole moment, etc., may emerge which may carry unique signatures of specific phase transitions involved. (For example, continuous transitions will lead to critical density fluctuations, and topological defects will induce characteristic density fluctuations depending on the specific symmetry breaking pattern.) If that happens then this method can provide a rich observational method of probing the physics of strongly interacting matter in the naturally occurring laboratory, that is interiors of neutron star. It will be interesting to see if any other astrophysical body, such as white dwarf, can also be probed in a similar manner.

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