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Computers and Mathematics with Applications 47 (2004) 419–425
An International Journal
computers & mathematics with applications
www.elsevier.com/locate/camwa

Absolute Stability of Lurie Systems with Impulsive Effects

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(Received and accepted August 2002)

Abstract—This paper studies absolute stability of Lurie systems with impulsive effects. Using the method of Lyapunov functions and the variation of parameters technique, we establish sufficient conditions for absolute stability.

Keywords—Impulsive Lurie systems, Absolute stability, Lyapunov function, Variation of parameters.

1. INTRODUCTION

There have been many papers dealing with dynamical systems with impulsive effects in recent years. The dynamic behavior of such systems is more complex than that of the classical dynamical systems. Several works have been presented in the literature [1–5] analyzing the stability of dynamical systems with impulsive effects. However, the absolute stability analysis of Lurie systems has been studied for several decades [6–9], and the absolute stability analysis of Lurie systems with impulsive effects was not considered. In the present paper, we study the absolute stability of Lurie systems with impulsive effects.

2. PRELIMINARIES

Denote by R the set of real numbers, R_+ the set of nonnegative real numbers, N the set of positive integers, and R^n the real n-space. For $x \in R^n$, denote by $x^\top = (x_1, x_2, \dots, x_n)$ the transpose of x, $\|x\|$ the Euclidean vector norm, i.e., $\|x\| = \sqrt{x^\top x}$. Let $R^{n\times n}$ denote the set of real $n \times n$ matrices. For $A \in R^{n\times n}$ denote by $\|A\|$ the norm of A induced by the Euclidean vector norm, i.e., $\|A\| = \sqrt{\lambda_{\max}(A^\top A)}$. For $B \in R^{n\times n}$, denote by $B^\top$ the transpose of B. For a symmetric matrix P, we write $P > 0$ ($P < 0$) if P is a positive definite (negative definite)

This work was partially supported by National Science Foundation of China (Grant No. 69874016 and 60074008), and NSERC-Canada. The second author was on leave from the Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1.

0898-1221/04/$ - see front matter Typeset by $\mathcal{A}_\mathcal{M}\mathcal{S}$-TEX
doi: 10.1016/S0898-1221(04)00021-5

matrix. Denote by $\lambda_{\min}(P)$, $\lambda_{\max}(P)$, respectively, the smallest and the largest eigenvalues of the matrix P.

Consider the Lurie system

$$\dot{x}(t) = Ax(t) + bf(\sigma(t)), \qquad \sigma(t) = c^{\top}x(t),$$

where $x(t) \in R^n$ is the state variable, $A \in R^{n\times n}$, $b, c \in R^n$, A is asymptotically stable, and

$$f(\sigma) \in F_\kappa = \left\{f(\sigma) \mid f(0) = 0,\ 0 < \sigma f(\sigma) \le \kappa\sigma^2,\ \sigma \ne 0,\ f(\sigma) \in C[R, R]\right\}.$$

Consider a discrete set $\{t_k\}$ of time instants, where $0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$, $t_k \to \infty$, as $k \to \infty$. At time instants t_k, jumps in the state variable x are denoted by $\Delta x|_{t=t_k} = x(t_k^+) - x(t_k)$, where $x(t_k^+) = \lim_{t\to t_k^+} x(t)$, $k \in N$. Let $I_k \in C[R^n, R^n]$ denote the incremental change of the state at the time t_k, and $I_k(0) = 0$ for all $k \in N$. Then, we get the following impulsive Lurie system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bf(\sigma(t)), && t \ne t_k, \\ \sigma(t) &= c^{\top}x(t), && t \ne t_k, \\ \Delta x &= I_k(x), && t = t_k, \\ x\left(t_0^+\right) &= x_0, && t_0 \ge 0, \quad k \in N. \end{aligned} \tag{1}$$

It is clear that the impulsive Lurie system (1) has a trivial solution.

DEFINITION 1. *The impulsive Lurie system (1) is said to be absolutely stable if for any f in F_κ, the trivial solution of system (1) is globally asymptotically stable.*

To study the absolute stability of the impulsive Lurie system (1), we use the following definition and lemma [1].

DEFINITION 2. *Letting $V : R_+ \times R^n \to R_+$, for $(t, x) \in (t_{k-1}, t_k] \times R^n$, we define*

$$D^+V(t, x) = \lim_{h\to 0^+} \sup \frac{1}{h}\left\{V\left[t + h, x + h\left(Ax + bf\left(c^{\top}x\right)\right)\right] - V(t, x)\right\}.$$

LEMMA. *Assume that*

(i) *$m : R_+ \to R_+$ is continuous on $(t_{k-1}, t_k]$, $\lim_{t\to t_k^+} m(t) = m(t_k^+)$ exists for all $k \in N$, and satisfies the inequalities*

$$\begin{aligned} D^+m(t) &\le g(t, m(t)), && t \ne t_k, \\ m\left(t_k^+\right) &\le J_k(m(t_k)), && k \in N, \\ m(t_0) &\le u_0, \end{aligned}$$

where $g : R_+ \times R_+ \to R_+$ is continuous on $(t_{k-1}, t_k] \times R_+$, $\lim_{(t,v)\to(t_k^+,u)} g(t, v) = g(t_k^+, u)$ exists, and $J_k : R_+ \to R_+$ is nondecreasing for each $k \in N$.

(ii) *$\gamma(t)$ is the maximal solution of the following scalar impulsive differential equation:*

$$\begin{aligned} \dot{u} &= g(t, u), && t \ne t_k, \\ u\left(t_k^+\right) &= J_k(u(t_k)), && k \in N, \\ u(t_0) &= u_0 \ge 0, \end{aligned}$$

existing on $[t_0, \infty)$.

Then, we have $m(t) \le \gamma(t)$, $t \ge t_0$.

3. MAIN RESULTS

In this section, we shall establish some sufficient conditions for absolute stability of the impulsive Lurie system.

THEOREM 1. *Assume that*

(i) *there exist a matrix $P > 0$ and two numbers $\beta > 0$, $\gamma > 0$ such that*

$$\Omega = \begin{bmatrix} A^\top \left(P + \gamma\kappa cc^\top\right) + \left(P + \gamma\kappa cc^\top\right) A & \left(P + \gamma\kappa cc^\top\right) b + (\beta - \gamma) A^\top c + c \\ b^\top \left(P + \gamma\kappa cc^\top\right) + (\beta - \gamma) c^\top A + c^\top & 2(\beta - \gamma) c^\top b - \dfrac{2}{\kappa} \end{bmatrix} < 0; \tag{2}$$

(ii) *for $\beta_k \geq 0$, $k \in N$,*

$$\|x(t_k) + I_k(x(t_k))\| \leq \left[\frac{(1 + \beta_k)\lambda_{\min}(P)}{\lambda_{\max}(P) + \kappa(\beta + \gamma)\|c\|^2}\right]^{1/2} \|x(t_k)\|, \tag{3}$$

where $\sum_{k=1}^{\infty} \beta_k < \infty$.

Then, the impulsive Lurie system (1) is globally asymptotically stable.

PROOF. Consider the following Lyapunov function:

$$V(x) = x^\top P x + 2\beta \int_0^{c^\top x} f(s)\, ds + 2\gamma \int_0^{c^\top x} (\kappa s - f(s))\, ds.$$

Since $P > 0$, $\beta > 0$, $\gamma > 0$, $\int_0^{c^\top x} f(s)\, ds \geq 0$, and $\int_0^{c^\top x} (\kappa s - f(s))\, ds \geq 0$, we see that $V(x)$ is radially unbounded and

$$\lambda_{\min}(P)\|x\|^2 \leq V(x) \leq \left\lfloor \lambda_{\max}(P) + (\beta + \gamma)\kappa\|c\|^2 \right\rfloor \|x\|^2. \tag{4}$$

For any P in LMI (2), we find $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that

$$\Omega < \begin{bmatrix} -\varepsilon_1 I & 0 \\ 0 & -\varepsilon_2 \end{bmatrix} < 0. \tag{5}$$

From $f \in F_\kappa$ we know that

$$\kappa c^\top x f\left(c^\top x\right) - f^2\left(c^\top x\right) \geq 0. \tag{6}$$

When $t \neq t_k$ we have

$$\begin{aligned} D^+ V(x) &= x^\top \left(A^\top P + PA\right) x + \left(b^\top P x + x^\top P b\right) f\left(c^\top x\right) + 2\beta f\left(c^\top x\right) c^\top \left[Ax + bf\left(c^\top x\right)\right] \\ &\quad + 2\gamma \left(\kappa c^\top x - f\left(c^\top x\right)\right) c^\top \left[Ax + bf\left(c^\top x\right)\right] \\ &= \begin{bmatrix} x \\ f\left(c^\top x\right) \end{bmatrix}^\top \begin{bmatrix} A^\top \left(P + \gamma\kappa cc^\top\right) + \left(P + \gamma\kappa cc^\top\right) A & \left(P + \gamma\kappa cc^\top\right) b + (\beta - \gamma) A^\top c \\ b^\top \left(P + \gamma\kappa cc^\top\right) + (\beta - \gamma) c^\top A & 2(\beta - \gamma) c^\top b \end{bmatrix} \\ &\quad \times \begin{bmatrix} x \\ f\left(c^\top x\right) \end{bmatrix}. \end{aligned}$$

Using (5) and (6) we get

$$\begin{aligned} D^+ V(x) &= -\frac{2}{\kappa} \left[\kappa c^\top x f\left(c^\top x\right) - f^2\left(c^\top x\right)\right] + \begin{bmatrix} x \\ f\left(c^\top x\right) \end{bmatrix}^\top \Omega \begin{bmatrix} x \\ f\left(c^\top x\right) \end{bmatrix} \\ &\leq -\varepsilon_1 x^\top x - \varepsilon_2 f^2\left(c^\top x\right) \leq -\varepsilon_1 x^\top x \leq -\alpha V(x), \end{aligned} \tag{7}$$

where

$$\alpha = \frac{\varepsilon_1}{\lambda_{\max}(P) + (\beta+\gamma)\kappa\|c\|^2} > 0.$$

When $t = t_k$, by Condition (ii) and (4), we have

$$\begin{aligned} V(x + I_k(x)) &= (x + I_k(x))^\top P(x + I_k(x)) + 2\beta \int_0^{c^\top(x+I_k(x))} f(s)\,ds \\ &\quad + 2\gamma \int_0^{c^\top(x+I_k(x))} (\kappa s - f(s))\,ds \\ &\le \left\lfloor \lambda_{\max}(P) + (\beta+\gamma)\kappa\|c\|^2 \right\rfloor \|x + I_k(x)\|^2 \\ &\le (1+\beta_k)\lambda_{\min}(P)\|x\|^2 \le (1+\beta_k)V(x), \\ \Delta V(x)|_{t=t_k} &= V(x+I_k(x)) - V(x) \le \beta_k V(x). \end{aligned} \tag{8}$$

Let $\varepsilon > 0$ and $t_0 \in R_+$ be given. Let $M = \prod_{k=1}^{\infty}(1+\beta_k)$. Since $V(0) = 0$, there exist $\delta = \delta(t_0) > 0$ such that $V(x) < ((\lambda_{\min}(P))/M)\varepsilon^2$, for $\|x\| < \delta$. Let $x(t) = x(t, t_0, x_0)$ be any solution of system (1) with $\|x_0\| < \delta$. We are going to show that $\|x(t)\| < \varepsilon$ for $t \ge t_0$.

If this is not true, then there exist $t^* > t_0$ such that

$$\|x(t^*)\| \ge \varepsilon \quad \text{and} \quad \|x(t)\| < \varepsilon, \qquad t \in [t_0, t^*), \tag{9}$$

then for $t \in [t_0, t^*]$, define $m(t) = V(x(t))$. Then, by (7) and (8), we get

$$\begin{aligned} &D^+ m(t) \le -\alpha m(t) \le 0, && t \in [t_0, t^*], \quad t \ne t_k, \\ &\Delta m(t_k) \le \beta_k m(t_k), && t_0 < t_k < t^*, \end{aligned}$$

which implies $m(t) \le \prod_{t_0<t_k<t}(1+\beta_k)m(t_0)$, $t \in [t_0, t^*]$.

Hence, $m(t^*) \le Mm(t_0) = MV(x_0) < \lambda_{\min}(P)\varepsilon^2$, $\|x_0\| < \delta$. But, (4) and (9) imply $m(t^*) = V(x(t^*)) \ge \lambda_{\min}(P)\|x(t^*)\|^2 \ge \lambda_{\min}(P)\varepsilon^2$, which is a contradiction. Thus, we must have $\|x(t)\| < \varepsilon$ for $t \ge t_0$, and hence, the trivial solution of system (1) is stable. Now, consider the impulsive system

$$\begin{aligned} \dot{u} &= -\alpha u, && t \ne t_k, \\ \Delta u|_{t=t_k} &= \beta_k u(t_k), && k \in N, \\ u(t_0) &= V(x_0). \end{aligned} \tag{10}$$

For any $t \ge 0$ and $V(x_0) \ge 0$ the solution $u(t, t_0, u_0)$ of system (10) is given by

$$u(t) = u(t, t_0, u_0) = V(x_0) \prod_{t_0<t_k<t} (1+\beta_k)e^{-\alpha(t-t_0)} \le MV(x_0)e^{-\alpha(t-t_0)}, \qquad t \ge t_0. \tag{11}$$

For any solution $x(t, t_0, x_0)$ of system (1), define $m(t) = V(x(t, t_0, x_0))$, then by (7) and (8), we get

$$\begin{aligned} D^+ m(t) &\le -\alpha m(t), && t \ne t_k, \\ m\left(t_k^+\right) &\le (1+\beta_k)m(t_k), && k \in N, \\ m(t_0) &= V(x_0). \end{aligned}$$

By the lemma, we have $m(t) \le \gamma(t)$, $t \ge t_0$, where $\gamma(t)$ is the maximal solution of system (10).

Hence, from (11) we know that $m(t) \le MV(x_0)e^{-\alpha(t-t_0)}$, $t \ge t_0$.

Let $\varepsilon > 0$, $t_0 \in R_+$, and $x_0 \in R^n$ be given, since $MV(x_0)e^{-\alpha(t-t_0)} \to 0$ as $t \to \infty$. There exist $T = T(\varepsilon, t_0, x_0) > 0$, such that $m(t) \le MV(x_0)e^{-\alpha(t-t_0)} \le \lambda_{\min}(P)\varepsilon^2$ for $t \ge t_0 + T$, then it follows from (4) that $\lambda_{\min}(P)\|x(t, t_0, x_0)\|^2 \le m(t) < \lambda_{\min}(P)\varepsilon^2$ for $t \ge t_0 + T$, which implies $\|x(t, t_0, x_0)\| < \varepsilon$ for $t \ge t_0 + T$.

Thus, the trivial solution of system (1) is globally attractive. Therefore, the trivial solution of system (1) is globally asymptotically stable and Theorem 1 is proved. ∎

COROLLARY 1. *Assume that*

(i) *there exist a matrix $P > 0$ and a number $\beta > 0$ such that*

$$\begin{bmatrix} A^\top P + PA & Pb + \beta A^\top c + c \\ b^\top P + \beta c^\top A + c^\top & 2\beta c^\top b - \dfrac{2}{\kappa} \end{bmatrix} < 0;$$

(ii) *for $\beta_k \geq 0$, $k \in N$,*

$$\|x(t_k) + I_k(x(t_k))\| \leq \left[\frac{(1+\beta_k)\lambda_{\min}(P)}{\lambda_{\max}(P) + \kappa\beta\|c\|^2}\right]^{1/2} \|x(t_k)\|,$$

where $\sum_{k=1}^{\infty} \beta_k < \infty$.

Then, the impulsive Lurie system (1) is absolutely stable.

COROLLARY 2. *Assume that*

(i) *there exists a matrix $P > 0$ such that*

$$\begin{bmatrix} A^\top P + PA & Pb + c \\ b^\top P + c^\top & -\dfrac{2}{\kappa} \end{bmatrix} < 0;$$

(ii) *for $\beta_k \geq 0$, $k \in N$,*

$$\|x(t_k) + I_k(x(t_k))\| \leq \left[\frac{(1+\beta_k)\lambda_{\min}(P)}{\lambda_{\max}(P)}\right]^{1/2} \|x(t_k)\|,$$

where $\sum_{k=1}^{\infty} \beta_k < \infty$.

Then, the impulsive Lurie system (1) is absolutely stable.

THEOREM 2. *Assume that*

(i) $\sup\{t_{k+1} - t_k\} = \lambda < \infty$;

(ii) $\|x(t_k) + I_k(x(t_k))\| \leq \alpha\|x(t_k)\|$, $\alpha \geq 0$, $k \in N$;

(iii)

$$\left\|e^{A(t_{k+1}-t_k)}\right\| \leq q < \frac{1}{\alpha\left(1 + \|b\|\,\|c\|\kappa\lambda e^{(3\|A\|+\|b\|\,\|c\|\kappa)\lambda}\right)}, \qquad q \geq 0, \quad k \in N.$$

Then, the impulsive Lurie system (1) is absolutely stable.

PROOF. Since for any $t \in (t_k, t_{k+1}]$,

$$x(t) = x\left(t_k^+\right) + \int_{t_k}^{t} \left[Ax(r) + bf\left(c^\top x(r)\right)\right] dr,$$

which implies that

$$\|x(t)\| \leq \left\|x\left(t_k^+\right)\right\| + \int_{t_k}^{t} (\|A\| + \|b\|\,\|c\|\kappa)\|x(r)\|\, dr, \qquad t \in (t_k, t_{k+1}].$$

By the Gronwall inequality, we obtain

$$\|x(t)\| \leq \left\|x\left(t_k^+\right)\right\| e^{(\|A\|+\|b\|\,\|c\|\kappa)(t-t_k)} \leq \left\|x\left(t_k^+\right)\right\| e^{(\|A\|+\|b\|\,\|c\|\kappa)\lambda}, \qquad t \in (t_k, t_{k+1}]. \tag{12}$$

So, we get

$$\begin{aligned}\left\|\int_{t_k}^{t_{k+1}} e^{A(r-t_k)} bf\left(c^\top x(r)\right)\, dr\right\| &\le \int_{t_k}^{t_{k+1}} e^{\|A\|\lambda}\|b\|\,\|c\|\kappa\|x(r)\|\, dr \\ &\le \int_{t_k}^{t_{k+1}} e^{\|A\|\lambda}\|b\|\,\|c\|\kappa\left\|x\left(t_k^+\right)\right\| e^{(\|A\|+\|b\|\,\|c\|\kappa)\lambda}\, dr \\ &= \left\|x\left(t_k^+\right)\right\|\,\|b\|\,\|c\|\kappa\lambda e^{(2\|A\|+\|b\|\,\|c\|\kappa)\lambda}, \qquad k\in N.\end{aligned} \tag{13}$$

From (1) and [5] we know that

$$x(t_{k+1}) = e^{A(t_{k+1}-t_k)} x\left(t_k^+\right) + \int_{t_k}^{t_{k+1}} e^{A(r-t_k)} bf\left(c^\top x(r)\right)\, dr,$$

then, by Conditions (ii), (iii), and (13) we obtain that

$$\begin{aligned}\left\|x\left(t_{k+1}^+\right)\right\| &= \|x(t_{k+1}) + I_{k+1}(x(t_{k+1}))\| \le \alpha\|x(t_{k+1})\| \\ &= \alpha\left\|e^{A(t_{k+1}-t_k)}\left(x\left(t_k^+\right) + e^{-A(t_{k+1}-t_k)}\int_{t_k}^{t_{k+1}} e^{A(r-t_k)} bf\left(c^\top x(r)\right)\, dr\right)\right\| \\ &\le \alpha\left\|e^{A(t_{k+1}-t_k)}\right\|\left(\left\|x\left(t_k^+\right)\right\| + \left\|e^{-A(t_{k+1}-t_k)}\right\|\left\|\int_{t_k}^{t_{k+1}} e^{A(r-t_k)} bf\left(c^\top x(r)\right)\, dr\right\|\right) \\ &\le \alpha q\left(1+\|b\|\,\|c\|\kappa\lambda e^{(3\|A\|+\|b\|\,\|c\|\kappa)\lambda}\right)\left\|x\left(t_k^+\right)\right\|, \qquad k\in N.\end{aligned}$$

Letting $\gamma = \alpha q(1+\|b\|\,\|c\|\kappa\lambda e^{(3\|A\|+\|b\|\,\|c\|\kappa)\lambda})$, then $0<\gamma<1$ and the above relation implies

$$\left\|x\left(t_{k+1}^+\right)\right\| \le \gamma\left\|x\left(t_k^+\right)\right\|, \qquad k\in N. \tag{14}$$

Letting $t_0 > 0$ be given, without loss of generality, we assume that $0 < t_0 < t_1$. Then, from (1) we get

$$x(t_1) = x\left(t_0^+\right) + \int_{t_0}^{t_1}\left[Ax(r) + bf\left(c^\top x(r)\right)\right] dr,$$

which implies that

$$\|x(t_1)\| \le \left\|x\left(t_0^+\right)\right\| + \int_{t_0}^{t_1}(\|A\| + \|b\|\,\|c\|\kappa)\|x(r)\|\, dr.$$

By the Gronwall inequality, we obtain

$$\|x(t_1)\| = \|x(t_0)\| e^{(\|A\|+\|b\|\,\|c\|\kappa)\lambda}. \tag{15}$$

By Condition (ii) and (15), we have

$$\left\|x\left(t_1^+\right)\right\| = \|x(t_1) + I_1(x(t_1))\| \le \alpha\|x(t_1)\| \le \alpha\|x_0\| e^{(\|A\|+\|b\|\,\|c\|\kappa)\lambda}. \tag{16}$$

For each time point $t \ge t_0$ there exists a k such that $t_k < t \le t_{k+1}$, so, from (12) we get $\|x(t)\| \le \|x(t_k^+)\| e^{(\|A\|+\|b\|\,\|c\|\kappa)\lambda}$, and applying (14) and (16), we get

$$\|x(t)\| \le \alpha\|x_0\|\gamma^{k-1} e^{2(\|A\|+\|b\|\,\|c\|\kappa)\lambda}, \qquad \text{for } t_k < t \le t_{k+1}. \tag{17}$$

For any solution $x(t) = x(t, t_0, x_0)$ of system (1), let $\varepsilon > 0$, $t_0 \ge 0$ be given, and from $0<\gamma<1$ and (17), we know that there exist $\delta = (\gamma\varepsilon/\alpha)e^{-2(\|A\|+\|b\|\,\|c\|\kappa)\lambda} > 0$ such that $\|x(t)\| < \varepsilon$ for $\|x_0\| < \delta$, $t \ge t_0$, i.e., the trivial solution of system (1) is stable.

Now, as $t\to\infty$, $k\to\infty$, by (17) and $0<\gamma<1$, we have $\lim_{t\to\infty}\|x(t)\| = 0$. There the result holds. ∎

COROLLARY 3. *Assume that*

(i) $\sup\{t_{k+1} - t_k\} = \lambda < \infty$;

(ii)

$$\left\|(I + B_k)e^{A(t_{k+1}-t_k)}\right\| \le q < \frac{1}{1 + \|b\|\,\|c\|\kappa\lambda e^{(3\|A\|+\|b\|\,\|c\|\kappa)\lambda}}, \qquad k \in N.$$

Then, the impulsive Lurie system

$$\begin{aligned} \dot{x} &= Ax + bf\left(c^\top x\right), & t &\ne t_k, \\ \Delta x|_{t=t_k} &= B_k x, & t &= t_k, \quad k \in N, \end{aligned} \tag{18}$$

is absolutely stable.

COROLLARY 4. *Assume that*

(i) $\sup\{t_{k+1} - t_k\} = \lambda < \infty$;

(ii)

$$\lim_{k\to\infty} \sup \left\|(I + B_k)e^{A(t_{k+1}-t_k)}\right\| < \frac{1}{1 + \|b\|\,\|c\|\kappa\lambda e^{(3\|A\|+\|b\|\,\|c\|\kappa)\lambda}}.$$

Then, the impulsive Lurie system (18) is absolutely stable.

4. CONCLUSION

In this paper, we have presented some sufficient conditions that guarantee absolute stability of Lurie systems with impulsive effects by using Lyapunov functions and the method of variation of parameters. Suitable conditions have to be placed on the impulses in order to maintain absolute stability.

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