Numerical Analysis on “Trap of Credit Creation” in Macroeconomics under Zero-Interest-Rate Policy

Yu Murata\textsuperscript{a}, Tomohiro Inoue\textsuperscript{b}, Moto Kamiura\textsuperscript{a,c,*}

\textsuperscript{a} Graduate School of Science and Engineering, Tokyo Denki University, Ishizaka, Hatoyama-cho, Hiki-gun, Saitama, 350-0394, Japan
\textsuperscript{b} Faculty of Political Science and Economics, Waseda University, 1-6-1 Nishi-waseda, Shinjuku-ku, Tokyo, 169-8050, Japan
\textsuperscript{c} Research Institute of Electrical Communication, Tohoku University, 2-1-1 Katahira, Aoba-ku, Sendai 980-8577, Japan

Abstract

Protracted recession in Japan for the last twenty years is characterized by persistent deflation and negative output gap. Recently, Inoue et al. presented the concept of “trap of credit creation” which represents mechanism of the persistent deflation based on zero-interest-rate economy. Three new Keynesian DGE models by Inoue et al. explain this type of deflation as a steady state of a system. The DGE models, which are based on optimization for utility function, derive simultaneous nonlinear differential equations.

In the present paper, we study dynamical features on the steady states of the three DGE models (i.e. Model I, II, and III), using numerical calculation: i.e. we analyze stability of steady states, using Jacobian matrices. Model I and II correspond to models for positive-interest-rate economy, and Model III suggests mechanism of the recession characterized as trap of credit creation under zero-interest-rate economy. We show each of the models have a stable steady state. This result supports adequacy of the present models, which can shed light on mechanism of the protracted recession.

Keywords: Zero interest rate, New Keynesian DGE model, Trap of credit creation, Numerical analysis.

1. Introduction

1.1. Recession Characterized as “Trap of Credit Creation”

Protracted recession in Japan for the last twenty years is characterized by two features: i.e. persistent deflation and negative output gap. In aspect of monetary policy, the Bank of Japan (i.e. the central bank of Japan) has...
implemented quantitative easing policy against the persistent deflation. However, bank lending and money stock have not increased enough, and just excess reserves have increased. Inoue et al.\(^1\) calls the situation in which bank lending has not increased but excess reserves have increased via quantitative easing policy bound by zero-interest-rate “trap of credit creation”.

On the other hand, a new Keynesian Dynamic Generalized Equilibrium (DGE) model proposed by Benhabib et al.\(^2\) expresses persistent deflation as not a transient state but a steady state. The DGE model has two steady states: i.e. one is an unstable steady state in high inflation, and the other is a stable steady state in low inflation (i.e. persistent deflation). Tsuzuki and Inoue\(^3,4\) show that a negative output gap are derived from growth rate of money lower than rate of technical progress. Consequently, we can think that if quantitative easing policy cannot increase money stock then it is not effective against this type of recession.

It is necessary to study mechanism by which inhibits the effectivity of quantitative easing policy. For this objective, Inoue et al.\(^1\) proposes three new Keynesian DGE models including commercial banks which play an essential role on credit creation. Inoue et al. pays attention to two modes on interest rates: i.e. positive-interest-rate economy and zero-interest-rate economy. The former derives two DGE models: i.e. Model I is a system under positive-interest-rate policy, and Model II is a system under positive-interest-rate policy and quantitative easing policy. The latter derives the other DGE model: i.e. Model III is a system under zero-interest-rate policy and quantitative easing policy. It is known that each of these DGE models has non-trivial steady state. However dynamical features of the steady states have not been known yet.

In the present paper, we study dynamical features on the steady states of the three DGE models by Inoue et al., using numerical calculation: i.e. we analyze stability of steady states, using Jacobian matrices. Especially, Model III suggests mechanism of the recession characterized as trap of credit creation. Model I and II are compared to Model III to research effect of zero-interest-rate.

1.2. Numerical Analysis on DGE Models

Implication of analysis in macroeconomics is different from that in physics. A model of macroeconomics has two types of variable: i.e. predetermined variables and non-predetermined variables (i.e. jump variables). The former corresponds to an independent variable in physics. The latter is characteristic of an economic model with forward-looking agents, by which the system can select a path leading to a steady state based on the following conditions:

- (C1) If (the number of positive eigenvalues on a steady state) = (the number of non-predetermined variables of the system), then there is unique path leading to a steady state.
- (C2) If (the number of positive eigenvalues on a steady state) < (the number of non-predetermined variables of the system), then there are an infinite number of paths leading to a steady state.
- (C3) If (the number of positive eigenvalues on a steady state) > (the number of non-predetermined variables of the system), then there is no path leading to a steady state.

See also Blanchard and Karn\(^5\) for details on predetermined and non-predetermined variables. The discussion is on a difference equation system, but it can be translated into the above conditions on a differential equation system.

2. Model definition

2.1. Basic System Configuration Deriving Three Types of Models

In this study, we numerically analyze the three types of models by Inoue et al. These three models (i.e. Model I, II, and III) are derived from a system which consists of the following sectors and relationships: i.e. there are multiple households and commercial banks which are indexed by continuous numbers \(i\) and \(j\). The
households supply heterogeneous labor forces via a temporary help agency, which constructs integrated labor and supplies it to a company. The commercial banks buy final goods from the company, and they transform the final goods to heterogeneous capital goods. A capital broker integrates the capital goods borrowed from the commercial banks, and lends the integrated capital goods to the company. When the commercial banks buy the final goods, they create bank money and pay out it to the company. The bank money is transferred as wages from the company to the households, which circulate deposits back into bank accounts. Credit creation is generated in the circulation of money. There also exist a central bank and a government. Monetary base consists of only bank reserves, and money stock consists of only deposits. Government bonds are possessed by the central bank, the commercial banks and the households. The whole system is shown as the following Fig.1.

The above description for the system is translated into the following mathematical model. A company has nominal profit \[ \Pi_c = py - Wh - Rp k \] under Cobb-Douglas type production functions \[ y = k_1 h_1 \], where \( R \) is nominal interest rate, \( y \) is final goods, and \( k \) is integrated real capital input. The company is under perfect competition, thus nominal profit is zero. Government bonds fulfill \[ B_{all} = B + B_C = B + \text{Res} \] (i.e. all-bond \( B_{all} \) is the sum of bond \( B \) in the commercial banks and bond \( B_C \) in the central bank.) and \[ dB_{all}/dt = \zeta py \], where \( \text{Res} \) is bank reserves and \( \zeta (>0) \) is a parameter.

The household \( i \) (for all \( i \in [0,1] \)) optimizes the following utility function:

\[
\max_{c,d,\omega} \int_0^\infty \left[ \ln c + \delta \ln d - \frac{1}{1+\psi} \frac{1}{1+\psi} - \frac{\zeta}{2} \omega^2 \right] e^{-\rho t} dt, \tag{1}
\]

subject to \( da/dt = ra + w_i h_i - c - Rd \) (i.e. budget constraint on the households), \( \omega_k = (dW/dt) / W \) (i.e. definition on change rate of nominal wage rate), \( h_i = (W/W)^{-(\delta)} h \) (i.e. results of cost optimization on the temporal help agency), and \( h_i = z_i \) (i.e. definition of labor input).

A commercial bank \( j \) (for all \( j \in [0,1] \)) optimizes the following utility function:
\[
\max_{R_j} \left[ R^B + (R_j - \nu)K_j \right],
\]

subject to \( K_j = \left( R/R \right)^{\tau} K \) (i.e. lending bank \( j \) on nominal capital), and \( D_j = \text{Res} + B + K_j < D_M := \text{Res}/\bar{\sigma} \) (i.e. condition for deposit \( D_j \) of bank \( j \)), where \( R_j \) is lending rate of bank \( j \), \( K = kp \) is integrated nominal capital input, \( D_M \) is maximum deposit of each bank, and \( \bar{\sigma} \) is reserve requirement. In this system, any bank follows the same optimization, therefore we can describe \( D_j = D \) and \( R_j = R \) for all \( j \).

The central bank implements a policy on interest rate based on the Taylor rule\(^6\), \( R(\pi) = R + \tau(\pi - \bar{\pi}) \), where \( \tau (>1) \) is a parameter.

The meanings and ranges of symbols are shown in the following Table. 1. The right side columns, I, II and III, of the table express the values used in simulations of the following sections.

<table>
<thead>
<tr>
<th>Meaning</th>
<th>Symbol</th>
<th>Range</th>
<th>Setting of model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter of Cobb-Douglas type production function</td>
<td>( \alpha )</td>
<td>( 0 &lt; \alpha &lt; 1 )</td>
<td>0.4</td>
</tr>
<tr>
<td>Rate of technical progress</td>
<td>( g )</td>
<td>-</td>
<td>0.01</td>
</tr>
<tr>
<td>Parameter of wage adjustment cost</td>
<td>( \gamma )</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>Parameter of alternative among labor</td>
<td>( \phi )</td>
<td>( 1 &lt; \phi )</td>
<td>21</td>
</tr>
<tr>
<td>Parameter of marginal disutility of labor</td>
<td>( \psi )</td>
<td>( 0 &lt; \psi )</td>
<td>1</td>
</tr>
<tr>
<td>Subjective discount rate on household budget</td>
<td>( \rho )</td>
<td>( 0 &lt; \rho )</td>
<td>0.01</td>
</tr>
<tr>
<td>Parameter of Taylor rule</td>
<td>( \tau )</td>
<td>( 1 &lt; \tau )</td>
<td>2, -</td>
</tr>
<tr>
<td>Growth rate of bank reserves</td>
<td>( \theta )</td>
<td>( g \leq \theta )</td>
<td>0.01, -</td>
</tr>
<tr>
<td>Target inflation rate</td>
<td>( \bar{\pi} )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Target interest rate</td>
<td>( \bar{R} )</td>
<td>( \bar{R} = g + \rho + \bar{\pi} )</td>
<td>0.02, -</td>
</tr>
<tr>
<td>Interest rate of zero-interest-rate economy</td>
<td>( R_0 )</td>
<td>( R_0 \sim 0 )</td>
<td>-</td>
</tr>
<tr>
<td>Consumption per integrated capital input</td>
<td>( \rho (:= c/k) )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Integrated labor input per integrated capital input</td>
<td>( \hat{h} (:= h/k) )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Level of employment</td>
<td>( l )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Change rate of nominal wage rate</td>
<td>( \omega )</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The above system is mainly characterized as the systems to which the roles of commercial banks are introduced to discuss “trap of credit creation”. The macroeconomic system is divided into the two types of modes: i.e. positive-interest-rate economy (Model I and II) and zero-interest-economy (Model III).

The two types of modes are based on the data\(^7\) plotted on a graph of inflation versus nominal interest rate. In the positive-interest-rate economy, the data are basically on the solid line of Fig.2 which expresses Taylor rule. On the other hand, in the zero-interest-rate economy, the data are on the dashed line of Fig.2. In this mode, the economic system is bound by zero or very low interest rate (i.e. a central bank cannot manipulate interest rate as a policy). Thus a steady state as an intersection point of Fisher equation might not generate on the side of positive inflation, in zero-interest-rate economy.

Mathematically, positive-interest-rate economy is characterized by \( D = D_M \) and \( R > R_0 \), and zero-interest-rate economy is characterized by \( D < D_M \) and \( R = R_0 \). In addition, difference of the two economies rises up to excess reserves, \( ES := \text{Res} - \bar{\sigma} D = (\sigma - \bar{\sigma}) D \), where \( \sigma \) is actual reserve ratio. In positive-interest-rate economy, \( D = D_M = \text{Res}/\bar{\sigma} \), therefore \( \text{Res} = \bar{\sigma} D \), and \( ES = 0 \) (i.e. there is no excess reserves). In zero-interest-rate economy, \( D < D_M = \text{Res}/\bar{\sigma} \), therefore \( \text{Res} > \bar{\sigma} D \), and \( ES > 0 \) (i.e. there exist excess reserves).
2.2. Model I: Positive-Interest-Rate Policy

Under the condition of positive-interest-rate policy, we obtain the following simultaneous nonlinear differential equations (3)-(6), which are derived from the optimization for utility functions defined in the section 2.1.

\[
\begin{align*}
\frac{\dot{c}}{\dot{c}} &= \alpha \hat{h}^{1-a} - \rho - \hat{h}^{1-a} + \dot{c} \\
\frac{\dot{i}}{\dot{i}} &= \frac{\dot{h}}{\dot{h}} + \hat{h}^{1-a} - \dot{c} - g \\
\frac{\dot{h}}{\dot{h}} &= \frac{1}{\alpha} \left( R - \alpha \hat{h}^{1-a} - \omega \right) \\
\dot{\omega} &= \omega \left[ \rho - \phi \frac{\hat{h}^{1-a}}{\gamma \omega} + (\phi - 1) \frac{(1 - \alpha) \hat{h}^{1-a}}{\gamma \omega \hat{c}} \right].
\end{align*}
\]

The steady state of the system is expressed as the following asterisked variables (7)-(10).

\[
\begin{align*}
\dot{c}^* &= \frac{g + \rho}{\alpha} - g \\
\dot{i}^* &= \left[ \frac{\gamma \pi^* \rho + \varphi - 1}{\varphi} \left( g + \rho - \alpha \omega^* \right) \right]^{\frac{1}{1+\varphi}} \\
\dot{h}^* &= \left( g + \rho \right)^{\frac{1}{1-a}} \\
\omega^* &= \pi^* = \bar{\pi}
\end{align*}
\]

In a dynamical system \( \dot{x}_i = f_i(x) \) (1 \( \leq i \leq n \)), stability of steady state \( x^* \) fulfilling \( f(x^*) = 0 \) is analyzed by Jacobian matrix \( J = [ J_{xy} ] = [ df_i/dx_j ] \). Jacobian matrix of the Model I, \( J_1 \), is given by the following (11):
2.3. Model II: Positive-Interest-Rate and Quantitative Easing Policy

Under the condition of positive-interest-rate and quantitative easing policy, we obtain the following equations (12)-(16). Note that the dimension of \( R \) is added.

\[
\dot{R} := R(R - (\theta + \rho)) \tag{12}
\]

\[
\dot{\hat{h}} := \frac{1}{\alpha} \left( R - \omega \right) - \hat{c} - g \tag{13}
\]

\[
\dot{\hat{c}} := \hat{c} \left( 1 - \alpha \right) \hat{h}^{1-\alpha} - \rho \tag{14}
\]

\[
\dot{\omega} := \frac{1}{\gamma} \left[ \gamma \pi \omega - \omega \phi^{1+\psi} + (\varphi - 1)(1 - \alpha) \right] \tag{15}
\]

\[
\dot{\hat{h}} = \frac{\hat{h}}{\alpha} \left( R - \alpha \hat{h}^{1-\alpha} - \omega \right) \tag{16}
\]

The steady state of the system is expressed as the following asterisked variables (17)-(21).

\[
R^* = \theta + \rho \tag{17}
\]

\[
l^* = \frac{\gamma \pi^* \omega + \varphi - 1 (1 - \alpha) (g + \rho)}{\varphi g + \rho - \alpha g} \tag{18}
\]

\[
\hat{c}^* = \frac{g + \rho}{\alpha} - g \tag{19}
\]

\[
\omega^* = \pi^*_2 = \theta - g \tag{20}
\]

\[
\hat{h}^* = \left( \frac{g + \rho}{\alpha} \right)^{\frac{1}{1-\alpha}} \tag{21}
\]

Jacobian matrix of the Model II, \( J_2 \), is given by the following (22):
2.4. Model III: Zero-Interest-Rate Policy

Under the condition of zero-interest-rate policy, we obtain the following equations (23)-(26).

\[ \dot{c} := \hat{c} - \frac{1}{\alpha - 1} \hat{c}^{1-\alpha} - \rho \]  
\[ \dot{\hat{h}} := \frac{\hat{h}}{\alpha} \left( R_0 - \alpha \hat{h}^{1-\alpha} - \omega \right) \]  
\[ \dot{l} := l \left[ \frac{1}{\alpha} \left( R_0 - \omega \right) - \hat{c} - g \right] \]  
\[ \dot{\omega} := \frac{1}{\gamma} \left[ \gamma \rho - \phi \right] \left( 1 - \alpha \right) \hat{h}^{1-\alpha} \]  

The steady state of the system is expressed as the following asterisked variables (27)-(30).

\[ \hat{c}^* = \frac{g + \rho}{\alpha} - g \]  
\[ \hat{h}^* = \left( \frac{g + \rho}{\alpha} \right)^{1-\alpha} \]  
\[ \hat{l}^* = \left[ \frac{\gamma \rho}{\phi} + \frac{\phi - 1}{\phi} \left( 1 - \alpha \right) \left( g + \rho - \alpha g \right) \right]^{1+\psi} \]  
\[ \omega^* = \pi^*_3 = R_0 - g - \rho \]  

Jacobian matrix of the Model III, \( J_3 \), is given by the following (31):
3. Numerical Analysis on Steady States

In the previous section, we obtained the steady states and the Jacobian matrices on Model I-III. Here we substitute the values of parameters of Table I into them, and study the features of the steady states.

3.1. Model I: Positive-Interest-Rate Policy

When we substitute the values of parameters of Table I into the Jacobian matrix of Model I, \( J_1 \), we obtain a Jacobian matrix in the steady state, \( J_1^* \), as the following (32).

\[
J_1^* = \begin{bmatrix}
0.04 & -0.10610011 & 0 & 0 \\
0 & 0.11 & 0 & -0.01696511 \\
-8.4515425 & 7.47257975 & 0 & -2.1128564 \\
-1.875 & 6.6312567 & -0.17748239 & 0.01 \\
\end{bmatrix}
\]  

In addition, eigenvalues of \( J_1^* \) is given by the following (33).

\[
(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (-0.4968124, 0.06174859 - 0.00606493i, 0.06174859 + 0.00606493i, 0.5333153)
\]  

I.e. the steady state of Model I is characterized as a saddle which consists of three positive eigenvalues and one negative eigenvalue in real parts. On the other hand, Model I has three non-predetermined variables and one predetermined variable. \( \hat{c} \) and \( \hat{\omega} \) are non-predetermined variables. Either \( \hat{h} \) or \( \hat{l} \) is a non-predetermined variable, and the other is predetermined variable, since \( \hat{h} \) and \( \hat{l} \) are connected as the constraint \( \hat{h} = h/k = zl/k \). Consequently, Model I fulfills (the number of positive eigenvalues on a steady state) = (the number of non-predetermined variables of the system), thus there is unique path leading to a steady state.
Fig 3. The values of $c^*$, $h^*$, $l^*$ and $\omega^*$ for parameter $\alpha$.

Fig 4. The value of $\det J_1^*$ for parameter $\alpha$.

Fig 5. The eigenvalues of $J_1^*$ for parameter $\alpha$.

Fig 3-5 show the values of steady state (i.e. $c^*$, $h^*$, $l^*$, $\omega^*$), $\det J_1^*$ and the eigenvalues of $J_1^*$ for parameter $\alpha$, where $\alpha$ and $1-\alpha$ expresses capital’s and labor’s shares of output $y$, under the Cobb-Douglas type production functions $y = k^\alpha h^{1-\alpha}$. On the Fig 5, increase in capital’s share $\alpha$ (i.e. decrease in labor’s share $1-\alpha$) implies gradient on landscape of phase space get shallower.

3.2. Model II: Positive-Interest-Rate and Quantitative Easing Policy

When we substitute the values of parameters of Table 1 into the Jacobian matrix of the Model II, $J_2$, we obtain a Jacobian matrix in the steady state, $J_2^*$, as the following (34).

$$J_2^* = \begin{bmatrix}
-0.02 & 0 & 0 & 0 & 0 \\
0 & 0.04 & -0.10610011 & 0 & 0 \\
0.01696511 & 0 & -0.08 & 0 & -0.01696511 \\
2.11288564 & -0.84515425 & -0.017748239 & -0.05 & -2.11288564 \\
0 & -1.875 & 6.6312567 & -0.17748239 & 0.01
\end{bmatrix}$$  \hspace{1cm} (34)

In addition, eigenvalues of $J_2^*$ is given by the following (35).

$$\lambda = (-0.5330985, -0.0810735, -0.02, 0.0410735, 0.4930985)$$  \hspace{1cm} (35)

I.e. the steady state of Model II is characterized as a saddle which consists of two positive eigenvalues and three negative eigenvalues. On the other hand, Model II has three non-predicted variables and two predetermined variable. $\hat{c}$ and $\omega$ are non-predicted variables. Either $\hat{h}$ or $l$ is a non-predicted variable, and the other is predetermined variable, for the same reason in Model I. $R$ is a predetermined variable. Consequently, Model II fulfills (the number of positive eigenvalues on a steady state) < (the number of non-predicted variables of the system), thus there are an infinite number of paths leading to a steady state.
Fig 6. The values of $\hat{c}^*, \hat{h}^*, \hat{l}^*, \hat{\omega}^*$ and $R^*$ for parameter $\alpha$.

Fig 7. The value of $\det J^*_2$ for parameter $\alpha$.

Fig 8. The eigenvalues of $J^*_2$ for parameter $\alpha$.

Fig 6-8 show the values of steady state (i.e. $\hat{c}^*, \hat{h}^*, \hat{l}^*, \hat{\omega}^*, R^*$), $\det J^*_2$ and the eigenvalues of $J^*_2$ for parameter $\alpha$. The behaviors of the variables and the eigenvalues of Model II are basically similar to that of Model I.

### 3.3. Model III: Zero-Interest-Rate Policy

When we substitute the values of parameters of Table 1 into the Jacobian matrix of the Model III, $J_3$, we obtain a Jacobian matrix in the steady state, $J^*_3$, as the following (36).

$$
J^*_3 = \begin{bmatrix}
0.04 & -0.10610011 & 0 & 0 \\
0 & -0.03 & 0 & -0.01696511 \\
-0.84402663 & 0 & 0 & -2.11006658 \\
-1.875 & 6.6312567 & -0.17724559 & 0.01
\end{bmatrix}
$$

(36)

In addition, eigenvalues of $J^*_1$ is given by the following (37).

$$
(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (-0.5068983, -0.02164868, 0.03164868, 0.5168983)
$$

(37)

I.e. the steady state of Model III is characterized as a saddle which consists of two positive eigenvalues and two negative eigenvalues. On the other hand, Model III has three non-predetermined variables and one predetermined variable. $\hat{c}$ and $\omega$ are non-predetermined variables. Either $\hat{h}$ or $\hat{l}$ is a non-predetermined variable, and the other is predetermined variable, for the same reason in Model I. Consequently, Model III fulfills (the number of positive eigenvalues on a steady state) < (the number of non-predetermined variables of the system), thus there are an infinite number of paths leading to a steady state.
Fig 9-11 show the values of steady state (i.e. \( \hat{c}^*, \hat{h}^*, l^*, \omega^* \)), \( \det J_2^* \) and the eigenvalues of \( J_3^* \) for parameter \( \alpha \). The behaviors of the variables and the eigenvalues of Model III are also basically similar to that of Model I and II.

4. Discussion

We can see that the present DGE models have non-trivial steady states, by the analyses in the section 2 and 3. Inflation \( \pi \) and nominal interest rate \( R \) of them are the following: Model I: \((\pi_1^*, R_1^*) = (\pi, R) = (0, 0.02)\), Model II: \((\pi_2^*, R_2^*) = (\theta - g, \theta - \rho) = (0, 0.02)\), Model III: \((\pi_3^*, R_3^*) = (R_0 - \theta - g, R_0) = (-0.02, 0)\).

Consumption per integrated capital input \( \hat{c}^* \), Integrated labor input per integrated capital input \( \hat{h}^* \), Level of employment \( l^* \), Change rate of nominal wage rate \( \omega^* \), which are variables in the steady states, show qualitatively same behavior for a given parameter \( \alpha \).

Each of the steady states on the Model I-III is a saddle point. The eigenvalues of \( J_k^* \) \((k=1,2,3)\) show that the saddle structures are stable against change of parameter \( \alpha \). The steady states are also characterized by the conditions in the section 1.2, (C1)-(C3): i.e. the present three models fulfill (the number of positive eigenvalues on a steady state) \( \leq \) (the number of non-predetermined variables of the system), thus there is one or more paths leading to a steady state. In other words, these steady states are stable in macroeconomics. Note that the model by Benhabib et al.\(^2\) has an unstable steady state as high inflation and a steady state as low inflation. By contrast, the present models are divided into the two modes, positive-interest-rate economy (i.e. Model I, II) and zero-interest-rate economy (i.e. Model III). Consequently, both high and low inflation are realized as the steady states.

5. Conclusion

Protracted recession in Japan for the last twenty years is characterized by persistent deflation and negative output gap. Persistent deflation corresponds to a stable steady state in low inflation, in the model by Benhabib et al.\(^2\). New Keynesian DGE models by Inoue et al.\(^1\) suggest the existence of “trap of credit creation” which is a mechanism of the persistent deflation based on zero-interest-rate economy. It has been known that the DGE models have non-trivial steady states, but it has not been known of dynamical features of the steady states.

In the present paper, we study dynamical features on the steady states of the three DGE models by Inoue et al. (i.e. Model I, II, and III), using numerical calculation: i.e. we analyze stability of steady states, using Jacobian matrices. These models are divided into the two modes, positive-interest-rate economy (i.e. Model I, II) and zero-interest-rate economy (i.e. Model III). Especially, Model III suggests mechanism of the recession
characterized as trap of credit creation. In the present study, we showed that each of the present models by Inoue et al. has a stable steady state. In addition, the steady states are structurally stable against change of parameter $\alpha$. These results support adequacy of the present models, which can shed light on mechanism of the protracted recession.

Acknowledgements

Part of this work was carried out under the Cooperative Research Project Program of the Research Institute of Electrical Communication, Tohoku University.

References