Vibration and stability of an axially moving beam immersed in fluid

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Abstract

Vibration and stability are investigated for an axially moving beam in fluid and constrained by simple supports with torsion springs. The equations of motion of the beam with uniform circular cross-section, moving axially in a horizontal plane at a known rate while immersed in an incompressible fluid are derived first. An “axial added mass coefficient” and an initial tension are implemented in these equations. Based on the Differential Quadrature Method (DQM), a solution for natural frequency is obtained and numerical results are presented. The effects of axially moving speed, axial added mass coefficient, and several other system parameters on the dynamics and instability of the beam are discussed. Particularly, natural frequency in terms of the moving speed is presented for fixed–fixed, hinged–hinged and hybrid supports with torsion spring. It is shown that when the moving speed exceeds a certain value, the beam becomes subject to buckling-type instability. The variations of the lowest critical moving speed with several key parameters are also investigated.

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1. Introduction

A theory for the vibration of axially moving beams is of considerable interest in many fields. Some examples of the engineering applications include, but are not limited to, extrusion processes, deployment of appendages in space (Paidoussis, 2003), robotic manipulators, telescopic members of loading vehicles, machine tools (Al-Bedoor and Khulief, 1996), and chain-saw blades. In all those applications, a dynamical model is essential to design and control lightweight, high precision processes or mechanisms.

In the last decades, the dynamics of an elastic or viscoelastic beam axially moving with a certain speed has been a topic under intensive study. Most of the investigations concerned the vibration characteristics and dynamical behaviors of an axially moving beam. An extensive review of the early literature in the domain of axially moving materials is provided by Mote (1972). It includes the work that was done in the areas of band-saws, belts, chain drives, pipes conveying fluid, and other similar systems which all have in common

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the same (or similar) linear equation of motion and boundary conditions. More recently, Wickert (1989) presented a review of more recent research in this area; more recently still, Paidoussis (2003) presented a condensed review of all the work on the problem of an extruding beam. Mote (1965) first calculated the natural frequencies of an axially moving elastic beam by solving approximately the frequency equation with hinged–hinged and clamped–clamped boundary conditions, while the tension effects were not considered. Öz and his co-workers (Öz, 2001; Öz, 2003; Öz and Pakdemirli, 1999; Özkaya and Öz, 2002) obtained the modal functions and natural frequencies of axially moving elastic beams, both for hinged–hinged and clamped–clamped ends. Most recently, Chen and Yang (2006) investigated the natural frequency and stability of an axially moving viscoelastic beam with hybrid supports, i.e., the moving beam is constrained by simple supports with torsion springs at both ends.

In all the studies mentioned in the foregoing, the problem was formulated such that there was no fluid surrounding the moving beam or, if there was, it was ignored. Motivated by the quest for a fundamental understanding of fluid–structure interactions as well as by applications in several areas of engineering, the dynamical behaviors of axially moving flexible beams immersed in fluid have attracted the attention of several investigators. Taleb and Misra (1981) have investigated the vibration of a cantilevered beam of uniform circular cross-section being deployed in a dense incompressible fluid (see Fig. 1). Subsequently, in the work by Gosselin et al. (2007), it was proved that the fluid-dynamic forces were not correctly accounted for in analysis performed by Taleb and Misra (1981). Gosselin et al. (2007) introduced an “axial added mass coefficient” to better approximate the force of the surrounding fluid acting on the cantilevered beam. Based on Galerkin’s procedure, the vibration frequency and dynamical response of the cantilevered beam moving axially were obtained.

It is well known that boundary conditions have significant influence on vibration and stability of distributed parameter systems. In all available studies on axially moving beams immersed in fluid, boundary conditions under consideration were limited to be cantilevered. However, in some engineering circumstances, the boundary conditions of a beam on two ends may be hinged–hinged, clamped–clamped, hinged–clamped, or hybrid. For that purpose, this paper study vibration and stability of an axially moving beam in fluid and constrained by simple supports with torsion springs. Moreover, the axial tension in axially moving beams with both ends supported is different from that in the cantilevered beam, as will be seen later.

This paper is organized as follows. It first derives the equation of motions and presents the boundary conditions. Then the Differential Quadrature Method (DQM) is proposed to discretize the equation of motion as well as the boundary conditions. For a beam constituted by the Kelvin model, the effects of axially moving speed, mass ratio, and several other system parameters on vibration are analyzed through calculating the natural frequencies and the lowest critical moving speeds.

2. Problem formulation

The system under consideration (Fig. 2a) consists of a uniformly cylindrical beam with both ends supported of diameter \( D \), area moment of inertia \( I \), mass per unit length \( m \). The beam moves at an axial speed \( V(t) \) between two motionless ends separated by distance \( L \). A complex modulus of elasticity of the material is utilized, \((E + E' \delta / \partial t)I\), where \( E' \) is a coefficient of internal dissipation which was assumed to be viscoelastic and

![Fig. 1. The moving cantilever system developed by Gosselin et al., 2007 and Taleb and Misra (1981).](image-url)
of the Kelvin–Voigt type. Consider this beam to be immersed in an incompressible fluid of density \( \rho \). In the present study, only small lateral motions are considered, and it is assumed that no separation occurs in the cross-flow around the beam, and that the forces of the fluid acting on a beam element are the same as those acting on a corresponding element of a long undeformed beam of the same cross-sectional area and inclination.

We denote the transverse displacement as \( w(x, t) \). Thus, the transverse velocity of the beam can be found using the total derivative with respect to time

\[
c = \frac{D w(x, t)}{D t} = \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + V \frac{\partial w}{\partial x}
\]

in which the partial derivative of \( x \) with time is equal to the axially moving speed, \( V \).

In this study, we denote the lateral-direction virtual mass as \( M \), which can be defined as

\[
M = \rho \left( \frac{1}{4} \pi D^2 \right)
\]

The forces related to the interactions between the beam and the fluid are calculated in two parts, namely the inviscid forces analytically and viscous forces semi-empirically. This simplified approach has proven to be successful (Païdoussis, 2003).

Therefore, in the axial direction of the moving beam, the layer of fluid which stays attached to the beam is considerably smaller than that of the lateral-direction virtual mass. As noted by Païdoussis (2003) and Gosselin et al. (2007), this mass should have a value between zero and \( M \) and is accepted to be \( \beta M \), where \( \beta \) is an “axial added mass coefficient”. Thus, the total momentum of the fluid per unit length is found to be \( M(\partial \tilde{c}/\partial t + \beta V \partial \tilde{c}/\partial x)w \), and the rate of change of this momentum per unit length is \( M(\partial \tilde{c}/\partial t + \beta V \partial \tilde{c}/\partial x)^2 w \), which produces an equal and opposite lateral force on the moving beam. This lateral force is the inviscid forces and is expressed analytically.

As noted by Gosselin et al. (2007), however, the viscous drag acting on the beam can be calculated using the equations derived by Taylor (1952). This viscous drag accounts for the normal and longitudinal components of the force exerted on the cylindrical beam placed at an angle \( \theta \) in a flow of velocity \( U \), which is exactly equal
to \( U = (v^2 + w^2)^{1/2} \) and can be linearized to \( U \approx V \). Let \( F_N \) and \( F_L \) be the normal and longitudinal viscous forces per unit length, respectively. By using some linearization, these two components of viscous forces exerted on the cylindrical beam can be written as

\[
F_N = \frac{1}{2} C_N \left( \frac{M}{D} \right) \left( \frac{\partial w}{\partial t} + V \frac{\partial w}{\partial x} \right) + \frac{1}{2} \tilde{C}_N \left( \frac{M}{D} \right) \left( \frac{\partial w}{\partial t} + V \frac{\partial w}{\partial x} \right)
\]

\[
F_L = \frac{1}{2} C_T \left( \frac{M}{D} \right) V^2
\]

where

\[
C_N = C_T = \frac{4}{\pi} C_f, \quad \tilde{C}_N = \frac{4}{\pi}(8c_{\text{max}}/3\pi)C_D
\]

In the above equation, \( C_D \) and \( C_T \) are, respectively, the form and friction drag coefficients for a cylindrical beam in cross-flow. It is noted that \( c_{\text{max}} \) is given by the relationship (Gosselin et al., 2007)

\[
c^3 \approx c^2 c_{\text{max}} (8/3\pi)
\]

Now, consider a small beam element of length \( \delta x \) as shown in Fig. 2b. Let \( T \) and \( Q \) be the axial tension and shear force respectively. The differential equations of motion are found by summing forces in the \( x- \) and \( y- \) directions, yielding

\[
\frac{\partial T}{\partial x} - F_L = (m + \beta M) \frac{\partial V}{\partial t}
\]

\[
\frac{\partial Q}{\partial x} - F_L \frac{\partial w}{\partial x} - F_N - m \left( \frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right)^2 w - M \left( \frac{\partial}{\partial t} + \beta V \frac{\partial}{\partial x} \right)^2 w + \frac{\partial}{\partial x} \left( T \frac{\partial w}{\partial x} \right) = 0
\]

In the present study, only small transverse displacements are considered. Therefore, the \( x- \) direction components of normal viscous force and the inertia force, and the second-order terms involving \( w \) have been neglected or discarded, as can be seen in Eqs. (7) and (8).

By assuming the effects of shear deformation and rotary inertia to be negligible, the shear force can be written as

\[
Q = - \left( E + E' \frac{\partial}{\partial t} \right) I \frac{\partial^3 w}{\partial x^3}
\]

Combining Eqs. (7)–(9) one obtains

\[
\left( E + E' \frac{\partial}{\partial t} \right) I \frac{\partial^4 w}{\partial x^4} + \left[ (m + M \beta^2) V^2 - T(x) \right] \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left( C_N V + \tilde{C}_N \right) V \frac{M}{D} \frac{\partial w}{\partial x} + 2(m + \beta M) V \frac{\partial^2 w}{\partial x \partial t} + (m + M) \frac{\partial^2 w}{\partial t^2} + \frac{1}{2} \left( C_N V + \tilde{C}_N \right) \frac{M}{D} \frac{\partial w}{\partial t} = 0
\]

It is noting that the axial tension \( T(x) \) in the beam should be formulated. For that purpose, Eq. (4) is substituted into Eq. (7) and the whole is integrated over \( x \), while the integration constant is replaced by the value of initial tension \( (P_0) \) at the end of the beam; i.e.,

\[
T(x) = -(m + \beta M) \frac{\partial V}{\partial t} (L - x) - \frac{1}{2} C_T \left( \frac{M}{D} \right) V^2 (L - x) + P_0
\]

It is worth noting that the formula of the total axial tension \( T(x) \) is different from that in the cantilevered moving beam derived by Gosselin et al. (2007). In that study, a nonzero value of \( T(L) \) arises from drag-induced compression at the free end. Moreover, in the present paper, an initial tension \( P_0 \) is additionally considered. Now, substituting Eq. (11) into Eq. (10) gives the final equation of small lateral motions.
At both ends, the beam is constrained by hybrid supports (simple supports with torsion springs whose spring stiffness constant are $K_1$ and $K_2$ respectively). Setting the transverse displacements to zero and balancing the bending moment at both ends lead to the boundary conditions

\[
\begin{align*}
\left( E + E_{\alpha} \frac{\partial}{\partial t} \right) I \frac{\partial^4 w}{\partial x^4} &+ \left[ (m + M \beta^2) V^2 + \frac{1}{2} C_T \left( \frac{M}{D} \right) V^2 (L - x) + (m + \beta M) \frac{\partial V}{\partial t} (L - x) - P_0 \right] \frac{\partial^3 w}{\partial x^3} \\
+ \left[ \frac{1}{2} (C_N V + \tilde{C}_N) V \frac{M}{D} \frac{\partial w}{\partial x} + 2(m + \beta M) V \frac{\partial^2 w}{\partial x \partial t} + (m + M) \frac{\partial^3 w}{\partial t^3} + \frac{1}{2} (C_N V + \tilde{C}_N) \frac{M}{D} \frac{\partial w}{\partial t} \right] & = 0
\end{align*}
\]  

Fig. 3. (a) The imaginary component of the dimensionless frequency, $\Omega$, as functions of the moving speed, $v$, for the lowest three modes of a pinned–pinned beam. (b) The real component of the dimensionless frequency, $\Omega$, as functions of the moving speed, $v$, for the lowest three modes of a pinned–pinned beam.
Upon introducing the dimensionless variables and parameters

\[ \eta = \frac{w}{L}, \quad \xi = \frac{x}{L}, \quad \tau = \left( \frac{EI}{M + m} \right)^{1/2} t = \sigma t, \quad v = \left( \frac{M}{EI} \right)^{1/2} VL, \quad \varphi = \frac{M}{EI}, \quad \phi = \frac{m}{M} \]

\[ \Gamma = \frac{P_o L^2}{EI}, \quad \alpha = \sigma E^*, \quad C_N = \frac{C}{\alpha L}, \quad \varepsilon = \frac{L}{D}, \quad k_1 = \frac{K_1 L}{EI}, \quad k_2 = \frac{K_2 L}{EI} \]

Eqs. (12) and (13) become

\[ (1 + \alpha \frac{d n}{d z} + \frac{1}{2} C_T \varphi(1 - \xi)^2 + \frac{1}{2} \varphi^{1/2} (\varphi + \beta) \frac{d n}{d z} (1 - \xi) - \Gamma \right) \frac{d^2 \eta}{d z^2} + \left[ \frac{1}{2} e (C_N \varphi^{1/2}) \frac{d n}{d z} + \frac{1}{2} e (C_N \varphi^{1/2}) \right] \frac{d \eta}{d z} + \frac{1}{2} e (C_N \varphi^{1/2}) \frac{d \eta}{d z} = 0 \] \[(16) \]

3. Solutions by the DQM

The DQM approach (Bert and Malik, 1996; Ni et al., 2005; Ni et al., 2006; Ni and Huang, 2000; Wang and Ni, 2003) is used to formulate solutions to Eqs. (15) and (16). This approach of numerical analysis was introduced to problems of solid mechanics in 1996 by Bert and Malik (1996). An extended exposition of this approach is given in the first of these studies.

The DQM has been applied to the vibrations of beam-type structures (beams, pipes conveying fluid, etc.) (Ni et al., 2005; Ni et al., 2006; Ni and Huang, 2000; Wang and Ni, 2003). In the DQM, the partial derivative of a function with respect to a space variable at a given discrete point can approximately be expressed by a weighted linear sum of the function values at all discrete points. Consider a one-dimensional function \( f(x) \), the approximate values of \( L_k \{ f(x) \} \) at the \( i \)th discrete points are given by

\[ L_k \{ f(x) \}_{x=x_i} = \sum_{j=1}^{N} A_{ij}^{(k)} f(x_j) \] \[(17)\]

where \( L_k \) is linear differential operator, \( k \) is \( k \)th order of derivative, \( x \) is the independent variable, \( x_j \) \((j = 1, 2, \ldots, N)\) are the sample points obtained by dividing the \( x \)-variable into \( N \) discrete values, \( f(x_j) \) are the function values at point \( x_j \), and \( A_{ij}^{(k)} \) are the matrix elements of weighting coefficient matrix attached to these function values. Thus, for the function \( f(x) \), explicit formulas for the weighting coefficients given by Ni and Huang (2000) may be used. For the first-order derivative, the formulas are

\[ A_{ij}^{(1)} = \frac{\prod(x_i)}{\prod(x_j)(x_i - x_j)} \quad (i, j = 1, 2, \ldots, N; \ j \neq i) \] \[(18)\]

where

\[ \prod(x_i) = \prod_{k=1, k \neq i}^{N} (x_i - x_k), \quad \prod(x_j) = \prod_{k=1, k \neq j}^{N} (x_j - x_k) \] \[(19)\]

The off-diagonal elements of the weighting coefficient matrix corresponding to the second- and higher-order derivatives may be obtained through the following recurrence relationship

\[ A_{ij}^{(r)} = r \left[ A_{ii}^{(r-1)} A_{ij}^{(r-1)} - A_{ij}^{(r-1)} \right] \quad (i, j = 1, 2, \ldots, N; \ j \neq i) \] \[(20)\]

and their diagonal elements are given by
\[ A_i^{(i)} = - \sum_{k=1, k \neq i}^N A_k^{(i)}, \quad (i = 1, 2, \ldots, N) \]  

For beams there are two conditions at each end. It is necessary to enforce one of the boundary equations at an interior point. This point, a ‘\( \delta \) point’, is taken a short distance (\( \delta \approx 10^{-5} \) on a unit domain) from the boundary point. The sampling points on a unit domain (\( \xi(0 \leq \xi \leq 1) \)) are thus taken as

\[
\begin{align*}
\xi_1 &= 0, \quad \xi_2 = \delta, \quad \xi_{N-1} = 1 - \delta, \quad \xi_N = 1 \\
\xi_i &= \frac{1}{2} \left[ 1 - \cos \left( \frac{i - 3}{N - 4} \pi \right) \right] \quad (i = 3, 4, \ldots, N - 2)
\end{align*}
\]  

Substitution of Eq. (17) into Eqs. (15) and (16) leads to the DQM domain equations

\[
\begin{align*}
&\sum_{j=1}^N \left[ A_{ij}^{(4)} + \left( (\phi + \beta^2)v^2 + \frac{1}{2} C_T v (1 - \xi_i) v^2 + \phi^{1/2} (\phi + \beta) \frac{\partial v}{\partial \xi} (1 - \xi_i) - \Gamma \right) A_{ij}^{(2)} + \left( \frac{1}{2} \frac{\partial v}{\partial \xi} (C_N v + \overline{C_N} \phi^{1/2}) \right) A_{ij}^{(1)} \right] \eta_j \\
&+ \sum_{j=1}^N \left[ 2 \phi^{1/2} (\phi + \beta) v A_{ij}^{(1)} + z A_{ij}^{(4)} \right] \dot{\eta}_j + \frac{1}{2} v (C_N v \phi^{1/2} + \overline{C_N} \phi) \dot{\eta}_i + \dot{\eta}_i = 0 \\
&\quad (i = 3, 4, \ldots, N - 2) \\
&\eta_1 = 0; \quad \sum_{j=1}^N [A_{ij}^{(2)} - k_1 A_{ij}^{(1)}] \eta_j = 0, \quad i = 2 \\
&\eta_N = 0; \quad \sum_{j=1}^N [A_{ij}^{(2)} + k_2 A_{ij}^{(1)}] \eta_j = 0, \quad i = N - 1
\end{align*}
\]  

By rearranging Eqs. (23)–(25), an assembled form is given as follows:

\[
\begin{bmatrix}
K_{bb} & K_{bd} \\
K_{db} & K_{dd}
\end{bmatrix}
\begin{bmatrix}
\eta_b \\
\eta_d
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 \\
G_{db} & G_{dd}
\end{bmatrix}
\begin{bmatrix}
\dot{\eta}_b \\
\dot{\eta}_d
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 \\
M_{db} & M_{dd}
\end{bmatrix}
\begin{bmatrix}
\ddot{\eta}_b \\
\ddot{\eta}_d
\end{bmatrix}
= 0
\]  

where the subscript \( b \) denotes elements associated with the boundary points while \( d \) the remainder, such as

\[
\begin{align*}
\eta_b &= \{ \eta_1, \eta_2, \eta_{N-1}, \eta_N \}^T, \quad \eta_d &= \{ \eta_3, \ldots, \eta_{N-2} \}^T
\end{align*}
\]  

Obviously, all the sub-matrixes in Eq. (26) can be determined by Eqs. (23)–(25). For a self-excited vibration, the solution of Eq. (26) can be written in the following form:

\[
\{ \eta \} = \{ \dot{\eta} \} \exp(\Omega \tau)
\]  

where

\[
\{ \dot{\eta} \} = \{ \dot{\eta}_b \}^T, \quad \{ \dot{\eta}_d \}^T
\]  

and \( \{ \dot{\eta} \} \) is an undetermined function of amplitude, \( \Omega \) is a dimensionless frequency.

Substituting Eq. (28) into Eq. (26) and after having eliminated \( \{ \dot{\eta}_b \} \), one obtains a homogeneous equation, which corresponds to a generalized eigenvalue problem

\[
(\Omega^2 [M] + \Omega [G] + [K]) \{ \eta_d \} = \{ 0 \}
\]  

To obtain a non-trivial solution of the above equation, it is required that the determinant of the coefficient matrix vanishes, namely,

\[
\det(\Omega^2 [M] + \Omega [G] + [K]) = 0
\]  

where \([M], [G]\) and \([K]\) denote the structural mass matrix, damping matrix and stiffness matrix, respectively. The matrix elements of \([G]\) and \([K]\) are associated with several parameters of the beam system. Therefore, one can compute the eigenvalue numerically from Eq. (31) and obtain the natural frequency of the beam with various parameter values.
It ought to be mentioned that Eq. (31) forms a generalized eigenvalue equation and should be reduced to a standard eigenvalue equation. For that purpose, we let
\[
\{\Phi\} = (\{\ddot{\eta}_d\}, \{\ddot{\eta}_d\})^T
\]
and one obtains
\[
[A]\{\Phi\} = \Omega\{\Phi\}
\]
in which
\[
[A] = \begin{bmatrix}
-|M|^{-1}|G| & -|M|^{-1}|K| \\
[I] & [0]
\end{bmatrix}
\]

4. Results

Numerical studies have been conducted to investigate the effects of several key parameters on the dynamics and stabilities of the axially moving beam immersed in fluid.

For the numerical studies, a beam with constant moving speed shown in Fig. 2a is considered. Thus, the case of \( k_1 = 0 \) and \( k_2 = 0 \) corresponds to pinned–pinned boundary conditions; the case of \( k_1 = \infty \) and \( k_2 = \infty \) corresponds to clamped–clamped boundary conditions; and the case of \( k_1 = 0 \) and \( k_2 = k_0 \) (\( 0 < k_0 < \infty \)) corresponds to hybrid supports. These three cases mentioned will be of our interest. For that purpose, the natural frequency Im(\( \Omega \)) is determined numerically from Eq. (31). Unless otherwise stated, several parameters are chosen to be
\[
\begin{align*}
N &= 19, \quad \delta = 0.00001, \quad \beta = 0.1, \quad \varphi = 0.2, \quad \varepsilon = 20, \quad \tilde{\gamma} = 0.001 \\
\Gamma &= 1, \quad C_N = 0.1, \quad C_N = C_T = 1.0, \quad k_1 = 0, \quad k_2 = 0
\end{align*}
\]

4.1. Natural frequency

Fig. 3 shows the effect of moving speed on the variation of the lowest three eigenvalues (\( \Omega_1, \Omega_2 \) and \( \Omega_3 \)) of the moving beam with pinned–pinned supports (\( k_1 = 0 \) and \( k_2 = 0 \)). When \( v < v_{cr} \approx 1.06 \), the imaginary parts of eigenvalues (i.e., natural frequencies) are reduced in magnitude as the moving speed is increased to the lowest critical moving speed \( v_{cr} \approx 1.06 \). Thus, if the moving speed is kept below the lowest critical moving speed, the first natural model, the second natural mode and the third natural mode corresponding to \( \Omega_1, \Omega_2 \) and \( \Omega_3 \) respectively, are always stable. It is interesting to observe that only the imaginary part of \( \Omega_1 \) (i.e., the first natural frequency) vanishes completely as the moving speed reaches the lowest critical moving speed while its real part turns to two different values (see Fig. 3b). Physically this implies that the first natural mode becomes unstable by the divergence instability when the moving speed becomes equal or larger than the lowest critical moving speed \( v_{cr} \approx 1.06 \). The lowest critical moving speed is called the divergence speed and denoted by \( v_{D1} = v_{cr} \).

For \( 1.06 < v < 2.27 \), the first natural mode is unstable with divergence instability, while the second natural mode and the third natural mode keep stable.

For \( 2.27 < v < 3.1496 \), as the moving speed is increased, the first natural frequency increases while the second and third keep positive. It is noted that the two values of the real part of \( \Omega_1 \) merge to each other at \( v_c \approx 2.27 \) (i.e., the critical moving speed at which the flutter instability occurs in the first mode). This implies, in the range \( 2.27 < v < 3.1496 \), that the second and third modes are always stable, and the first natural mode will be unstable (flutter instability).

However, for \( 3.1496 < v < 3.4 \), as the moving speed is increased, the second natural frequency becomes zero. Similarly as before, its real part turns to two different values (see Fig. 3b). Physically this implies that the second natural mode becomes unstable by the divergence instability when the moving speed becomes equal or larger than \( v_{D2} \approx 3.1496 \) (i.e., the critical moving speed at which the divergence instability occurs in the second mode).
In Fig. 4, the axially moving beam is clamped at both ends \((k_1 = \infty \text{ and } k_2 = \infty)\). It can be seen that the lowest critical moving speed is \(v_{cr} \approx 2.072\), which is higher than that for the beam with pinned–pinned supports. The critical moving speed \(v_F\) becomes \(v_F \approx 3.006\), which is still higher than that for the pinned–pinned case.

As shown in Fig. 4, it is interesting that the second mode is stable in the range \(0 < v < 4.3\). However, the third mode loses stability at \(v_{D3} \approx 4.18\) (i.e., the critical moving speed at which the divergence instability occurs in the third mode).

For the case of axially moving beam with hybrid supports \((k_1 = 0 \text{ and } k_2 = 10)\), the evolution of the eigenvalues with increasing moving speed is shown in Fig. 5. It is clear that, in this case, divergence in the first mode occurs at \(v_{cr} \approx 1.298\), which is higher than the one for the pinned–pinned supports while is lower than the one
for the clamped-clamped supports. At $v_F \approx 2.679$, flutter instability can be detected in the first mode. At $v_D \approx 3.393$, divergence instability occurs in the second mode. However, in the range $0 < v < 3.5$, the third mode keeps stable, as can be seen in Fig. 5.

4.2. The effects of several key parameters on the lowest critical moving speed

It is instructive to look at the effects of several key parameters on the lowest critical moving speed (i.e., $v_{cr}$). The results are shown in Figs. 6–9 for the pinned–pinned boundary conditions. Fig. 6 shows the variations of $v_{cr}$ with the mass ratio, $\varphi$. When the mass ratio is small, the lowest critical moving speed is increased as $\varphi$ increases. However, as the mass ratio is increased beyond a critical value, the lowest critical moving speed

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Fig. 5. (a) The imaginary component of the dimensionless frequency, $\Omega$, as functions of the moving speed, $v$, for the lowest three modes of a beam with hybrid supports. (b) The real component of the dimensionless frequency, $\Omega$, as functions of the moving speed, $v$, for the lowest three modes of a beam with hybrid supports.
Fig. 6. Variations of $v_{cr}$ with $\varphi$.

Fig. 7. Variations of $v_{cr}$ with $\beta$.

Fig. 8. Variations of $v_{cr}$ with $\varepsilon$. 
is reduced with increasing $\varphi$. In Fig. 7, the curve shows that the lowest critical moving speed is reduced as the axial added mass coefficient increases. Also to be remarked is that the lowest critical moving speed decreases with increasing $\varepsilon$ (see Fig. 8). From Fig. 9, it is seen that the lowest critical moving speed is associated with the stiffness ($k_2$) of the torsion spring. When $k_2$ is increased, the lowest critical moving speed becomes higher. Moreover, as can be expected, the lowest critical moving speed will keep at a fixed value with sufficiently large values of $k_2$, which corresponds to the pinned-clamped boundary conditions.

5. Conclusion

The equations of motion of a flexible slender beam with uniform circular cross-section, moving axially in horizontal plane at a known rate while immersed in an incompressible fluid are derived. To formulate the axial tension in the moving beam, an axial added mass coefficient is introduced in these equations and the initial tension considered. Numerical studies are conducted to investigate the effect of the moving speed on the natural frequency and stability of axially moving beams for three end conditions. It is also found that the divergence in the first mode occurs as the moving speed of the beam is increased. As the result, only the first natural mode may become unstable with flutter.

Moreover, the lowest critical moving speeds for the pinned–pinned end conditions have been presented. The results can be used in the analysis of axially moving beams in fluid for checking the stability. Since the deformation may be large, the nonlinear theory has to be developed. Therefore, the nonlinear theory of axially moving beam in fluid will be the subject of further study.

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