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Nonlinear thermal convection in a viscoelastic nanofluid saturated porous medium under gravity modulation

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Abstract This paper carried out a nonlinear thermal convection in a porous medium saturated with viscoelastic nanofluid under vibrations. The Darcy model has been used for the porous medium, while the nanofluid layer incorporates the effect of Brownian motion along with thermophoresis. An Oldroyd-B type constitutive equation was used to describe the rheological behavior of viscoelastic nanofluids. The non-uniform vertical vibrations of the system, which can be realized by oscillating the system vertically, is considered to vary sinusoidally with time. In order to find the heat and mass transports for unsteady state, a nonlinear analysis, using a minimal representation of the truncated Fourier series of two terms, has been performed. Effect of various parameters has been investigated on heat and mass transport and then presented graphically. It is found that gravity modulation can be used effectively to regulate either heat or mass transports in the system.

1. Introduction

As it is well known fact that thermal conductivity of solids is greater than fluids, in general fluids in heat transfer has applications, such as water, ethylene glycol and engine oil have low thermal conductivity when compared to thermal conductivity of solids, especially metals. Hence an addition of solid particles in a fluid can increase the conductivity of fluids. Due to the Brownian motion of nano-particles through fluids, better results obtained for heat transport. Brownian motion increases the mode of heat transfer or mass transfer in the system. The word nanofluid to represent the dispersion of nanoparticles and the enhancement of higher thermal conductivity of nanofluids were introduced by Masoud et al. [1] and Choi [2]. Numerous attempts have been made to find the enhanced behavior of high thermal conductivity of nanofluids, some of them are Chen et al. [3], Vadász [4,5]. However, a satisfactory explanation has yet to be found as emphasized by Eastman et al. [6] in their recent comprehensive review of the nanofluid literature. This enhanced behavior of thermal conductivity

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implies an enormous potential of nanofluids for device miniaturization and process intensification which could have impacts on many industrial sectors including chemical processing, transportation, electronics, medical, energy, and the environment. The ballistic nature of heat transport within porous media was analyzed by Chen [7]. Further 10–30% increase of the effective thermal conductivity in alumina/water nanofluids with 1–4% of alumina was reported by Das et al. [8]. These reports led Buongiorno and Hu [9] to suggest the possibility of using nanofluids in advanced nuclear systems. Non-Newtonian rheological behavior of nanofluids is significant, but for large Lewis number, the effect was small. Wu and Kao [21] found that engine oil with TiO2 nanoparticle additive exhibited lower friction force as compared to the original oil. Their experiment showed that a smaller particle size exhibits better friction reduction with particle size ranging from 59 to 220 nm. Some other studies related to the nanofluids are given by [22–32] and corresponding introduction therein.

The above literature deals with nanofluids as Newtonian fluids. Non-Newtonian rheological behavior of nanofluids is indicated by many investigators like Chen et al. [34], Schmidt et al. [35]. Thermal convection of non-Newtonian fluids in a porous medium received considerable importance in several fields of application such as food processing, oil recovery, and the spread of contaminants in the environment, and in various processes in the chemical and materials industries. The onset of convection in a Non-Newtonian nanofluid saturated porous medium was briefly discussed by Nield [36]. He noticed that, the Horton-Rogers-Lapwood problem becomes singular when a Newtonian fluid is replaced by a standard power-law fluid. This singularity can be removed when the nanofluid effects due to thermophoresis and Brownian motion become independent of the power-law index. The concept of thermal convection in a viscoelastic fluid-saturated porous medium was investigated by many authors given in [37–47].
Here most of the authors investigated onset of thermal convection and nonlinear thermal instability for binary fluid saturated porous medium. Some of them are considered rotating porous medium, and effective results obtained for linear and nonlinear studies. But, modulation work has not been evaluated in their studies which is important in regulation of convective phenomenon in the medium.

The gravity modulation is one consisting of varying acceleration term in the gravitational Rayleigh number around the gravitational acceleration, i.e., by vertically oscillating a horizontal porous layer. This modulation leads to the variable coefficients in the momentum equation and involves the vertical time-periodic vibrations of the system. Also, this leads to the appearance of a modified gravity, collinear with actual gravity, in terms of a time-periodic gravitational perturbation and it is known as g-jitter. The modulated gravity field which can be used to modify the momentum equation in order to control the convective phenomenon in the form of amplitude and frequency of modulation is an important phenomenon in thermal and engineering sciences. An application can be seen in materials processing under reduced gravity conditions when convection due to buoyancy forces is strongly reduced.

For zero gravity the desired basic state may be set up in a melt when convection due to buoyancy forces is strongly reduced. Also for any residual gravity and in particular fluctuations of effective gravity due to orientation changes of the vehicle and on-board activities introduce notable perturbations. The residual acceleration fields on board of a spacecraft are nonstationary and the measured oscillation frequencies are from $10^{-2}$ Hz to 100. In crew activity or orbital maneuvers (in spaceflight, an orbital maneuver is the use of propulsion systems to change the orbit of a spacecraft) give rise to time dependent accelerations with high amplitudes and fluctuating direction. In space, the gravity effect is suppressed and hence buoyancy effect also reduces. However, microgravity environment is helpful in suppressing convective flows. The effect of g-jitter which originates from crew motions, mechanical vibrations (motors, pumps, excitations of natural frequencies of spacecraft structure), atmospheric drag and the earth’s gravity gradient have shown to make it difficult to realize a diffusion controlled growth from melts in micro-gravity. The topology of the neutral curves is more complex than that encountered in constant gravity multiply diffusive layers, leading to new types of behavior not possible in the absence of modulation.

Gresho and Sani [48] was the first to study the gravity modulation on the stability of a heated fluid layer. They studied the impact of the two-dimensional gravity modulation on the convective threshold of a stable and an unstable motionless state. Malashetty and Padmavathi [49] investigate the effect of gravity modulation on the onset of convection in fluid and porous layers. Recently, Umavathi [50] studied both temperature and gravity modulation of convection in a porous medium saturated by a nanofluid by using a linear stability analysis. For nonlinear case of thermal instability in fluid saturated porous medium with vibrations was studied by Bhadaura and Kiran [51–52]. They show that, the gravity modulation can be used to alter the heat and mass transport in the medium. The effect of nonlinear throughflow on binary viscoelastic fluid saturated porous medium under gravity modulation is investigated by Kiran [53]. He shows that, throughflow plays a dual role, for outflow enhances the heat and mass transfer and inflow diminishes the heat and mass transfer. The modulation frequency diminishes or modulation amplitude increases the heat and mass transfer in the system. He also found that, oscillatory flows strengthen the heat and mass transfer in terms of oscillatory frequency than stationary flows, Bhadaura and Kiran [54–57], studied a nonlinear convection under gravity modulation for stationary and oscillatory modes, they found that, heat or mass transfer rates are better for oscillatory mode than stationary mode of convection.

The onset of thermal convection in a viscoelastic nanofluid saturated porous medium is studied by Shue [58] using modified Darcy model. They derived analytically the onset criterion for stationary and oscillatory convection and found that, the oscillatory case is possible in both bottom and top heavy nanoparticle distributions. They found that, the viscoelasticity and nanofluid properties cause the convection to set in through oscillatory rather than stationary modes. This gives only onset criteria but, missing finite amplitude convection which is an important phenomenon for nonlinear theories. The convection of non-Newtonian fluids in a porous medium has a wide range of applications; some of them mentioned in 2nd paragraph, and in various processes in the chemical and material industries. Since the elastic behavior of viscoelastic fluids and it is inherent in non-Newtonian fluids, oscillatory instabilities can set in before stationary modes. While considering the above literature, very few works are available for nanofluid saturated porous medium under modulation due to [50,51]. But, for
non-Newtonian nanofluid fluid case no study is being reported under vibrating porous medium, where vibrations of the system provides analysis to regulate heat or mass transport in terms of amplitude and frequency of modulation. This motivated, to study a nonlinear analysis of thermal instability in a viscoelastic nanofluid saturated porous medium under gravity modulation.

2. Mathematical formulation

An Oldroyd-B nanofluid-saturated horizontal layer of porous medium confined between \( z = 0 \) and \( z = d \) is considered. The physical configuration is represented by Fig. 1. Each boundary wall is assumed to be perfectly thermally conducting. The porous layer is extended infinitely in \( x \) and \( y \)-directions, and \( z \)-axis is taken vertically upward with the origin at the lower boundary. The temperatures at the lower and upper walls are taken to be \( T_b \) and \( T_r \), respectively, the layer is heated from below and cooled from above. The Darcys law is assumed to hold and the Oberbeck–Boussinesq approximation is employed. Homogeneity and local thermal equilibrium solid and fluid phases are assumed. The reference temperature is taken to be \( T_r \). For linear theory, the change in temperature of the nanofluid is assumed to be small in comparison to \( T_r \). Employing the Oberbeck–Boussinesq approximation, the governing equations to study the thermal instability are given by Sheu [58]:

\[
\begin{align*}
\nabla \cdot \mathbf{v}_b &= 0, \quad (1) \\
(1 + \frac{\partial}{\partial t}) \left( \frac{\rho_f}{\tau_1} \frac{\partial \mathbf{v}_b}{\partial t} \right) + \nabla p - \left[ \rho_p (1 - \mu \phi_0) + \frac{1}{(1 - \phi)} \right] + \nabla \psi &= \frac{K}{(1 + \frac{\partial}{\partial t})} \psi_b, \quad (2) \\
(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f \mathbf{v}_b \cdot \nabla T &= k_m \nabla^2 T + \frac{D_T}{T_r} \nabla \phi \cdot \nabla T, \quad (3) \\
\frac{\partial \phi}{\partial t} + \frac{1}{\delta_1} \mathbf{v}_b \cdot \nabla \psi &= D_T \nabla^2 \phi + \frac{D_T}{T_r} \nabla^2 T, \quad (4) \\
\tilde{g} = \tilde{g}_0 (1 + \epsilon \cos(\Omega t)) \tilde{k}, \quad (5)
\end{align*}
\]

where \( \mathbf{v}_b \) is the Darcy velocity, where \( D_T \) is the Brownian diffusion coefficient and \( D_T \) is the thermophoretic diffusion coefficient. The physical variables have their own meanings given in nomenclature. Assuming temperature and volumetric fraction of the nanoparticles to be constant at the stress-free boundaries, one may take the boundary conditions on \( T \) and \( \phi \) as:

\[
\begin{align*}
\mathbf{v} &= 0, \quad T = T_b, \quad \phi = \phi_0 \text{ at } z = 0, \quad (6) \\
\mathbf{v} &= 0, \quad T = T_r, \quad \phi = \phi_1 \text{ at } z = d, \quad (7)
\end{align*}
\]

where \( \phi_1 \) is greater than \( \phi_0 \). The dimensionless variables are considered as given below:

\[
\frac{(x', y', z')}{d} = (x, y, z)/d, \quad \tau' = \tau k_T / \gamma^2, \quad \Psi'(u', v', w') = (u, v, w) - \frac{\phi_1 - \phi_0}{\phi_1 - \phi_0} T, \quad \phi' = \phi - \frac{\phi_1 - \phi_0}{\phi_1 - \phi_0} T, \quad \frac{d/k_T}{\nabla} = p K_j/k T, \quad \frac{\gamma}{(\rho c)_f (\rho_f)} = \frac{\lambda}{\mu_0}, \quad \tilde{g} = \frac{g_0}{(1 + \epsilon \cos(\Omega t)) \tilde{k}}, \quad \epsilon = \frac{\lambda}{\mu_0}.
\]

The non-dimensionalized governing equations are (after dropping the asterisk for simplicity)

\[
\begin{align*}
\nabla \cdot \mathbf{v} &= 0, \quad (8) \\
\left(1 + \frac{\partial}{\partial t} \right) \left( \frac{1}{P_r} \frac{\partial \mathbf{v}}{\partial t} + \nabla p + g_0 (Ra - T + Ra \phi_0) \right) &= - \left(1 + \frac{\partial}{\partial t} \right) \nonumber \mathbf{v}, \quad (9) \\
\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T &= \nabla^2 T + \frac{N_A}{Le} \nabla \phi \cdot \nabla T + \frac{N_4 N_2}{Le} \nabla^2 T \cdot \nabla T, \quad (10) \\
\frac{1}{Le} \frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi &= \frac{1}{Le} \nabla^2 \phi + \frac{N_4}{Le} \nabla T^2 \sin T, \quad (11) \\
\mathbf{v} &= 0, \quad T = 1, \quad \phi = 0 \text{ at } z = 0, \quad \text{and} \quad \mathbf{v} = 0, \quad T = 0, \quad \phi = 1 \text{ at } z = 1, \quad (12)
\end{align*}
\]

where \( g_0 = (1 + \epsilon \cos(\Omega t)) \). The non-dimensionalized parameters in the above equations have their usual meanings given in nomenclature, \( N_A \) is the modified diffusivity ratio, which is similar to the Soret parameter that arises in cross diffusion in thermal instability. At the basic state, the nanofluid is assumed to be at rest, therefore the quantities at the basic state will vary only in \( z \)-direction, and are given by:

\[
\mathbf{v} = 0, \quad p = p_b(z), \quad T = T_b(z), \quad \phi = \phi_b(z). \quad (13)
\]

Substituting the Eq. (13) in Eqs. (10) and (11), one can get:

\[
\begin{align*}
\frac{d^2 T_b}{dz^2} + \frac{N_A}{Le} \frac{dT_b}{dz} + \frac{N_4 N_2}{Le} \left( \frac{dT_b}{dz} \right)^2 &= 0, \quad (14) \\
\frac{d^2 \phi_b}{dz^2} + \frac{d^2 T_b}{dz^2} &= 0. \quad (15)
\end{align*}
\]

According to Buongiorno [12], for most of the nanofluid studies the value of \( Le/\phi_1/\phi_0 \) is large of order \( 10^5 \), since the nanoparticle fraction decrement \( (\phi_1 - \phi_0) \) is typically no smaller than \( 10^{-3} \) this means that \( Le \) is large of order \( 10^2 - 10^3 \), while \( N_4 \) is no greater than about 10. Using the above analysis, Tzou [16,17], Nield and Kuznetsov [18] showed that the second and third terms in equation Eq. (14) are small and hence obtain the following:

\[
\begin{align*}
\frac{d^2 T_b}{dz^2} &= 0, \quad \frac{d^2 \phi_b}{dz^2} = 0. \quad (16)
\end{align*}
\]

The boundary conditions for solving Eq. (16) can be obtained from Eq. (12) as:

\[
\begin{align*}
T_b &= 1, \quad \phi_b = 0 \text{ at } z = 0, \quad (17) \\
T_b &= 0, \quad \phi_b = 1 \text{ at } z = d. \quad (18)
\end{align*}
\]

Solving the Eq. (16), subject to the above conditions given in Eqs. (17) and (18), obtain the following the solution:

\[
\begin{align*}
T_b &= 1 - z, \quad \phi_b = z. \quad (19) \\
T_b &= 0, \quad \phi_b = 1. \quad (20)
\end{align*}
\]

3. Nonlinear stability

Now superimpose the perturbations on the basic state as given below:

\[
\mathbf{v} = \mathbf{v}', \quad p = p_b + p', \quad T = T_b + T', \quad \phi = \phi_b + \phi'. \quad (21)
\]

Substituting the above Eq. (21) in Eqs. (8)–(11), and using the expressions (19) and (20), eliminating the pressure and introducing the stream function, one can arrive at
\[
\left(1 + \frac{\partial}{\partial \tau}\right) \left(1 \frac{\partial}{\partial \tau} \left(\nabla^2 \psi\right) + g_m \frac{\partial T}{\partial x} - g_m \frac{\partial \phi}{\partial x}\right)
= - \left(1 + \varepsilon \frac{\partial}{\partial \tau}\right) \nabla^2 \psi, \tag{22}
\]
\[
\frac{\partial \psi}{\partial x} \nabla^2 T = - \frac{\partial T}{\partial x} \frac{\partial (\psi, T)}{\partial (x, z)}, \tag{23}
\]
\[
- \frac{1}{\rho_1} \frac{\partial \psi}{\partial x} - N_s \nabla^2 T = \frac{1}{\rho_e} \nabla^2 \phi - \frac{1}{\gamma} \frac{\partial \phi \phi}{\partial (x, z)}. \tag{24}
\]
A local nonlinear stability analysis shall be performed and hence consider the following Fourier expressions:

\[
\psi = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm}(\tau) \sin(mx) \sin(nz), \tag{25}
\]
\[
T = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} B_{nm}(\tau) \cos(mx) \sin(nz), \tag{26}
\]
\[
\phi = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} C_{nm}(\tau) \cos(mx) \sin(nz), \tag{27}
\]

In general the following modes (1, 1) is for stream function, (0, 2) for temperature and (1, 1) for nanoparticle concentration, which means only two terms have been considered (also see the studies of [27–33]) in order to study heat and mass transfer. The reader may note that, here is first nonlinear effects are accounted and further terms may slightly be addition to the nonlinear effects.

\[
\psi = A_{11}(\tau) \sin(2x) \sin(2z), \tag{28}
\]
\[
T = B_{11}(\tau) \cos(2x) \sin(2z) + B_{02}(\tau) \sin(2\pi z), \tag{29}
\]
\[
\phi = C_{11}(\tau) \cos(2x) \sin(\pi z) + C_{02}(\tau) \sin(2\pi z), \tag{30}
\]

where the amplitudes \(A_{11}(\tau), B_{11}(\tau), B_{02}(\tau), C_{11}(\tau)\) and \(C_{02}(\tau)\) are functions of time and are to be determined. Substituting the Eqs. (28)–(30) in Eqs. (22)–(24), and taking the orthogonality condition with the eigenfunctions, associated with the considered minimal mode, obtain the following simultaneous differential equations:

\[
\frac{d}{d\tau} A_{11}(\tau) + \frac{Pr \rho_0}{\rho_1} \left[ \sum_{n=1}^{\infty} C_{11}(\tau) \right] - \frac{Pr \rho_0}{\rho_1} \left[ \sum_{n=1}^{\infty} C_{02}(\tau) \right] \frac{d}{d\tau} A_{11}(\tau) = - \left[ A_{11}(\tau) - \frac{\rho_0}{\rho_1} A_{02}(\tau) B_0(\tau) \right], \tag{31}
\]
\[
\frac{d}{d\tau} B_{11}(\tau) = - \left[ A_{11}(\tau) + \frac{\rho_0}{\rho_1} A_{02}(\tau) B_0(\tau) \right], \tag{32}
\]
\[
\frac{d}{d\tau} B_{02}(\tau) = \frac{\pi \rho_0}{\rho_1} A_{11}(\tau) B_0(\tau) - 4 \pi^2 B_0(\tau), \tag{33}
\]
\[
\frac{d}{d\tau} C_{11}(\tau) = - \left[ A_{11}(\tau) + \frac{\rho_0}{\rho_1} A_{02}(\tau) C_0(\tau) + \frac{N_s}{\rho_1} \frac{d}{d\tau} B_1(\tau) \right], \tag{34}
\]
\[
\frac{d}{d\tau} C_{02}(\tau) = \frac{\pi \rho_0}{\rho_1} A_{11}(\tau) C_0(\tau) - 4 \pi^2 C_0(\tau) + \frac{N_s}{\rho_1} \frac{d}{d\tau} B_1(\tau). \tag{35}
\]

The above system of simultaneous autonomous ordinary differential equations can be subsequently solved numerically using NDSolve Mathematic 8.

4. Heat and mass transport

The Nusselt number for heat transport \(Nu(\tau)\) is defined as

\[
\text{Nu}(\tau) = \frac{\text{Heat transport by} (\text{conduction} + \text{convection})}{\text{Heat transport by conduction}}
= 1 + \left[ \int_0^{2\pi/\kappa} \left( \frac{\partial \phi}{\partial x} \right) dx \right]_{\tau=0}. \tag{36}
\]

Substituting the Eqs. (19) and (29) in Eq. (36), obtain the Nusselt number

\[
\text{Nu}(\tau) = 1 - 2\pi B_0(\tau). \tag{37}
\]
The Sherwood number, Sh(τ), is defined similar to the Nusselt number, as follows

\[
Sh(\tau) = \frac{\text{Mass transport by (molecular diffusion + advection)}}{\text{Mass transfer by molecular diffusion}},
\]

\[
= 1 + \frac{1}{2} \int \left[ \frac{\partial}{\partial z} \left( \frac{1}{z} \frac{\partial T}{\partial z} \right) + \nu \frac{\partial^2 T}{\partial z^2} \right] dz \bigg|_{z=0},
\]

\[
= (1 - 2\pi C_0(\tau)) + N_A(1 + 2\pi B_0(\tau)).
\]

5. Results and discussion

Since transport phenomena associated with nanofluids have received numerous applications in many fields such as the delivery of nanodrug solar collectors, thermal management, transportation, the environment and national security, nanofluids can be optimized during manufacture using sheet processing. Many superior lubricants as well as thermal working fluids may develop for applications in aerospace, medical engineering, energy systems, etc. Moreover modulated flows provide a way that is external to the system helps us to control heat or mass transfer. More recent motivation for the present work has been provided by the development of space experiments and the use of mechanical vibration in industrial processes requiring control of convective motions. In this paper the effect of gravity modulation in a horizontal layer of a porous medium saturated with a viscoelastic nanofluid is investigated. Using Darcy model in the momentum equation a nonlinear stability analysis is performed to study heat and mass transport. A linear theory has been investigated by Umavathi [50] while considering temperature and gravity.
modulations for ordinary nanofluid saturated porous medium. Sheu [58] investigated a linear stability analysis for a layer of porous medium saturated with a viscoelastic nanofluid. Keeping in mind of both the papers, the aim of the present paper is to study a nonlinear thermal instability under gravity modulation to control heat and mass transport in the medium. According to Buongiorno [12], for most nanofluids investigated so far $Le$ is large, but Bhadauria and Agarwal [31] considered $Le = 10$ in order to show the parameter effect clearly for nanoparticle concentration Rayleigh number. In this problem the value of $Le$ has taken around 30.

The effect of gravity modulation on heat transport has been depicted in Figs. 2–15. The following parameters $PrD$, $Rn$, $Na$, $Le$, $\varepsilon$, $\gamma$, $\epsilon$ and $\Omega$ occur in the present study, and influence the convective heat and mass transport. The amplitude $\varepsilon$ and frequency $\Omega$ of modulation are chosen to be...
external to the system of controlling convection. Because of small amplitude of modulation, the values of $\varepsilon$ are considered to be small. Further, the gravity modulation assumed to be of low frequency, as at low range of frequencies, the effect of frequency on onset of convection as well as on heat and mass transport is maximum. The coefficient of heat transport, i.e. Nusselt number and the coefficient of nanoparticle concentration transport, i.e. Sherwood number are calculated as function of time and other parameters of the system. The obtained results are depicted in Figs. 2–15 for Nu and Sh versus time $\tau$. In the figures the values of Nu and Sh start with 1 and 2 respectively, and remain constant for a quite some time, showing the conduction state. Then the values of Nu and Sh increase as time passes, thus showing that the convection is taking place. These values oscillate and then approach constant values thus showing the steady state.

While keeping the parameter values as $(Ra = 10^4, Pr_D = 1.0, \lambda_1 = 0.6, \lambda_2 = 0.1, Ra = 5.0, Na = 1.0, Le = 30, \gamma = 1.0, \varepsilon = 0.1)$ and $\omega = 2.0$, fixing them, each individual effect of parameter on heat and mass transport is discussed and the results are presented in the graphs. From Figs. 2 and 3, it is found that initially when time $\tau$ is small the vibrations become of high amplitudes as the value of Prandtl–Darcy number $Pr_D$, Nusselt and Sherwood numbers increases as $Pr_D$, thus increasing the rate of heat and mass transport. But, at large values of time $\tau$, the vibrations become smaller and subsequently the values of Nu and Sh approach steady state values. The value of $Pr_D$ can be

![Figure 16](image-url)  
Figure 16  Streamlines at (a) $\tau = 0.1$, (b) $\tau = 0.12$, (c) $\tau = 0.14$, (d) $\tau = 0.16$, (e) $\tau = 0.2$, (f) $\tau = 0.6$. 
taken more than one, in that case the effect of local acceleration term which appears in momentum equation will be disappeared. Taking $Pr_D = 0.5, 1.0, 3.0$ one can see that, there is increment in heat transfer, the same effect can be seen in the case of Sherwood number. The effect of the stress relaxation parameter, $\lambda$, on heat and mass transport is shown in Figs. 4, 5. The Nu and Sh increase with an increase in the stress relaxation parameter which indicates that the effect of stress relaxation parameter is to advance the onset of convection in viscoelastic nanofluid-saturated porous media and increases heat and mass transport. Figs. 6, 7 depicts the effects of the strain retardation parameter, $\varepsilon$, Nu and Sh. It is found that with increasing the value of the strain retardation parameter, both Nu and Sh increase, indicating that it delays the onset of convection in viscoelastic nanofluid-saturated porous media and decrease heat and mass transport. The present results are comparable with the results obtained by Malashetty et al. [39–44], Kumar and Bhaduria [45,46] for ordinary viscoelastic fluids.

It was also noted that a negative value of $Rn$ indicates a bottom-heavy case, while a positive value indicates a top-heavy case. Further, the influence of the concentration Rayleigh number $Rn > 0$ on both Nusselt and Sherwood numbers is found to enhance the heat and mass transport as given in Figs. 8 and 9 for top-heavy case, while opposite effect can be seen for bottom-heavy case. The same results were obtained by Agarwal et al. [27,28].

On the contrary

Figure 17  Isotherms (a) $\tau = 0.0$, (b) $\tau = 0.07$, (c) $\tau = 0.14$, (d) $\tau = 0.16$, (e) $\tau = 0.2$, (f) $\tau = 0.4$. 
in the case of concentration Sherwood number both $Le$ and $Na$ have increasing effect, given in Figs. 10 and 11, and so the heat and mass transport. One can see that, in Fig. 9 when $Le$ takes 50 there is an increment in nanoparticle concentration. Also the frequency and magnitude of oscillations increases. The value of $Le$ is taken as 30 for showing the effect of parameters clear in the figures. The reader may note that, the values of $Le$ may consider more than 30 but, for author convenient it is taken as 30. Since $Na$ and $Le$ do not have significant effect on $Nu$, the corresponding figures are not presented to avoid recreation of figures. These results for unmodulated case earlier reported by Bhadauria and Agarwal [30–32],

Figs. 12 and 13 show that, the effect of amplitude of gravity modulation on heat and mass transport is to increase the values of $Nu$ and $Sh$ and hence transport phenomena in both the cases. The comparison is also made for with or without modulation, modulation case is more in heat and mass transport than in unmodulated case, these results conform the results obtained by Bhadauria and Kiran [51]. For un-modulation case one can see the paper of Agarwal et al. [29]. But, for Newtonian fluid saturated porous medium case Srivastava et al. [60] show the quite opposite results for weak nonlinear convection using Ginzburg–Landau model. Also Kiran [53] shows the same for viscoelastic fluid saturated porous media using complex Ginzburg–Landau equation. The reader may

Figure 18  Isohalines at (a) $\tau = 0.0$, (b) $\tau = 0.07$, (c) $\tau = 0.14$, (d) $\tau = 0.16$, (e) $\tau = 0.2$, (f) $\tau = 0.4$. 
note that these opposite results are due to non-Newtonian viscoelastic nanofluid saturated porous media. Similarly in Figs. 14 and 15 show that the effect of frequency of gravity modulation on heat and mass transport is to decrease the values of Nu and Sh, and hence stabilize the system. Thus the classical results are obtained by Gresho and Sani [48]. Also the results correspond to heat and mass transfer for gravity modulation one can see [54–57] for viscoelastic fluids. The effect of heat capacity ratio γ is to decrease the values of Nu and Sh and porosity δ is to increase the values of Nu and Sh these are the results earlier obtained by Bhadauria et al. [32]. In order to avoid more number the figures corresponding to the figures of γ and δ have not presented. It is observed that, in most of the cases there is significant effect of parameters on Nu and Sh at low values of time, but less effect at large time, since vibrations become smaller in magnitude, and disappear as Nu, Sh reach steady state value. The results of gravity modulation on the system preserve the results obtained by Bhadauria and Kiran [59] for rotational speed modulation.

In Figs. 16–18 show the time-dependent fields for different values of ψ, T, φ, at different times. It is clear that with increasing in time, magnitudes of stream lines increases and further in time achieves steady state. But, for the case of isotherms and isohalines as time passes loses their evenness showing the flow of heat and mass transports through conduction to convection. In all Figs. 16–18 for ψ, the sense of motion in the subsequent cells is alternately identical with and opposite to that of the adjoining cell. In case of isotherms, at the starting time, conduction occurs which approaches to convection stage very soon, with the magnitude of isotherms increasing with time. In the intermediate time range, uniform convection cells are observed which change to strong convection with the passage of time. For the isohalines, it is observed that the concentration is more near the walls and less in the middle of the system. The particles remain concentrated toward the walls and enhance the convection toward the walls. The trend observed for steady and unsteady streamlines, isotherms, and isohalines is well in agreement with each other.

6. Conclusions

A nonlinear stability of a horizontal layer of porous medium saturated with viscoelastic nanofluid is investigated, which is heated from below and cooled from above, while incorporates the effects of Brownian motion along with thermophoresis. The results have been obtained in terms of Nusselt and Sherwood numbers with the help of finite amplitude equation. The effect of various parameters have been obtained and depicted graphically. The following conclusion are drawn from the above study.

1. It is found that, the Gravity modulation can be used to regulate the heat and mass transports effectively.
2. The effect of viscoelastic parameters have significant effect on heat and mass transport.
3. Increase in concentration Rayleigh number $R_n$, Modified diffusivity ratio $N_A$ and Lewis number $Le$ increases the effect of gravity modulation.
4. An increment in Prandtl–Darcy number $Pr_d$ is to increase the values of Nu and Sh at small values of time $\tau$ but no effect at large values of time $\tau$.
5. The effect of increased nanoparticle concentration $Rn$ positively (indicates a top-heavy nanoparticle distribution) is to enhance the heat and mass transport, but increased nanoparticle concentration $Rn$ negatively (bottom-heavy nanoparticle distribution) is to reduce the heat and mass transport.
6. There is no significant effect of $N_A$ and $Le$ on heat transport, but on mass transport.
7. Increasing $\epsilon$, is to increase the both Nu and Sh values, whereas an increase in $\Omega$ decreases the same.

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