Formation model of cathode surface structure in contact with plasma flows of high-current low-inductance vacuum spark

S.A. Sarantsev*

National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe shosse 31, 115409, Moscow, Russia

Abstract

This paper presents a model describing formation of a submicron surface structure on electrodes of high-current low-inductance vacuum spark. The model is based on the development of the Kelvin-Helmholtz instability, which occurs at the boundary of tangential discontinuity between plasma and melt. This model has been used to determine the most probable lengths of instability waves, the rate of which conforms to the available sizes of structural elements on electrodes surface.

Keywords: high-current low-inductance vacuum spark; submicron structure; electrode surface; micropinch; Z-pinch; Kelvin-Helmholtz instability.

1. Introduction

High-current low-inductance vacuum spark (HCLIVS) [Kuznetsov et al. (2014), Beilis et al. (1999), Krasov et al. (2007), Bashutin et al. (2013), Astrakhantsev et al. (1995)] offers high plasma parameters \( n_e > 10^{21} \text{ cm}^{-3}, T \sim \text{keV} \), long operational life, and a simple and quite inexpensive design, which makes it a promising source of dense...
impulse high-temperature plasma for technological applications and other purposes. This discharge belongs to Z-pinch discharges utilizing electrode material as working substance.

Researches dealing with vacuum spark barely touch upon the processes occurring on electrode surface, although they significantly influence discharge development. Relation between these processes and the processes in interelectrode space is poorly investigated. Therefore, it is impossible fully describe HCLIVS formation and development from the moment of discharge initiation up to the moment of a micropinch disintegration. The same reason explains the absence of HCLIVS mathematical model and scaling principles, which makes it difficult to use the spark in applications requiring high reproducibility of plasma parameters and radiative characteristics discharge by discharge.

This paper describes formation of periodic submicron structure on HCLIVS cathode. This description is based on hydrodynamic model of the Kelvin-Helmholtz instability development occurring upon contact between flowing plasma and melt surface. Processes on electrodes surface were described using the data obtained at the Pion unit [Sarantsev et al. (2015)] of the Plasma Physics Department of the National Nuclear Research University MEPhI.

2. Formation model of submicron structure on metal surface upon the contact with plasma flows

Upon the contact of plasma flows formed during the development of a micropinch in HCLIVS, the electrode surface undergoes significant heat and impact loads, which transform the surface. The raster electronic microscopy of the HCLIVS cathode showed that the periodic submicron structure with ~150 to ~600 nm cells appeared on the cathode surface [Sarantsev et al. (2015)]. The size of cells depends on conditions of discharge development (charge voltage, discharge current, and trigger configuration).

Fig. 1 shows a photo of the HCLIVS cathode with circled area, where the periodic submicron structure is formed. Fig. 2 showcases a photo of this structure.

Fig. 1. Cathode Surface photo at the Pion unit after 50 Discharges. Black circled is the area, where periodic submicron structure is formed.

Fig. 2. Periodic Structure on Cathode Surface.

Structures formation can be explained as follows. The melt front in course of plasma action extends to the cathode material together with plasma radial wash along the surface, which results in flowing of the melt top layers. At the same time, the tangential discontinuity of Kelvin-Helmholtz instability occurs in a thin transition layer between recently melted and yet static metal and a moving melt. This process is well known in hydrodynamics. Such fluid dynamics occurs, for instance, when a thin liquid layer is exposed to an airflow moving parallel to the free surface. In general, this type of dynamics develops when two liquid layers with different velocity and density move relative to each other. Instability mechanism results from the Bernoulli effect, i.e. when there is an agitation
on a boundary line, for example, melt below the line rises, current flow lines warp and transverse pressure gradients occur at their condensation points boosting the agitation (Fig. 3). Thus, in order to describe formation of a layer with periodic submicron structure, it is essential to determine plasma flow parameters extent, which facilitate unstable waves appearance with length of about a linear size of the structural surface elements.

Let us consider a formation model of periodic structures in the area of plasma contact with cathode surface. The area, where plasma and melt flow parallel at different velocity, appears when plasma flows over the cathode surface. When the agitation occurs on the boundary surface, it grows rapidly, i.e. the Kelvin-Helmholtz hydrodynamic instability develops. Studies [Sarychev et al. (2010), Granovskii et al. (2013)] describing the mathematical model of the Kelvin-Helmholtz instability, which is based on the linear analysis if speaking about parallel flow of two ideal liquids. Researches utilized this math model to explain the occurrence of a nanocrystal layer on the samples surface exposed to electroexplosive alloying.

For simplicity, taking the research model [Sarychev et al. (2010)] we assume that melt is a viscous liquid with density \( \rho_1 \), kinematic viscosity \( \nu \), and slip velocity \( u_1 \). Plasma flowing over is an ideal liquid with density \( \rho_2 \) and slip velocity \( u_2 \). For the boundary line, we assume that stress tangential components between viscous and ideal liquids show a leap due to surface tension \( \sigma_0 \).

The \( x \)-axis shall be directed along the boundary line between layers \( y=\eta(x,t) \) (Fig. 4), and the \( y \)-axis – transverse to \( x \) towards the second layer. The first layer \( (-\infty<x<\infty; -\infty<y<\eta(x,t)) \) contains viscous liquid with kinematic viscosity \( \nu \), density \( \rho_1 \), and constant slip velocity \( u_1 \) in stable state. The second layer \( (-\infty<x<\infty; \eta(x,t)<y<\infty) \) contains ideal liquid with density \( \rho_2 \) and constant slip velocity \( u_2 \) in stable state, which is directed along the \( x \)-axis.

We use linear approximation in equations and boundary conditions. The agitations shall be set as follows: \( U_n \) for axial velocity, \( V_n \) for transverse velocity, \( P_n \) for pressures in the \( n \)-th liquid, which correspond to the Navier-Stokes and Euler differential equations \( (n=1 – \text{viscous liquid (melt)}, n=2 – \text{ideal liquid (plasma)}) \).
In the range of \(-\infty<x<\infty; -\infty<y<\eta(x,t)\):

\[
\frac{\partial U_1}{\partial t} + u_1 \frac{\partial U_1}{\partial x} = -\frac{1}{\rho_1} \frac{\partial P_1}{\partial x} + \sqrt{\left(\frac{\partial^2 U_1}{\partial x^2} + \frac{\partial^2 U_1}{\partial y^2}\right)},
\]

\[
\frac{\partial V_1}{\partial t} + u_1 \frac{\partial V_1}{\partial x} = -\frac{1}{\rho_1} \frac{\partial P_1}{\partial y} + \sqrt{\left(\frac{\partial^2 V_1}{\partial x^2} + \frac{\partial^2 V_1}{\partial y^2}\right)} \frac{\partial U_1}{\partial x} + \frac{\partial V_1}{\partial y} = 0. \tag{1}
\]

In the range of \(-\infty<x<\infty; \eta(x,t)<y<\infty\):

\[
\frac{\partial U_2}{\partial t} + u_1 \frac{\partial U_2}{\partial x} = -\frac{1}{\rho_2} \frac{\partial P_2}{\partial x} + u_2 \frac{\partial V_2}{\partial x} = -\frac{1}{\rho_2} \frac{\partial P_2}{\partial y} + \frac{\partial U_2}{\partial x} + \frac{\partial V_2}{\partial y} = 0. \tag{2}
\]

Boundary conditions for (1) and (2) are as follows:

\[
at \ y \to \pm \infty : U_n, V_n, P_n \to 0; n = 1, 2. \tag{3}
\]

At the boundary line when \(y = 0\) they include kinematic and dynamic conditions:

\[
\frac{\partial \eta}{\partial t} + u_1 \frac{\partial \eta}{\partial x} = V_1, \quad \frac{\partial \eta}{\partial t} + u_2 \frac{\partial \eta}{\partial x} = V_2, \tag{4}
\]

\[
-R_1 + 2\rho_1 V_1 \frac{\partial V_1}{\partial y} + P_2 = \sigma_0 \frac{\partial^2 \eta}{\partial x^2}, \frac{\partial U_1}{\partial y} + \frac{\partial V_1}{\partial x} = 0. \tag{5}
\]

Let us present the solution of (1)-(5) as follows:

\[
U_n (x, y, t) = U_{0n} (y) \exp \left[\omega t - ik x\right], V_n (x, y, t) = V_{0n} (y) \exp \left[\omega t - ik x\right], \tag{6}
\]

\[
P_n (x, y, t) = P_{0n} (y) \exp \left[\omega t - ik x\right], \eta (x, t) = \eta_0 \exp \left[\omega t - ik x\right]. \tag{7}
\]

where \(k\) is the wave number, \(\omega\) is the cyclic frequency, \(U_{0n}, V_{0n}, P_{0n}\) are the peak values of velocities and pressures, respectively, \(\eta_0\) is the maximum deviation of boundary line when \(y=0\).

Let us add (6) to (1)-(5) and after transformations we obtain a boundary problem for transverse velocities:

\[
\frac{d^4 V_1}{dy^4} - \left(k^2 + k_1^2\right) \frac{d^2 V_1}{dy^2} + k^2 k_1^2 V_1 = 0, \frac{d^2 V_2}{dy^2} + k^2 V_2 = 0.
\]

\[
\rho_1 V_1 \frac{d^2 V_1 (0)}{dy^2} - \left(k_1^2 + 2k^2\right) \frac{d V_1 (0)}{dy} + \rho_2 \Omega_2 \frac{d V_2 (0)}{dy} = \frac{\sigma_0 k^4}{\Omega_1} V_1 (0), \tag{8}
\]

\[
\frac{d^2 V_1 (0)}{dy^2} + k^2 V_1 (0) = 0, \frac{V_1 (0)}{\Omega_1} = \frac{V_2 (0)}{\Omega_2},
\]

\[
\frac{d V_1}{dy} = 0, V_1 (-\infty) = 0, V_2 (\infty) = 0.
\]
The following symbols are used:

\[ \Omega_1 = \omega - iku_1; \Omega_2 = \omega - iku_2; k_1^2 = k^2 + \frac{\Omega_1}{v}. \]  

(9)

Under the condition of no agitations at \(-\infty\), it follows that

\[ \text{Re}(k_1) > 0. \]  

(10)

Subject to a nonzero solution of a boundary problem (7), we obtain a characteristic equation:

\[ \rho_1 v^2 \left( k_1^4 + 2k_1^2 k^2 - 4k_1 k^3 + k^4 \right) + \rho_2 v^2 \left( k_1^2 - k^2 + ik(u_2 - u_1)v^{-1} \right)^2 + \sigma_0 k^3 = 0. \]  

(11)

To simplify (9) we use a substitute

\[ z = \frac{k_1}{k} = \sqrt{1 + \frac{\omega - iku_1}{vk^2}}. \]  

(12)

We obtain a quadratic algebraic equation with complex coefficients against \( z \), which is analyzed within a wide range of parametric variations:

\[ (z - 1) \left( z^3 + z^2 + 3z - 1 \right) + \mu \left( z^2 - 1 + i\omega_0 \right)^2 + \omega_0^2 = 0. \]  

(13)

Dimensionless parameters in this equation are

\[ \mu = \frac{\rho_2}{\rho_1}, \omega_0 = \frac{\omega_0}{\omega}, \omega_l = \frac{\omega_l}{\omega}. \]  

(14)

where

\[ \omega_0 = \sqrt{\frac{\sigma_0 k^3}{\rho_1}}, \omega_l = v k^2, \omega_l = (u_2 - u_1) k. \]  

(15)

where \( \omega_0 \) is the capillary wave frequency, \( \omega_l \) is the reverse time of viscous agitations relaxation, and \( \omega_r \) is determined by flow-over plasma spray parameters and melt properties.

To simplify the equations analysis we used a parametric space \((u_0, \lambda)\), the relative slip velocity of layers and wavelength considering the plasma spray impact on the pure iron surface with \( \rho_2 \approx 10 \text{ kg/m}^3 \), for which \( v = 6 \times 10^{-7} \text{ m}^2/\text{s} \), \( \rho_1 = 6.3 \times 10^3 \text{ kg/m}^3 \), and \( \sigma_0 = 1.7 \text{ Nm} \) [Sarychev et al. (2010)]. Using relative velocity \( u_0=10^5 \div 10^6 \text{ cm/s} \) and wavelength of the Kelvin-Helmholtz instability \( \lambda=20 \div 800 \text{ nm} \), we obtained following dependencies (Fig. 5 and Fig. 6).
3. Conclusion

Based on the obtained dependencies for different values of the relative velocities (Fig. 5), we can state that the increment depends on the wavelength on the non-monotonic basis. The maximum value $\alpha_{\text{max}}$ is achieved given the wavelength of the Kelvin-Helmholtz instability $\lambda_{\text{max}}$, which depends on the relative velocity (Fig. 6). The obtained values of the wavelength and the respective relative velocities correspond to the experimental data by the order of magnitude. Thus, the obtained dependency $\alpha(\lambda)$ allows to state that the proposed formation mechanism of the near-surface layer of the contact area between plasma and cathode surface, the formation being based on the development of the Kelvin-Helmholtz instability in the viscous layer of the melt, properly reflects the experimental data.

References


