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Decay constants in soft wall AdS/QCD revisited

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A B S T R A C T

Phenomenological AdS/QCD models, like hard wall and soft wall, provide hadronic mass spectra in reasonable consistency with experimental and (or) lattice results. These simple models are inspired in the AdS/CFT correspondence and assume that gauge/gravity duality holds in a scenario where conformal invariance is broken through the introduction of an energy scale.

Another important property of hadrons: the decay constant, can also be obtained from these models. However, a consistent formulation of an AdS/QCD model that reproduces the observed behavior of decay constants of vector meson excited states is still lacking. In particular: for radially excited states of heavy vector mesons, the experimental data lead to decay constants that decrease with the radial excitation level.

We show here that a modified framework of soft wall AdS/QCD involving an additional dimensionfull parameter, associated with an ultraviolet energy scale, provides decay constants decreasing with radial excitation level. In this version of the soft wall model the two point function of gauge theory operators is calculated at a finite position of the anti-de Sitter space radial coordinate.

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1. Introduction

The discovery of the AdS/CFT correspondence [1–3] – an exact duality between string theory in certain higher dimensional geometries and supersymmetric $SU(N)$ conformal gauge theories with large number of colors – motivated the development of the AdS/QCD phenomenological models. One of the ideas behind the AdS/QCD approach is to assume that there is an approximate duality between a field theory living in an anti-de Sitter background deformed by the introduction of a dimension-full parameter and a gauge theory where the parameter plays the role of an energy scale.

The idea of associating a cut off in anti-de Sitter space with an infrared energy scale in the gauge theory appeared in [4], where the AdS/CFT duality between a scalar field in anti-de Sitter space and a scalar operator (glueball) in a gauge theory was supplemented with the introduction of a cut off in the radial AdS coordinate. The amplitudes for fixed angle scattering calculated in this way show a "hard" behavior – they decrease with a power

of the energy – as it is observed experimentally for hadrons and is obtained in QCD [5,6]. One remarkable outcome of ref. [4], was to show that, contrarily to what happens in flat space, where the amplitudes decrease exponentially with the energy, string theory in anti-de Sitter space is consistent with the physics of strong interactions, concerning the large energy behavior of fixed angle scattering amplitudes.

This idea of placing a geometrical cut off in AdS space was then used in [7,8] as a model for calculating the masses of glueballs. The same approach was then extended to other hadrons and named as hard wall AdS/QCD model (for a recent review and a wide list of related references, see [9]).

The hard wall model fits the available spectrum of radial excitations predicted by lattice for scalar glueballs [7,8] and experimentally observed for light mesons and baryons [10]. However, the asymptotic behavior of the spectrum for the hard wall is that the masses grow linearly with the excitation number n for large n. In contrast, the available experimental data indicate that there is an approximate linear relation between mass squared and radial excitation number.

Another AdS/QCD model, the soft wall, where the square of the mass grow linearly with the radial excitation number was introduced in [11]. In this case, the background involves AdS space and a scalar field that acts effectively as a smooth infrared cut

off. Many interesting improvements of this model appeared then in the literature. In particular, for fermions the soft wall, as formulated in ref. [11], does not lead to a discrete spectrum because the dilaton introduced in the action factorizes out in the equations of motion. This problem can be overcome, as shown in [12–14], considering that the mass of the fermionic field depends on the radial coordinate of AdS space. This change in the model is interpreted as effectively capturing the dependence of the dimension of the gauge theory operator on the energy scale, as it happens in a non conformal theory. Another important improvement was the discovery that using light-front coordinates, the fifth coordinate of AdS space can be interpreted as representing the impact parameter [15,16]. Along the same line, the equivalence between QCD light-front and the equations of both hard and soft wall models was show in ref. [17]. It is interesting to mention also that the soft wall model was extended to heavy quark hadrons in [18].

AdS/QCD models provide also a tool for calculating another important property of hadrons: the decay constant. The decay of mesons, that in contrast to baryons are not subjected to a number conservation condition, is represented as a transition from the initial state to the hadronic vacuum. The amplitude for the decay process is thus calculated from the holographic dual of the transition matrix between the vacuum and the one particle state. For the soft wall model, expressing the two point correlator as a sum over transition matrix elements, one finds the decay constants [11] (for a detailed discussion see [19]). The decay constant f_n of a meson at radial excitation level n is related to the transition matrix between the one particle state and the vacuum. We use in this article the definition: $\langle 0| J_\mu(0) |n\rangle = \epsilon_\mu f_n m_n$.

The results of the soft wall are degenerate: all the decay constants of the radial excitations of a vector meson are equal. For the hard wall model the decay constants of radial excitations are not degenerate but they increase with the excitation level. In contrast, the experimental results available for heavy vector mesons show that higher excited radial states have smaller decay constants. For the excited states of the light vector meson ρ the available experimental data does not provide an estimate for the decay constants.

For the hard wall model the description of the decay process of vector mesons, as done in ref. [20] leads to $f_n \propto 1/J_1(\gamma_n)$ where J_1 is the Bessel function of order one and γ_n are the zeroes of the Bessel function of order zero: $J_0(\gamma_n) = 0$. These decay constants increase with the excitation level since the absolute value of J_1 evaluated at the zeroes γ_n of J_0 is a monotonically decreasing function of n. So, the hard wall model does not reproduce the observed behavior of the decay constants.

In the original soft wall model there is just one dimensionfull parameter, that plays the role of a mass scale. This infrared parameter is introduced in the dilaton background of the model and determines the mass spectrum. The main motivation of this letter is to propose that the description of decay processes in AdS/QCD should involve an additional dimensionfull parameter, or energy scale. In the usual AdS/QCD approach one describes the hadronic decays by means of field correlators calculated at the AdS boundary. Our proposal is to find the decay constants from the two point correlator calculated at some finite value $z = z_0$ of the radial coordinate of AdS space. This corresponds to introducing a new energy scale: $1/z_0$ in the model.

A consistent way of calculating field correlators of the gauge theory operators at a finite position $z = z_0$ of the radial AdS coordinate was presented in refs. [21,22]. One interesting thing about this procedure is that the introduction of a second energy scale improves the description of the mass spectra of vector mesons made of heavy quarks.

We specialize our discussion of the soft wall to the case of interest: vector mesons. The soft wall action for a vector field

$V_m = (V_\mu, V_z)$ ($\mu = 0, 1, 2, 3$), assumed to be dual to the gauge theory current $J^\mu = \bar{q}\gamma^\mu q$ reads:

$$I = \int d^4x dz \sqrt{-g}\, e^{-\Phi(z)} \left\{ -\frac{1}{4g_5^2} F_{mn} F^{mn} \right\}, \tag{1}$$

where $F_{mn} = \partial_m V_n - \partial_n V_m$ and $\Phi = k^2 z^2$ is the soft wall dilaton background, that plays the role of a smooth infrared cut off and k is a constant representing the mass scale.

The background geometry is anti-de Sitter AdS_5 space, with the metric:

$$ds^2 = e^{2A(z)}(-dt^2 + d\vec{x} \cdot d\vec{x} + dz^2), \tag{2}$$

where $A(z) = -log(z/R)$ and the Poincaré chart of AdS corresponds to $(t, \vec{x}) \in \mathcal{R}^{1,3}$, $z \in (0, \infty)$ and z is called radial coordinate.

We use the gauge $V_z = 0$. The boundary value of the remaining components of the vector field: $V_\mu^0(x) = \lim_{z\to 0} V_\mu(x, z)$, $\mu = 0, 1, 2, 3$ are assumed to be, as in the AdS/CFT correspondence, the sources of the correlation functions of the boundary current operator $J^\mu(x)$ ($= \bar{q}\gamma^\mu q(x)$). Namely:

$$\langle 0| J_\mu(x) J_\nu(y) |0\rangle = \frac{\delta}{\delta V^{0\mu}(x)} \frac{\delta}{\delta V^{0\nu}(y)} \exp\left(-I_{on\,shell}\right), \tag{3}$$

where the on shell action is given by the boundary term:

$$I_{on\,shell} = -\frac{1}{2\tilde{g}_5^2} \int d^4x \left[\frac{e^{-k^2 z^2}}{z} V_\mu \partial_z V^\mu \right]_{z\to 0}. \tag{4}$$

We have introduced $\tilde{g}_5^2 = g_5^2/R$ that is the relevant dimensionless coupling of the bulk vector field. For comparison with the proposal that we will present in the next section, let us emphasize that this action is calculated at the AdS boundary $z = 0$. One can then write the on shell action in momentum space and decompose the field as

$$V_\mu(p, z) = v(p, z) V_\mu^0(p), \tag{5}$$

where $v(p, z)$ is the bulk to boundary propagator that satisfies the equation of motion:

$$\partial_z \left(\frac{e^{-k^2 z^2}}{z} \partial_z v(p, z) \right) + \frac{p^2}{z} e^{-k^2 z^2} v(p, z) = 0. \tag{6}$$

In order that the factor $V_\mu^0(p)$, defined in the decomposition of eq. (5), works as the source of the correlators of gauge theory currents, one must impose the boundary condition: $v(p, z = 0) = 1$. On the other hand, the two point function in momentum space is related to the current–current correlator by

$$\left(p^2 \eta_{\mu\nu} - p_\mu p_\nu \right) \Pi(p^2) = \int d^4x\, e^{-ip\cdot x} \langle 0| J_\mu(x) J_\nu(0) |0\rangle. \tag{7}$$

Thus the two point function is holographically expressed in terms of the bulk to boundary propagator as:

$$\Pi(p^2) = \frac{1}{\tilde{g}_5^2 (-p^2)} \left[\frac{e^{-k^2 z^2} v(p, z) \partial_z v(p, z)}{z} \right]_{z\to 0}. \tag{8}$$

The two point function has the following spectral decomposition, in terms of masses m_n and decay constants f_n of the states of the gauge theory:

$$\Pi(p^2) = \sum_{n=1}^\infty \frac{f_n^2}{(-p^2) - m_n^2 + i\epsilon}. \tag{9}$$

In the soft wall model the states are described by the normalizable solutions $\phi_n(z)$ of the equation of motion (6). One can represent

the bulk to boundary propagator $v(p, z)$ in terms of the normalizable solutions by introducing the Green's function

$$G(p, z, z') = \sum_{n=1}^{\infty} \frac{\phi_n(z)\phi_n(z')}{(-p^2) - m_n^2}, \tag{10}$$

and writing:

$$v(p, z) = -\left[\frac{e^{-k^2 z'^2} \partial_{z'} G(p, z', z)}{z'}\right]_{z' \to 0}. \tag{11}$$

The normalized solutions have the form: $\phi_n(z) = \sqrt{2(n-1)!/n!} \times (kz)^2 L_{n-1}^1(k^2 z^2)$, where $L_n^1(x)$ are the generalized Laguerre polynomials of order one. They satisfy the boundary condition $\lim_{z \to 0} \phi_n(z) = 0$ and correspond to solutions of eq. (6) with momenta: $-p_n^2 = m_n^2 = 4k^2 n$, $n = 1, 2, \dots$.

The bulk to boundary propagator defined in eq. (11) satisfies the equation of motion (6) and also the boundary condition $v(p, 0) = 1$. It is important to discuss the subtle way in which this boundary condition is satisfied. If one simply takes the definition for the bulk to boundary propagator from the Green's function one finds an infinite sum of terms with the propagator poles $1/(-p^2 - m^2)$ that would in principle lead to a dependence of $v(p, 0)$ on the momentum p. However, the boundary condition $\lim_{z \to 0} \phi_n(z) = 0$ leads to a sum of singular terms with vanishing coefficients, that needs to be regularized. Such a regularization is presented in ref. [19], where an integral representation for the bulk to boundary propagator is found. This integral representation is well defined in the interval $0 \le z$. It is explicitly equal to one in $z = 0$ and coincides with the expression (11) when $z > 0$.

The two point function obtained holographically in eq. (8) has the same pole structure of eq. (9), with the residues corresponding to the decay constants. Thus:

$$f_n = \left[\frac{e^{-k^2 z^2} \partial_z \phi_n(z)}{m_n \tilde{g}_5 z}\right]_{z \to 0} = \frac{k\sqrt{2}}{\tilde{g}_5}. \tag{12}$$

This result shows that in the soft wall model all the radial excitations of a vector meson have the same decay constant. This degeneracy contrasts with the decrease with excitation level n in the decay constants obtained from experimental data for heavy vector mesons.

This problem was discussed in reference [23], where a finite temperature AdS/QCD model for quarkonium in a plasma was discussed. In order to describe the finite temperature behavior, the authors proposed a procedure to fit the zero temperature parameters. This was done by adding extra terms in the equation of motion for the normalizable modes, in such a way that one arrives at a model with four parameters that are fixed by the four quantities available: the masses and decay constants of J/ψ and ψ'.

2. Alternative description of the decay process

The soft wall model, as reviewed in the previous section, contains one dimensionfull parameter: k, introduced in the dilaton background. This parameter plays the role of an infrared (or mass) scale of the model and determines the spectrum of the vector mesons: $m_n^2 = 4k^2 n$ as well as the decay constants. The linear relation between the square of the mass and the excitation level is the expected asymptotic behavior. However, the observed behavior of decay constants of vector mesons is that they decrease with the excitation level while the soft wall model predicts degenerate values.

The relation between the vacuum expectation value of the product of two gauge currents and the decay constants

$$\int d^4 x \, e^{-ip \cdot x} \langle 0| J_\mu(x) J_\nu(0) |0\rangle$$

$$= \left(p^2 \eta_{\mu\nu} - p_\mu p_\nu\right) \sum_{n=1}^{\infty} \frac{f_n^2}{(-p^2) - m_n^2 + i\epsilon}, \tag{13}$$

comes from introducing on the lefthand side of this equation a basis of the states of the theory. The decay constants are related to the transition matrices between the one particle states and the vacuum as: $\langle 0| J_\mu(0) |n\rangle = \epsilon_\mu f_n m_n$. Our point of view is that, in order to incorporate in the model the effect of the interactions that govern the transition processes from the one particle state to the vacuum, corresponding to the hadron decay, one needs to introduce one additional dimensionfull parameter in the soft wall. This can be done by calculating holographically the operator product of currents of eq. (13) at a finite location $z = z_0$ of the radial coordinate. The new parameter, namely $1/z_0$, corresponds to an energy scale where the transition matrices are calculated.

The idea of introducing an ultraviolet cutoff in the soft wall model was previously considered in ref. [24]. In this reference normalized solutions $\tilde{\phi}_n$ satisfying boundary condition at a position corresponding, in our notation, to a minimum value of the z coordinate $z = 1/\Lambda$ were studied numerically. The condition imposed on the solutions was $\lim_{z \to 1/\Lambda} \tilde{\phi}_n(z) \sim z^2$; $\lim_{z \to 1/\Lambda} \tilde{\phi}_n'(z) \sim 2z$. The spectrum of masses obtained for the ρ meson from this approach showed a better fit to the experimental data, in comparison with the original soft wall model.

In this reference [24] the decay constants were also discussed. It was assumed that with the introduction of the UV cutoff they should be given (using our choice of radial coordinate and our convention for decay constants) by:

$$f_n = \left[\frac{\partial_z \tilde{\phi}_n(z)}{m_n \, g_5 \, z}\right]_{z \to 1/\Lambda}. \tag{14}$$

Using this relation, a decrease in the decay constants with the excitation level was found. However there is a problem with expression (14). The boundary value of the bulk field plays the role of the source of the gauge current correlators. When one takes the boundary to be at some finite value of the coordinate $z = 1/\Lambda$ and decompose the field, as in eq. (5), $V_\mu(p, z) = v(p, z) V_\mu^0(p)$, the source is $V_\mu^0(p) = \lim_{z \to 1/\Lambda} V_\mu(p, z)$ and the bulk to boundary propagator has to satisfy the condition: $\lim_{z \to 1/\Lambda} v(p, z) = 1$. In order to represent $v(p, z)$ in terms of the normalized solutions $\tilde{\phi}_n$ in the same way as in equations (10) and (11) it is necessary that the condition $\lim_{z \to 1/\Lambda} \tilde{\phi}_n(z) = 0$, holds for the normalized solutions. This way, one finds for the bulk to boundary propagator at $z = 1/\Lambda$ a sum of singular terms with vanishing coefficients that can be regularized to unity, as discussed in section 2.

For a non vanishing boundary condition, like the one used in ref. [24], the bulk to boundary propagator will be a sum of propagator pole terms with finite coefficients, keeping a momentum dependence. So, the condition: $v(p, 1/\Lambda) = 1$ is not satisfied and $V_\mu^0(p)$ is not the source of the correlators. Thus, in this case, one can not assume the expression (14) to represent the decay constants.

What comes out as a consistent approach to describe the decay constants by calculating the field correlators at a finite position $z = z_0$ of the radial coordinate is to follow a procedure presented in refs. [21,22]. In these references a cut off in the radial AdS coordinate was introduced so as to play the role of an ultraviolet cut off in the gauge theory. We will relate the same sort of cut off with

the decay process. The bulk to boundary propagator is written as a solution of the equation of motion (6) but now excluding the region $0 < z < z_0$. The solution is divided by a constant (in the z coordinate) so as to satisfy by construction the boundary condition $v(p, z_0) = 1$. Namely:

$$v(p, z) = \frac{U(p^2/4k^2, 0, k^2 z^2)}{U(p^2/4k^2, 0, k^2 z_0^2)}, \tag{15}$$

where $U(a, b, c)$ is the Tricomi function. Using the new on shell action given by:

$$I_{on\,shell} = -\frac{1}{2\tilde{g}_5^2} \int d^4x \, \frac{e^{-k^2 z^2}}{z} V_\mu \partial_z V^\mu \Big|_{z \to z_0}, \tag{16}$$

we calculate the two point function:

$$\Pi(p^2) = \frac{1}{\tilde{g}_5^2 (-p^2)} \frac{e^{-k^2 z^2} v(p, z) \partial_z v(p, z)}{z} \Big|_{z \to z_0}. \tag{17}$$

Then, using the new bulk to boundary propagator of eq. (15) we get:

$$\Pi(p^2) = \frac{1}{2\tilde{g}_5^2} \frac{e^{-k^2 z_0^2} U(1 + p^2/4k^2, 1, k^2 z_0^2)}{U(p^2/4k^2, 0, k^2 z_0^2)}, \tag{18}$$

where a recursion relation for the Tricomi functions has been used.

The spectrum of masses and decay constants comes from the analysis of the poles in the momentum variable p^2. The Tricomi function in the numerator is non singular in p^2, so the two point function of eq. (15) is only singular at the zeros of the denominator. Furthermore, the poles p_n^2 of the function $\Pi(p^2)$ are simple. So, in a neighborhood of $p^2 = p_n^2$ one can approximate the two point function by:

$$\lim_{p^2 \to p_n^2} \Pi(p^2) \approx \frac{f_n^2}{(-p^2) + p_n^2}. \tag{19}$$

We associate the coefficients of the approximate expansion near the pole with the decay constant f_n in analogy with the exact expansion shown in eq. (9). This way we get the masses from the localization of the poles of the two point function and the decay constants from the corresponding coefficient. More precisely, let χ_n be the roots of the Tricomi function:

$$U(\chi_n, 0, k^2 z_0^2) = 0, \tag{20}$$

then the holographic vector meson masses are: $m_n^2 = 4k^2 \chi_n$. The decay constants are calculated numerically from the fit to the approximate form presented in equation (19). That means:

$$f_n^2 = \lim_{p^2 \to p_n^2} \left((-p^2) + p_n^2 \right) \Pi(p^2). \tag{21}$$

The coupling $\tilde{g}_5 = g_5/\sqrt{R}$ of the vector field in the AdS bulk can be obtained by comparison with QCD, as explained in references [11,19], which gives: $\tilde{g}_5 = 2\pi$.

3. Confronting the model with the available data

The experimental values for the masses m_V and the electron positron decay widths $\Gamma_{V \to e^+ e^-}$ for vector mesons are taken from ref. [25]. The decay constant of a vector meson state is related to the corresponding mass and width by [26]:

$$f_V^2 = \frac{3 m_V \Gamma_{V \to e^+ e^-}}{4\pi \alpha^2 c_V}, \tag{22}$$

Table 1
Experimental masses and electron–positron widths from [25] and the corresponding decay constants for the charmonium S-wave resonances.

	Charmonium data		
	Masses (MeV)	$\Gamma_{V \to e^+ e^-}$ (keV)	Decay constants (MeV)
$1S$	3096.916 ± 0.011	5.547 ± 0.14	416.2 ± 5.3
$2S$	3686.109 ± 0.012	2.359 ± 0.04	296.1 ± 2.5
$3S$	4039 ± 1	0.86 ± 0.07	187.1 ± 7.6
$4S$	4421 ± 4	0.58 ± 0.07	160.8 ± 9.7

Table 2
Experimental masses and electron–positron widths from [25] and the corresponding decay constants for the bottomonium S-wave resonances.

	Bottomonium data		
	Masses (MeV)	$\Gamma_{V \to e^+ e^-}$ (keV)	Decay constants (MeV)
$1S$	9460.3 ± 0.26	1.340 ± 0.018	715.0 ± 2.4
$2S$	10023.26 ± 0.32	0.612 ± 0.011	497.4 ± 2.2
$3S$	10355.2 ± 0.5	0.443 ± 0.008	430.1 ± 1.9
$4S$	10579.4 ± 1.2	0.272 ± 0.029	340.7 ± 9.1

where $\alpha = 1/137$ and c_V for the two families that we consider are $c_{J/\Psi} = 4/9$ and $c_\Upsilon = 1/9$. The uncertainties in the decay constants are:

$$\delta f_V = \frac{3}{8\pi \alpha^2 c_V f_V} (m_V \delta \Gamma_{V \to e^+ e^-} + \Gamma_{V \to e^+ e^-} \delta m_V). \tag{23}$$

In Table 1 we show the experimental values of the masses and widths and the associated decay constants with the corresponding uncertainties for the charmonium vector meson J/ψ, made of a charmed quark anti-quark pair and for the first three radially excited S-wave resonances. This table shows that the decay constants decrease monotonically with the radial excitation level.

Then, in Table 2 we show the experimental data and the associated decay constants with the corresponding uncertainties for the bottomonium vector meson Υ, made of a bottom quark anti-quark pair and for the first three radially excited S-wave resonances. As it happens for the charmonium, the decay constants decrease monotonically with the radial excitation level.

Now we present the results for the masses, obtained from equations (20) and the decay constants obtained from equation (21) for charmonium and bottomonium S-wave states. A nice fit to the experimental data is obtained using the parameters

$$k_c = 1.2 \text{ GeV}; \ k_b = 3.4 \text{ GeV}; \ 1/z_0 = 12.5 \text{ GeV}, \tag{24}$$

where k_c and k_b are the values of the constants k used for charmonium and bottomonium, respectively. The values are different, reflecting the difference in the masses of charm and bottom quarks. The energy scale $1/z_0$ is taken to be flavor independent and represents a characteristic energy of the decay process. In the non hadronic decay a very heavy vector meson annihilates into light leptons. So, the ultraviolet scale is of order of the heavier masses that we consider. We show in Tables 3 and 4 the results obtained for the charmonium and bottomonium S-wave states, respectively. One can see that the model produces a decrease of the decay constants with the radial excitation level, as it happens with the available data for these particles.

One interesting way to characterize the model in terms of predictability is to define the rms error for estimating N quantities using a model with N_p parameters as:

$$\delta_{rms} = \sqrt{\frac{1}{(N - N_p)} \sum_i^N \left(\frac{\delta O_i}{O_i} \right)^2} \tag{25}$$

where O_i is the average experimental value and δO_i is the deviation of the value provided by the model. Considering all the

Table 3
Masses and decay constants in MeV obtained for the charmonium states.

	Results for charmonium states	
	Masses	Decay constants
1S	2410	258.8
2S	3409	251.7
3S	4174	245.9
4S	4819	241.0

Table 4
Masses and decay constants obtained for the bottomonium S-wave resonances.

	Results for bottomonium states	
	Masses	Decay constants
1S	7011	627
2S	9883	574
3S	12077	538
4S	13923	512

16 results of Tables 3 and 4, obtained using three free parameters: k_b, k_c and $1/z_0$ we find $\delta_{rms} = 30\%$. It is important to remark that we are fitting two different properties, the mass and the decay constant, of four resonances of two different hadrons using only three parameters in the model. So, the error of 30% is reasonable, considering the simplicity of the model and the complexity of the strong coupling physics involved in the heavy meson structure.

4. Final comments

The experimental data for the first four S-wave resonances of charmonium and bottomonium lead to decay constants that exhibit a clear decrease with the radial excitation level. In this article we proposed a consistent AdS/QCD approach where the decay constants are calculated from two point operator products at a finite value of the radial coordinate of anti-de Sitter space. The model was formulated with three independent parameters and was applied to the calculation of sixteen independent observables: eight masses and eight decay constants.

A consistent AdS/QCD method for finding decay constants that decrease with radial excitation was lacking in the literature. This property is reproduced in our model, in contrast to the original soft wall, providing a more realistic description of heavy vector mesons.

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