



# Weak gravity conjecture with large extra dimensions

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## Abstract

In the presence of large extra dimensions, the fundamental Planck scale can be much lower than the apparent four-dimensional Planck scale. In this setup, the weak gravity conjecture implies a much more stringent constraint on the UV cutoff for the U(1) gauge theory in four dimensions. This new energy scale may be relevant to LHC.

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Extra dimensions are naturally required in some fundamental theory. For example, the self-consistency condition for superstring theory requires that the critical dimension should be ten [1]; otherwise, the conformal anomaly on the world sheet cannot be canceled exactly. A new higher-dimensional mechanism for solving the hierarchy problem was proposed in [2]. Compactification from higher dimensions to four dimensions not only is necessary for connecting string theory with our real world, but also gives us many surprising insights.

In [3], Vafa pointed out that gravity and the other gauge forces cannot be treated independently and the vast series of semi-classically consistent effective field theories are actually inconsistent after gravity is included. Furthermore, the authors of [4] proposed the weak gravity conjecture which can be most simply stated as gravity is the weakest force. This conjecture implies that in a four-dimensional theory with gravity and a U(1) gauge theory, there is a new intrinsic UV cutoff

$$\Lambda \sim g M_4 \quad (1)$$

induced by four-dimensional gravity characterized by the four-dimensional Planck scale  $M_4$ , where  $g$  is the gauge coupling. Some related topics are discussed in [5–14].

In Standard Model, we have the perturbative U(1) gauge coupling at a very high energy scale and the weak gravity conjecture predicts a new intrinsic scale  $\Lambda \sim \sqrt{\alpha/G_4} \sim 10^{17}$  GeV which is much higher than the energy scale in colliders. If there

are new extremely weak gauge forces with coupling  $g \sim 10^{-15}$ , a new scale for gauge theory at TeV scale appears. However it is very difficult for us to detect such extremely weakly coupled gauge theory. It seems that the weak gravity conjecture is irrelevant to experiments.

On the other hand, the most exciting possibility raised by large extra dimensions [15–20] is that the fundamental Planck scale may be much lower than the apparent four-dimensional Planck scale. This implies that we may begin to experimentally access the dynamics of quantum gravity sooner than previously anticipated.

In this short Letter, we propose that a new intrinsic UV cutoff for U(1) gauge theory with large extra dimensions is proportional to the fundamental Planck scale, not the four-dimensional Planck scale. This new energy scale may be relevant for the physics at the LHC.

We compactify a  $d$ -dimensional theory to four dimensions, and the four-dimensional Planck scale is determined by the fundamental Planck scale  $M_d$  in  $d$  dimensions and the geometry of the extra dimensions,

$$M_4^2 = M_d^{n+2} R^n, \quad (2)$$

where  $n = d - 4$  and  $R$  is the average size of the extra dimensions. If  $M_d \sim 1$  TeV,  $R \sim 10^{13}$  cm for  $n = 1$  which is excluded,  $R \sim 1$  mm for  $n = 2$  which is roughly the distance where our present experimental measurement of gravitational strength forces stops, and  $R \sim 10^{-12}$  cm for  $n = 6$ . The gravitational interaction is unchanged over distances larger than

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the size of the extra dimensions  $R$  with the behavior of Newtonian potential  $1/r$ ; however, the Newtonian potential behaves as  $1/r^{n+1}$  for  $r \ll R$ . In this scenario, the Standard Model particles are always localized on a 3-brane embedded in the higher-dimensional space. A review is given in [17].

The mass of a lightest black hole is larger than the fundamental Planck scale; otherwise, a full quantum theory of gravity, such as string theory, is needed. Since the fundamental Planck scale can be much lower than the four-dimensional Planck scale, black holes not too much heavier than the fundamental Planck scale may be produced at lower energy scale [21]. Following the idea in [4], we take the monopole mass as a probe of the UV cutoff of a U(1) gauge theory in four dimensions. This U(1) gauge theory arise in the Higgsing of an SU(2). The order of mass of monopole is roughly

$$M_{\text{mon}} \sim \frac{\Lambda}{g^2}, \quad (3)$$

if the theory has a cutoff  $\Lambda$ , and the size of monopole is given by

$$L_{\text{mon}} \sim \frac{1}{\Lambda}. \quad (4)$$

For electro-weak scale  $10^2$  GeV, the size of monopole is roughly  $10^{-16}$  cm which is much smaller than the size of the extra dimensions if  $M_d \sim 1$  TeV. Here we want to stress that the scale  $1/R$  is the scale for the KK modes of the graviton, which means that we should take the  $(n+4)$ -dimensional gravity into account above the scale  $1/R$ . But this scale is not a scale for the gauge theories on a 3-brane. We can easily check that the electro-weak scale is higher than  $1/R$ . That is why we consider  $(n+4)$ -dimensional gravity, not four-dimensional gravity. The gravity for monopole propagates in  $d$ -dimensional space–time. The size of black hole  $R_{\text{bh}}$  in  $d$  dimensions with mass  $M$  takes the form, [21],

$$R_{\text{bh}}^{n+1}(M) \sim M_d^{-(n+2)} M. \quad (5)$$

Requiring that the monopole does not collapse to a  $d$ -dimensional black hole, or equivalently  $L_{\text{mon}} > R_{\text{bh}}(M_{\text{mon}})$ , yields

$$\Lambda \leq g^{\frac{2}{n+2}} M_d \sim \left( \frac{g^2}{G_d} \right)^{\frac{1}{n+2}}, \quad (6)$$

where  $G_d \sim M_d^{-(n+2)}$  is the Newton coupling constant in  $d$  dimensions. Otherwise black hole is contained and this U(1) gauge theory is not an effective field theory. The new intrinsic UV cutoff is proportional to the fundamental Planck scale, not the four-dimensional Planck scale. Our result is different from that in higher-dimensional space–time. In arbitrary dimensions, the new UV cutoff predicted by weak gravity conjecture is proportional to  $g$  (see [13]), not  $g^{2/(n+2)}$ . Using Eq. (2), we obtain

$$\Lambda \leq \frac{g^{\frac{2}{n+2}}}{(M_4 R)^{\frac{n}{n+2}}} M_4. \quad (7)$$

For  $n = 0$  which means there is no extra dimension, Eqs. (6) and (7) are just the same as (1). In our Letter, we only pay

attention to the case with  $n > 0$ . If the size of the extra dimension is much larger than four-dimensional Planck length, i.e.  $R \gg M_4^{-1}$ , the constraint on the UV cutoff in (6) or (7) is much more stringent than that in (1).

The above argument is consistent with the requirement that the gravity should be the weakest force. We take the charged particle with mass  $m$  into account. The interaction between the charged particles is described by a four-dimensional U(1) gauge theory. The repulsive gauge force is roughly  $\frac{g^2}{r^2}$ , where  $r$  is the distance between them. If  $r$  is much smaller than the size of extra dimensions, the gravitational force between them is  $\frac{G_d m^2}{r^{n+2}}$ .

Weak gravity conjecture says  $\frac{g^2}{r^2} \geq \frac{G_d m^2}{r^{n+2}}$ , or

$$m^2 \leq \frac{g^2 r^n}{G_d}. \quad (8)$$

This condition is easy to be satisfied if the charged particles are separated far away from each other. On the other hand, the Compton wavelength can be thought of as a fundamental limitation on measuring the position of a particle, taking quantum mechanics and special relativity into account. The reasonable distance between two charged particles should be greater than the Compton wavelength of the charged particle  $m^{-1}$ . Thus the most stringent constraint occurs when  $r \sim m^{-1}$ . Now Eq. (8) becomes

$$m \leq \left( \frac{g^2}{G_d} \right)^{\frac{1}{n+2}}. \quad (9)$$

Since the mass of the charged particle is naively proportional to the UV cutoff  $\Lambda$ , Eq. (9) is just the same as (6). So weak gravity conjecture with large extra dimensions is nothing but the condition for that gravity can be ignored.

According to Eq. (6), a new intrinsic UV cutoff for U(1) gauge theory is roughly given by

$$\Lambda \sim g^{2/(n+2)} M_d, \quad (10)$$

or a lower bound on the fundamental Planck scale takes the form

$$M_d \geq g^{-2/(n+2)} \Lambda. \quad (11)$$

At electro-weak energy scale  $\Lambda_{\text{ew}} \sim 10^2$  GeV, U(1) coupling is roughly  $g^2 \sim 10^{-2}$ . Since we did not find any UV cutoff for U(1) gauge theory under the electro-weak scale, the fundamental Planck scale should be higher than  $3 \times 10^2$  GeV for  $n = 2$ , or  $2 \times 10^2$  GeV for  $n = 6$ . If the fundamental Planck scale is 10 TeV, weak gravity conjecture predicts that a new intrinsic scale for U(1) gauge theory is roughly 3 TeV for  $n = 2$ , or 6 TeV for  $n = 6$ . LHC with a center of mass energy of 14 TeV offers an opportunity to approach the dynamics of quantum gravity and check weak gravity conjecture.

In this Letter, a much more stringent constraint on the effective low-energy theories containing gravity and U(1) gauge fields is obtained with large extra dimensions. Weak gravity conjecture induces a new intrinsic UV cutoff for U(1) gauge theory in four dimensions which is lower than the fundamental Planck scale. Compactification with large extra dimensions offers a complete natural understanding of the hierarchy in

Standard Model if the fundamental Planck scale is not much higher than  $1 \sim 10$  TeV. If so, not only the black holes are possibly produced, but also a new intrinsic UV cutoff lower than 10 TeV for U(1) gauge theory in Standard Model emerges. In Standard Model,  $U(1)_{\text{em}}$  breaks down above the electro-weak scale. There is an opportunity to test weak gravity conjecture for  $U(1)_Y$  at the LHC. We hope LHC bring us to some surprising facts in the near future.

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