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From generic towards a micromechanical fatigue model

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Abstract

Fatigue life formulas are still based on phenomenological models which adopt simple relations directly from experiments for different loading conditions and use fitted material parameters. The combination of enormous complexity of fatigue damage processes and simple, macro appearance of the formulas (usually power laws), are the source of Generic Fatigue Models (GFM). GFMs rely on minimal, but coherent, micro-details which are independent of the specific micro structure. Such a model has been developed, connecting analytically the S-N power law and endurance stress in terms of statistical strength distributions of material microelements and their neighbors.

This paper describes two types of generalizations of the basic GFM: a.) Two level (H-L and L-H) loading, in which a history dependent micro-damage evolution law is proposed, and b.) Multiaxial fatigue response by a simple 2D truss. Emphasize is on minimal parameters and capability of analytical predictions, in which every "material constant" has a physical or micro-geometrical meaning. The theoretical generalizations are compared with experimental data from the literature and show that the predictions are coherent with main experimental features.

Keywords: micromechanics; fatigue; heterogeneity; multiaxial

1. Generic Fatigue Model (GFM) – reasons, achievements and limitations

High cycle fatigue models applied in engineering design are mostly phenomenological, in which a relation between loading and fatigue life is identified directly from experiments. The classical Basquin Law (S-N), Paris Law (Crack growth rate), Goodman diagram (average load effect) and Miner types rule (multilevel load) are few well known examples:

$$\sigma = C_f \mathrm{N}_f^b \quad ; \quad a_{,N} = C_a \left(\Delta K\right)^{\alpha} \quad ; \quad \left(\frac{n_1}{N_1}\right)^{C_1} + \left(\frac{n_2}{N_2}\right)^{C_2} = 1 \tag{1}$$

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There are many others for effects like average stress, random loading, multiaxial fatigue etc. Such models are very useful but have limited physical insight and predictive capabilities. The collection of formulas includes a large number of "seemingly unrelated" material parameters (six are seen in (1)). On the other hand, direct "physical" models, based on surface extrusion-intrusion, or on dislocation pileups and vacancies (Mura-Tanaka [1]), are still short of "skipping the scales". Thus, analytical relations between the measured physical parameters and the constants in (1) are in demand. Continuum type models, combining macro plasticity theories and damage functions are also common [2-4]. These models have even more parameters; their prediction and insight are limited due to the lack of microstructure information. Recent studies simulate continuum, stochastic morphology and damage [5,6].

The fact that relations like in (1) are universal (Altus, 1991 [7], Kun et al, 2008 [8]), i.e., fit a variety of materials which are so different in their microstructures, calls for a more unified approach, in which only the generic material properties and mechanisms are implemented, while the specific details are ignored. Generic "rules" which constitute the building blocks for GFM are: a.) local heterogeneity, b.) a sufficiently general probabilistic damage or strength distribution, c.) irreversible processes during unloading, and d.) local interactions between broken/damaged micro elements and their neighbors.

The unique features of the GFM are by connecting analytically between the above generic micro failure mechanisms and Basquin Law (1a), proposing a relation between C_f and b and a physical interpretation of the endurance limit stress in terms of neighbor microelement interaction probability. A brief description and some characteristic results are shown in fig.1.

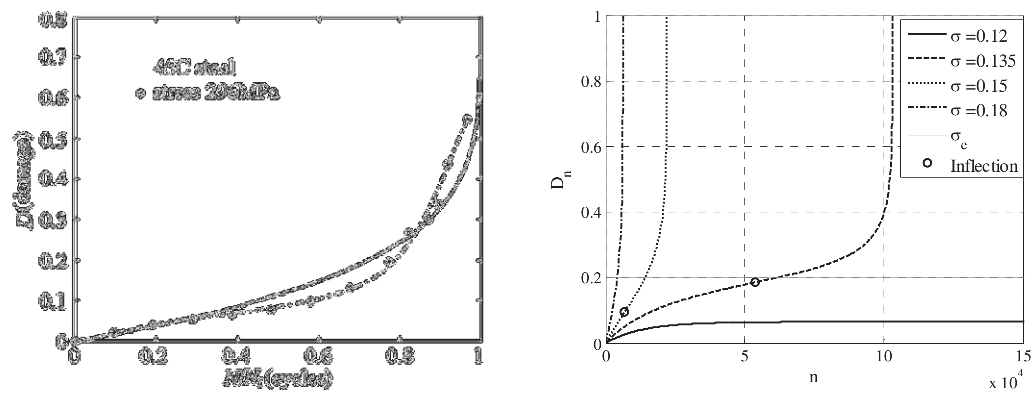

Fig.1., Damage accumulation during fatigue. Left: measurements by Electrical resistance increase for 45C Steel (Sun [9]) with a fitting curve function $D=1-(1-N/Nf)^{\beta}$ and a more accurate one (dashed line), revealing the three stage evolution. Right: GFM prediction for 4 levels of fatigue stresses (Altus, 2002 [10]).

The universality of the GFM is also the source of its limitation. By not specifying the microelements and their interactions in terms of direct measurable entities, the ability to independently validate the model by experiments decreases. Nevertheless, important indirect pieces of "evidence" can be collected as seen in Fig.1.

Another fundamental (and rare) validation can be achieved if a relation between some of the "seemingly independent" parameters in (1) can be predicted and verified experimentally. An example is the GFM prediction of a two level loading protocol [7]:

$$\left(\frac{n_H}{N_H}\right)+\frac{1}{\lambda}\cdot\left(\frac{n_L}{N_L}\right)=1 \tag{2}$$

Where λ depends on the loading order:

$$\begin{cases} H\to L: & \dfrac{1}{\lambda}=\left[1-\ln N_{H/L}\right]N_{H/L} \\ L\to H: & \lambda=1 \end{cases} \quad ; \qquad N_{H/L}\equiv N_H/N_L\,. \tag{3}$$

This result can be considered "universal", since no additional material parameter is involved. The current study generalizes the above in two ways: a.) improving the two level predictions by using a history dependent damage law, and b.) analyzing a simple 2D model to account for multiaxial fatigue response.

2. Implementing image analysis tools into the GFM

Here we follow some of the mathematical derivations which incorporate image analysis and set theory tools in order to follow the details of damage evolution. This more advanced analysis confirms the basic results of the original GFM on the macroscale, includes microcrack coalescence effects which were ignored in previous studies and enables to follow the microcrack size distribution at every step during fatigue life, keeping the analytical developments and simplicity of the macro results unchanged.

Consider the material as an ensemble of parallel, one dimensional elements, each with a stochastic, independent failure strain. All sets are denoted by large bold letters and their measures with regular ones. Let the set of parallel elements have a statistical cumulative distribution of strain to failure F(ε). Let $\mathbf{F}(\varepsilon)$ be the set composed of these elements. In the first cycle, damage is composed of the set $\mathbf{F}(\varepsilon_1)$,

$$\mathbf{D}_1=\mathbf{F}\left(\varepsilon_1\right)\equiv\mathbf{F}_1\,. \tag{4}$$

Assume that each micro-crack grows one element to the right at each cycle. The damage growth of micro-cracks after the first cycle is the union of $\mathbf{D}_1$ with its translation as one element to the right. In the second cycle, micro-cracks initiation is created by the union of the former with the set $\mathbf{F}(\varepsilon_2)\equiv\mathbf{F}_2$ that must be contained in the new morphology. Denote $\mathbf{F}_n^{\,k}$ as the set of elements with strength lower than ε_n translated k elements to the right. The total damage at the second cycle is:

$$\mathbf{D}_2=\mathbf{F}_2^0\cup\left(\mathbf{D}_1^0\cup\mathbf{D}_1^1\right)=\mathbf{F}_2^0\cup\mathbf{F}_1^0\cup\mathbf{F}_1^1=\mathbf{F}_2^0\cup\mathbf{F}_1^1\,. \tag{5}$$

Continuing this process and after some set manipulations, the damage at the nth cycle is,

$$\mathbf{D}_n=\bigcup_{k=1}^{n}\mathbf{F}_k^{n-k}\,. \tag{6}$$

(6) is valid for any monotonically growing fatigue strain history. Recalling that $\mathbf{F}_n^{\,k}$ and $\mathbf{F}_m^{\,p}$ are independent for k≠p, the damage measure can be derived,

$$D_n=1-\mu\left\{\bigcap_{k=1}^{n}\left(\mathbf{F}_k^{n-k}\right)^c\right\}=1-\prod_{k=0}^{n}\left(1-F_k\right). \tag{7}$$

Micro-crack arrest is added to the damage morphology by letting each crack a probability γ to arrest on every cycle. Details are involved and not given here. The macro stress σ at cycle n is,

$$\sigma = \left(1 - D_n\right) E_0 \varepsilon_n \,. \tag{8}$$

E_0 is the young modulus of the elements. For convenience, a Weibull strength distribution is chosen.

$$F\left(\varepsilon\right) = 1 - \exp\left(-\beta^{-1}\left(\varepsilon / \varepsilon_s\right)^{\beta}\right) \tag{9}$$

Using (7-9) a first-order nonlinear differential equation is obtained,

$$\varepsilon_{,k} - \beta^{-1}\varepsilon_s^{-\beta}\varepsilon^{\beta+1} = 0 \tag{10}$$

k is a continuous variable, equivalent to n. Previous studies [10] which have not considered microcrack coalescence resulted in a similar equation of a second order. For HCF the damage at the first cycle is negligible thus,

$$\varepsilon\left(0\right) \cong E_0^{-1}\sigma \,. \tag{11}$$

Solving (10) using (11) yields,

$$\varepsilon = \left(\left(E_0^{-1}\sigma\right)^{-\beta} - \varepsilon_s^{-\beta} k\right)^{-1/\beta} . \tag{12}$$

The S-N relation is extracted from (12) by finding the cycle in which $\varepsilon \to \infty$. The number of cycles to failure N takes the Basquin Law appearance:

$$\sigma / \sigma_s \cong A N^b \qquad ; \qquad A = e^{1/\beta} \;;\; b = -\beta^{-1} \tag{13}$$

When a crack arrest mechanism is active, the analysis is much more involved, leading to the following first order differential equation for the damage evolution measure:

$$\left(1 - D\right)^{\beta-1}\left(\gamma D + \left(1-\gamma\right) D_{,k}\right) = \beta^{-1}\left(\frac{\sigma}{E_0 \varepsilon_s}\right)^{\beta} \tag{14}$$

Numerical solutions reveal three damage evolution types according to the stress level (Fig.1). Interestingly, although an exact solution is not possible yet, the main characteristics of the solution, such as the endurance limit and inflection point can be extracted analytically [11].

3. Two-level Loading Response

So far, the strength of the non-broken elements was kept constant during the cyclic loading. We now release this restriction by replacing the current strain (ε_n) in (9) by accumulated strain power (ε_a), i.e.,

$$\varepsilon_a = \sum_{m=1}^{n}\left(\varepsilon_m\right)^b \tag{15}$$

The motivation of this step is studying the effect of a history dependent function. Indeed, Fig.2L shows a very realistic prediction. Comparing the predicted fatigue life left to failure at the second stress-level n_2/N_2 with Manson's experimental results [12], as seen in Fig.2R, demonstrates the power of the improved GFM.

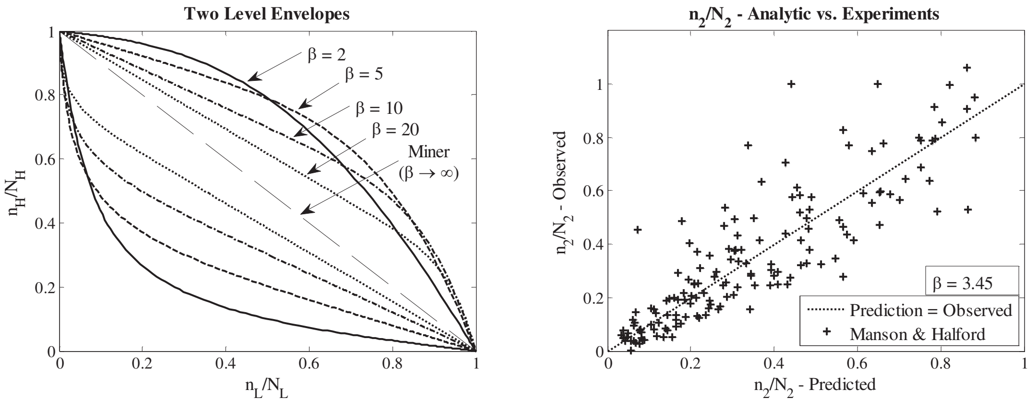

Fig.2., An improved GFM for two level loading. Left: general characteristics; Right: validation by comparing with Manson & Halford two level experiments.

4. Multiaxial Fatigue

As in (1), there are many formulas for multiaxial fatigue [12]. Generalization of the GFM to full multiaxial model is complicated. On the other hand, numerical studies [14] show the potential of simple 2D lattice simulations in understanding the mechanisms of fatigue damage evolution. We examine a simple 2D structure (truss) made of 1D elements of our previous analysis as seen in fig.3L. The advantages are: a.) Multiaxial loading is admissible, b.) Fatigue failure under unidirectional compressive loading is possible, and c.) Some analytical approximations based on the above 1D analysis are feasible.

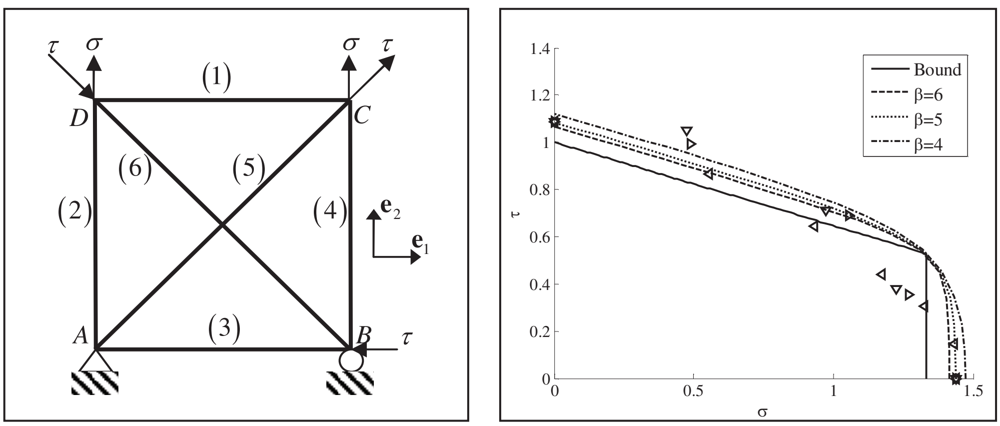

Figure 3., Left: Truss model for the 2D fatigue loading. Stresses are translated to forces. Right: Constant fatigue life σ−τ envelopes for different β values. The straight lines are for the analytical bounds based on the 1D model.

The stiffness of the two "cross elements" are κ times higher than those for the "box elements". The simulation is performed by calculating the strain in each element in each cycle and finding its damage increment as for the 1D case. Stiffness is updated after each cycle. A zero order analytical approximation is found by identifying the element with the highest strain and calculating its life without updating the truss stiffness. This approximation constitutes a lower bound for the fatigue life.

Fig.3R shows the $\sigma-\tau$ failure envelope by the 2D GFM for a constant fatigue life. The two straight lines describe the analytical bound and the curved lines are 2D simulations for different β values, which are the only material parameter used in the model. Note that the graphs coincide at the corner point of the analytical bound. The experimental data taken from [15] is presented by triangular markers where each marker orientation represents different specimen geometry. It is seen that the main characteristics of the multiaxial behavior is captured by the GFM. It is expected that a model made from many such "basic" structures will yield a smoother failure envelope.

5. Conclusions

Micromechanical features which can be followed analytically, and generalization to semi-2D models of fatigue, have promising potential to establish an improved fatigue prediction tools for design. The advantages of such models are: a.) their simplicity i.e., the minimal number of material parameters used; and b.) every parameter has a definite micro-geometrical or stochastic meaning. Comparisons with experimental data validate this general approach.

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