A Generalized Optimal Sensor Placement Technique for Structural Health Monitoring and System Identification

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Abstract

A structural health monitoring system consists of permanently installed sensors to collect structural information, and these sensors are required to be placed at ‘good’ positions for damage identification. Conventional sensor placement methods make use of dynamic characteristics of a structure, i.e., mode shapes and natural frequencies, to determine optimal sensor positions. The engineering community has traditionally relied on these modal methods and finite element analysis for dynamic predictions in the low frequency range. However, these techniques become ineffective in mid frequency range due to large number of modes and high modal density. In view of this, in this paper, an optimal sensor placement technique which can be employed for low as well as mid frequency range structures for structural health monitoring as well as structural system identification is presented. The frequency domain based optimal sensor placement technique (FEfi) presented in this paper makes use of principal components evaluated from frequency response functions at the desired frequency levels. Numerical examples with optimally placed sensors dictated by the proposed algorithm are presented and compared with the popular effective independence (Efi) technique based on mode shapes.

Keywords: Optimal sensor placement, Effective independence, frequency domain, structural health monitoring, system identification, Fisher information matrix

1. Introduction

The problem of parameter estimation of structural models using measured dynamic data is important in modal identification, structural model updating, structural health monitoring and structural control. The estimate of the parameter values involves uncertainties that are due to limitations of the mathematical models used to represent the behavior of the real structure, the presence of measurement error in the data and insufficient excitation and response bandwidth. In particular, the quality of information that can be extracted from the data for estimating the...
model parameters depends on the number and location of sensors in the structure as well as on the type and size of model and measurement error. The objective in an experimental design is to make a cost-effective selection of the optimal number and location of sensors such that the resulting measured data are most informative for estimating the parameters of a mathematical model of the structure. The issue of sensor placement attracts much attention from both academia and industry, especially due to increasing number of instrumented large structures for health monitoring in the last decade. The sensors installed in these structures are mostly permanent and are always sparse, in fact, far less than available positions. This is partly because of economic reasons, high cost of data acquisition systems (sensors and their supporting instruments), partly because of structural accessibility limitations. Furthermore, the wiring of sensors leading to monitoring room requires non-interference routing and special care to prepare their integrity, particularly for optical fiber sensors [1-3]. Although wireless sensors are coming into use, time synchronization and routing protocol problems currently limit their wide applications.

Several approaches have been reported in the literature [1-3] to solve the problem of optimal sensor placement. Among them, the Effective Independence (EfI) method is one of the most popular and commonly used technique [4]. The EfI method quantifies the independence between two or more reduced mode shapes, and has many attractive properties. In particular, it provides a natural criterion to differentiate truncated mode shapes for sensor placement, and has been applied to a wide range of large structural dynamic testing, as also recommend by Ewins [5], Rama Mohan Rao & Ganesh Anandakumar [6, 7] and Friswell and Mottershead [8] for modal testing, structural health monitoring and modal updating, and has already been embedded in the commercial software MSC/NASTRAN [9].

Effective independence approach for Sensor placement is based on a relatively small set of target modes within the range of interested frequency. These methods may encounter problems if modal density of the structure is higher. For example, a precision spacecraft require models that are valid to a much higher frequency range for accurate predictions. This higher frequency band, lie in the mid-frequency range. This requires high fidelity finite element models, and result in a large number of densely packed modes in the structure. In order to overcome the difficulty associated with effective independence approach based on mode shapes, it is proposed to use frequency response functions to arrive at optimal sensor locations. This gives generality to the optimal sensor placement algorithm, which can be applied to any range of frequencies or loading conditions. Analytical frequency response can be calculated employing finite element models for any specified frequency bands, damping, and input locations. The analytical frequency response can be decomposed into principal directions and singular values, which can be directly related to the system’s energy [10–12]. It can be shown that the principal directions with non-zero singular values span the same subspace as the excited modes [13], even though they do not generally coincide with mode shapes. The major advantage of principal components is that the system’s response is usually dominated by a relatively small number of principal directions, even for frequency bands with high modal density. Hence, using principal directions the difficult task of identifying the dynamically important mode shapes can be avoided. The effects from input location and damping are automatically accounted for in the principal directions. Further, the principal directions are always orthogonal, while mode shapes in general are not. This makes principal directions more robust to modeling errors and experimental noise [11]. Hence, principal directions can be treated as a more suitable basis for describing structural response.

2. Principal components using Frequency Response Functions (FRF)

The FRF of an n DOF system with structural damping, whose excitation and response are measured at DOFs p and q, respectively, is given by

\[
h_{pq}(\omega) = \frac{-\omega^2 \phi_{jp} \phi_{jq}}{\omega_j^2 - \omega^2 + 2i\zeta_j \omega \omega_j}
\]

where \(\omega\) is the forcing frequency in rad/sec, \(\omega_j\) is the undamped natural frequency of the \(j^{th}\) mode, \(\zeta\) is the damping coefficient, \(\phi_{jp}\) is the value of the \(j^{th}\) mode at the \(p^{th}\) output location, and \(\phi_{jq}\) is the value of the \(j^{th}\) mode at the \(q^{th}\) input location.
The frequency response data matrix, $H$, can then be defined as a collection of the individual frequency response matrices

$$H = \left[ h(\omega_1) \ h(\omega_2) \ h(\omega_3) \ \ldots h(\omega_f) \right]$$

(2)

in which $f$ is the number of data points in the frequency range of interest, and $h(\omega_i)$ is a matrix of data, $n_s$ data points in the frequency range of interest $n$ different measurements can be decomposed as

$$H = U \Sigma V^T$$

(3)

where $U$ and $V$ are orthogonal matrices (na x ns and ns x ns respectively), and $\Sigma$ is a diagonal matrix. The columns of the orthogonal complex matrix $V$ are the computed PCAs. The diagonal matrix $\Sigma$ is termed the singular matrix whose elements (along the diagonal) are non-negative numbers, called the singular values, arranged in decreasing order. These singular values each correspond to a single basis function, $v$, and it represent the level of ‘energy’ present in each mode. $V$ is a matrix with orthogonal columns containing normalised frequency response of the principal directions. Researchers have stated that the singular values are related to the total energy of the system contained in the corresponding principal directions [11, 12, 14]. The singular values are arranged from largest to smallest. Most of the system’s energy is usually concentrated in the first several singular values. The corresponding principal directions show how the energy is distributed in the system [15]. There are a variety of methods for determining how many singular values to retain to properly characterize a system. A common method is to retain the singular values and principal directions which correspond to the top 95% or 99% of the system’s total energy [8]. Like mode shapes, principal directions are the fundamental shapes that represent the system’s dynamics. However, principal directions automatically account for damping in the structure, effects of input locations, and out-of-band modes. Researchers have shown that principal directions converge to normal modes in symmetric linear systems if the mass matrix is diagonal and uniform, and the system is lightly damped [14, 15]. In general, principal directions do not coincide with the mode shapes, but it can be shown that the principal directions with non-zero singular values span the same subspace as the excited modes [13]. While there may be many vibrational normal modes in a frequency band, the response is usually dominated by a relatively small number of principal directions. Using principal directions thereby eliminates the difficult task of identifying the dynamically important mode shapes. The effects from input location and damping are automatically accounted for in the principal directions. The principal directions and singular values of the frequency response data matrix are also the eigenvectors and eigenvalues of the matrix

$$HH^* = U \Sigma^2 V^T$$

(4)

in which * represents the complex conjugate transpose. The real part of the $HH^*$ i.e., output covariance matrix is generally used to determine principal directions and singular values for the system. However, it can be shown that the sum of the k largest singular values for $HH^*$ will be greater than the sum of the k largest singular values for the covariance matrix. This implies that the system’s energy captured using fewer singular values from the complex covariance matrix is more than energy captured from the same number of singular values obtained using real part of $HH^*$. Therefore, we can capture the information in the frequency response much more accurately using the principal directors obtained from the complex covariance matrix

3. Frequency Effective Independence Method (FEfi)

The aim of the FRF based effective independence (FEfi) technique is to place sensors to maintain the dynamically important information contained in the frequency response data within the desired frequency band. This is accomplished by placing sensors such that the measured response is rich in the response of the active principal directions. The principal directions are analogous to mode shapes and they represent shapes that are fundamental to the structure’s dynamics. As in the case of modal based Efi, the FRF based sensor placement problem (FEfi) can be
written in the form of a state estimation problem

\[ H = \psi \bar{V} + N \]  \hspace{1cm} (5)

where \( \bar{V} = SV \) represents the frequency response of the dynamically important principal directions, and \( N \) is a matrix of Gaussian white noise. The sensors are to be placed on the structure in such a way that the measured frequency response; the response of the dynamically important principal directions is estimated accurately. The Fisher information matrix can be written as

\[ Q_c = \psi_c^T W \psi_c \]  \hspace{1cm} (6)

where \( \psi_c \) are the active principal directions partitioned to the candidate sensor set, and \( W \) is a weighting matrix i.e., inverse of the noise covariance matrix. In this paper, we choose \( W \) to be an identity matrix. Maximizing the information matrix in an appropriate norm results in minimizing the error covariance matrix and it provides the best state estimate. Sensors should be placed to provide the best estimate of the target modal response. Maximizing the determinant of the information matrix is chosen as the sensor placement criterion, because it results in maximizing signal strength and the independence of the target principal directions. Similar to the Effective Independence (Efi) method, the proposed FEfi is formulated by iteratively truncating the sensor locations that have the smallest impact on the value of the determinant of the information matrix. The FEfi value corresponding to the \( i^{th} \) sensor is given by

\[ FE_i = \psi_i Q^{-1} \psi_i^T \]  \hspace{1cm} (7)

Where \( FE_i \) represents the fractional reduction of the information matrix determinant, if the \( i^{th} \) sensor is removed from the candidate set. The candidate sensors are ranked based on their FEfi values and the sensor corresponding to the lowest FEfi value is removed. The FEfi values range from 0 to 1, where a zero valued sensor can be removed without impact to the information matrix determinant, while sensors with a value of 1 are vital to the independence of the target shapes and cannot be deleted. The candidate sensor locations are deleted in an iterative fashion to finally produce the desired number of sensors. In the final sensor set, the number of sensors must be at least equal to the number of target modes to ensure independence. New information matrices and FEfi values must be computed after truncation of each sensor as the FEfi value for each sensor changes as sensors are dropped from the candidate set.

4. Numerical Studies

Numerical investigations have been carried out to demonstrate the usefulness of FEfi. The first numerical example considered is a cantilever beam. The finite element idealization of the beam and material properties are shown in Figure 1. All the active 30 nodes in the beam are considered as possible candidate set of sensor locations and the number of sensors is gradually reduced to desired number of sensors. For the present problem, the desired number of sensors is considered as 10. The principal directions and singular values are extracted from the analytical frequency response. The singular values are truncated to 2, which retained 99% of the system’s total energy. The principal directions corresponding to the retained singular values are used in the FEfi calculations.

The beam is subjected to a single point excitation at the right hand end of the beam (node 30). The first five lower modes are excited using the forcing function. The optimal sensors obtained using modal based Efi with five modes are also computed for comparison. Numerical investigations are carried out with 1% and 10% modal damping. The final Efi and FEfi distributions, and sensor locations, for 1% and 10% modal damping are shown in Fig. 2. For the trial with 1% modal damping, the final sensor sets and effective independence distributions are indistinguishable between the modal and frequency response based techniques. However, when the modal damping is increased to 10% the FEfi technique skews the sensor positions towards the input location at the right end. This shows that FEfi is sensitive to changes in damping. As the damping is increased the response further away from the excitation point tend to decrease, and therefore the final sensors are skewed more toward the input location.

The second numerical example considered is a simply supported beam. The finite element idealization of the beam, material properties are shown in Figure 3. All the active 30 nodes in the beam are considered as possible candidate set of sensor locations and the number of sensors is gradually reduced to desired number of sensors. The
The desired number of sensors is considered as 10. The principal directions and singular values were extracted from the analytical frequency response and the number of principal components are considered as two based on the energy cutoff criterion discussed earlier. In the first trial, we have considered loading at the central node i.e node number 15 and it excites first, third and fifth modes. These three modes are only considered for computing analytical frequency responses. The damping is taken as 10%. Figure 4(a) shows the optimal sensor locations and it can be clearly observed that the sensor locations are clearly biased towards the loading point unlike Efi.

Investigations are carried out by changing the input force location to vary the excitation of the modes. Modal damping is taken as 10% and the input force excitation is given at the quarter-point of the beam. Since the input force excitation is given only at the quarter point of the beam, only the symmetric modes (2, 4, & 6) excites. If the target modes for modal Efi are chosen to be all the modes in the frequency band, then half of the target modes will never be excited for this input excitation. The FEfi technique requires only 2 singular values to make up 95% of the system’s total energy. This clearly demonstrates that FEfi automatically focuses resources on the 3 excited modes and ignores the unexcited modes in the frequency band. Figure 4(b) shows the optimal sensor locations using FEfi method. The results presented in Fig. 4 clearly demonstrate that the FEfi technique is sensitive to input excitation locations.

These two examples clearly demonstrate that on a simple structure, for the case of light damping with inputs that excite all of the modes in the frequency band, the FEfi scheme picks sensors in a similar fashion to modal Efi. This is due to the fact that principal directions converge to normal modes in symmetric linear systems if the mass matrix is diagonal and uniform, and the system is lightly damped [14,15]. These simple examples also show how the FEfi technique automatically accounts for input location and damping effects.

A rectangular concrete slab bridge of size 12 m x 3.5 m supported on the two short edges is considered as the third numerical example. The FE discretisation of the plate and the material properties are shown in Fig. 5. The load is assumed to be random (ambient data) and uniformly applied on the structure. Two hundred and thirty two nodes are considered as possible candidate set of sensor locations and the number of sensors considered is 30. Table 1 presents the optimal sensor locations obtained using FEfi and Efi algorithms with varied damping values. It can be observed from the results presented in Table 1 that the optimal sensor locations obtained using the FEfi varies with damping levels, while Efi remains constant. The number of sensor locations altered with 10% damping when compared with the corresponding Efi based approach is found to be twenty five. The number of sensor locations altered is found to be quite appreciable with increased damping, taking into account that the total number of sensors considered is only 30. Fig. 6 shows the optimal locations obtained using Efi and FEfi. The common and varying sensor locations suggested by the two algorithms are marked using distinguishable markers.

Finally, a bridge structure with multiple longitudinal girders is taken as a numerical example to demonstrate the effectiveness of FEfi algorithm for optimal sensor placement. The load is assumed to be random (ambient load) and uniformly applied on the structure. The bridge is of size 10.4 m x 8.43 m with four longitudinal girders. In order to accurately model the bridge, we preferred to use three dimensional finite elements to model the slab and also the longitudinal girders. The isometric view of the bridge and the material properties are shown in Fig. 7. The total number of brick elements in the FE model is 1643. Nine hundred and sixty nodes are considered as possible candidate set of sensor locations and the desired number of sensors is considered as 30 in the present work. Table 2 presents the optimal sensor locations obtained using Efi and FEfi algorithm with varied damping ratios. The results presented in Table 2 shows the variations in sensor locations obtained using FEfi algorithm with varied damping ratios. Fig. 8 shows optimal sensor locations obtained using Efi and FEfi algorithms. The common sensor locations and varying sensor locations suggested by Efi and FEfi algorithms are clearly marked in the figure using distinguishable markers.

5. Conclusions

In this paper, a frequency domain sensor placement technique is presented. This technique is developed by combining Principal Component Analysis (PCA) and Effective independence algorithm (Efi). Sensors are placed to
maintain the dynamically important information in the frequency response and the overall system energy within the frequency range of interest. The proposed FEfi technique eliminates the difficult task of identifying target modes as the frequency response data automatically accounts for input location and damping, the first numerical example presented in this paper clearly indicates that the frequency and modal based Efi methods provide comparable sensor configurations for systems with low damping and well separated modal frequencies. As damping levels increased, the frequency based Efi (FEfi) approach automatically skewed the sensor locations toward the input location. This is clearly demonstrated in this paper using four different numerical examples. The frequency domain optimal sensors placement technique discussed in this paper can be used for problems in all frequency ranges unlike modal based effective independence algorithm, which is applicable only for low frequency range. In that sense, FEfi is a more generalized version applicable to both low and mid frequency ranges.

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References

Fig. 1 Cantilever Beam and material properties

(a) Efi method with 1% and 10% damping

(b) FEfi method with 1% damping

(c) FEfi method with 10% damping

Fig. 2. Comparative performance of Efi and FEfi methods with varied damping levels and modes

Length of the Beam: 6 m
Cross section Area: 0.015 m$^2$
Number of Active nodes: 30
Elastic modulus: E=25.0 GPa
Poisson’s Ratio: 0.15

Length of the Beam: 10 m
Number of Active nodes: 30
Cross section Area: 0.015 m$^2$
Elastic modulus: E=25.0 GPa
Poisson’s Ratio: 0.15
Fig 3. Simply supported Beam

(a) Optimal sensors with FEfi (10% damping) and central load

(b) Optimal sensors with FEfi (10% damping) and load at quarter span

Fig 4. Simply supported Beam with varied damping and spatial loading points

Number of active nodes: 232  Thickness of plate: 0.25 m
Elastic modulus: 22.1 GPa  Poisson’s ratio: \( \nu = 0.15 \)

Fig 5. Slab Bridge of size 12m X 3.5
Fig. 6. Optimal sensor locations in the slab bridge using Efi and FEfi algorithms (30 sensors)

Fig. 7. Three-dimensional FE model of concrete girder bridge

Bridge Dimensions: 10.4 m X 8.436 m
Num. of active nodes: 960
Depth of the girder(s), slab: 0.505 m, 0.152 m
Elastic modulus: 23.65 GPa
Poisson’s ratio: $\nu = 0.20$
Fig. 8. Optimal sensor locations on concrete girder bridge using Efi and FEfi
(30 sensors)

Table 1. Optimal sensor locations on Slab Bridge with varied damping ratios with number of sensors limited to thirty

<table>
<thead>
<tr>
<th>SNO</th>
<th>Algorithm</th>
<th>Damping</th>
<th>Sensor locations</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Efi</td>
<td>1%</td>
<td>41, 48, 56, 57, 64, 65, 72, 73, 80, 88, 97, 105, 112, 113, 120, 121, 128, 136, 145, 153, 160, 161, 168, 169, 176, 177, 184, 185, 192</td>
</tr>
<tr>
<td>2</td>
<td>FEfi</td>
<td>1%</td>
<td>41, 48, 49, 56, 57, 64, 65, 72, 73, 80, 97, 104, 105, 112, 113, 120, 121, 128, 129, 152, 153, 160, 161, 168, 169, 169, 176, 177, 184, 185, 192</td>
</tr>
<tr>
<td>3</td>
<td>Efi</td>
<td>3%</td>
<td>41, 48, 49, 56, 57, 64, 65, 72, 73, 80, 88, 97, 105, 112, 113, 120, 121, 128, 136, 145, 153, 160, 161, 168, 169, 176, 177, 184, 185, 192</td>
</tr>
<tr>
<td>4</td>
<td>FEfi</td>
<td>3%</td>
<td>42, 47, 50, 55, 58, 63, 66, 71, 74, 79, 87, 98, 106, 111, 114, 119, 122, 127, 135, 146, 154, 159, 162, 167, 170, 175, 178, 183, 186, 191</td>
</tr>
<tr>
<td>5</td>
<td>Efi</td>
<td>8%</td>
<td>41, 48, 49, 56, 57, 64, 65, 72, 73, 80, 88, 97, 105, 112, 113, 120, 121, 128, 136, 145, 153, 160, 161, 168, 169, 176, 177, 184, 185, 192</td>
</tr>
<tr>
<td>6</td>
<td>FEfi</td>
<td>8%</td>
<td>41, 48, 50, 55, 57, 63, 66, 72, 73, 88, 89, 97, 104, 105, 114, 119, 121, 128, 137, 144, 153, 160, 161, 167, 170, 176, 177, 183, 185, 192</td>
</tr>
</tbody>
</table>

Table 2. Optimal sensor locations on Girder Bridge with varied damping ratios with number of sensors limited to thirty

<table>
<thead>
<tr>
<th>Method</th>
<th>Damping</th>
<th>Optimal Sensor Locations</th>
</tr>
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