Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Procedia Engineering 63 (2013) 225 – 233
Procedia Engineering
www.elsevier.com/locate/procedia

The Manufacturing Engineering Society International Conference, MESIC 2013

Development of a calibration model for optical measuring machines

J. Caja*, E. Gómez, P. Maresca, M. Berzal

Department of Mechanical Engineering and Construction – Universidad Politécnica de Madrid, Ronda de Valencia 3, 28012 Madrid, Spain

Abstract

This paper presents the problem of optical measuring machine calibration, emphasizing the calibration of the "optical system", omitting the calibration of the "machine system". The calibration of an optical measuring machine is the first step before using the instrument for any application. For this purpose, a mathematical model has been developed to transform the coordinates of a point in space (3D) into coordinates of a point in an image (2D). Using this camera model, a calibration procedure has been developed using a grid distortion pattern. Finally, a procedure for calculating the uncertainty of the camera and geometric distortion parameters based on the Monte Carlo method has been developed.

Selection and peer-review under responsibility of Universidad de Zaragoza, Dpto Ing Diseño y Fabricacion

Keywords: Dimensional metrology; Uncertainty calculus; Monte Carlo method; Optical Measuring Machine.

1. Introduction

The digital optical measurement systems have achieved wide spread in recent years in industry, reaching a share of around 20 % of the world market. These systems present interesting advantages over the coordinate measuring machines (CMM) with probing mechanical methods, mainly the speed of data acquisition, automation of measurement functions and, above all, the absence of contact.

Conceptually an optical measuring machine can be divided into two subsystems, a "machine system" and an "optical system". The "machine system" consists of a monoblock structure holding the measuring table and allows

* Corresponding author. Tel.: +34-91-336-5585; fax: +34-91-336-7676.
E-mail address: jesus.caja@upm.es

Selection and peer-review under responsibility of Universidad de Zaragoza, Dpto Ing Diseño y Fabricacion
doi:10.1016/j.proeng.2013.08.187

the displacement of the axes. The "optical system" consists of a charged-coupled device (CCD) which allows the acquisition and transfer of images to a computer connected to the "machine system" as well as the necessary lenses and objectives to obtain images of a given resolution. The system made up of CCD camera replaced the "contact sensor system" used in a CMM. This subdivision allows us to analyze separately the two systems in order to achieve a model as a basis for the evaluation of uncertainty.

The science of metrology encounters with the continuous challenge of innovating in procedures to adapt to increasingly demanding requirements. This progress has been made on several levels: the material and the theoretical. In the first instance through the development of new measuring instruments or the improvement of traditional, use of new materials, electronic solutions incorporated in the amplification levels, indication, and automation. On the theoretical level, new methods of measurement has been developed and a deeper understanding of the condition of influence quantities has been incorporated.

2. Camera Model

A camera model transforms the coordinates of a point in space (3D) to the coordinates of a point in an image (2D), i.e., explains the process of forming an image with a camera. In the first instance the Pinhole camera model (Fig. 1) is used. It is the most simple and specialized camera model, which represents an ideal camera distortion-free as shown by Tsai (1987), Weng et al. (1992) and Hartley et al. (2004). It will serve to explain other models.

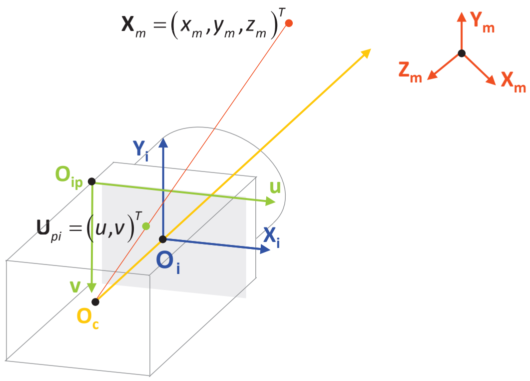

Fig. 1. Pinhole camera model.

This camera model can be expressed by the following matrix expression:

$$\begin{pmatrix} w\cdot u \\ w\cdot v \\ w \end{pmatrix} = \begin{pmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & r_{13} & -t_x \\ r_{21} & r_{22} & r_{23} & -t_y \\ r_{31} & r_{32} & r_{33} & -t_z \end{pmatrix} \cdot \begin{pmatrix} x_m \\ y_m \\ z_m \\ 1 \end{pmatrix} \qquad \begin{cases} \mathbf{U}_{hpi} = \mathbf{K}\left[\mathbf{R} \;\middle|\; -\mathbf{t}\right] \cdot \mathbf{X}_{hm} \\ \mathbf{U}_{hpi} = \mathbf{P} \cdot \mathbf{X}_{hm} \end{cases} \tag{1}$$

where $\mathbf{U}_{hpi} = (w\cdot u, w\cdot v, w)^T$ represents the coordinates in pixels of a point in the *"digital image system"* and $\mathbf{X}_{hm} = (x_m, y_m, z_m, 1)^T$ are the coordinates of the same point in the *"world coordinate system"*.

The parameters α_x (horizontal pixel size), α_y (vertical pixel size), s (skew parameter) and (x_0, y_0) (coordinates of the principal point of the image) define the characteristics the camera. These parameters will not depend on the position or orientation of the camera in the scene. For this reason they are called intrinsic parameters. The matrix $\mathbf{R}$ and vector $\mathbf{t}$ represent the position and orientation of the camera relative to the reference system of the scene. As not depend on the characteristics of the camera, the matrix $\mathbf{R}$ and the vector $\mathbf{t}$ contain the extrinsic parameters of the camera model.

Taking into account that the equipment of study has a telecentric optics, the affine camera model will be employed. In this model, the optical center is a point in the infinity, and the matrix $\mathbf{P}$ of the camera presents its last row equals to $(0, \ 0, \ 0, \ 1)$ as shown by Hartley et al. (2004). Conveniently operate, equation 1 becomes:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{pmatrix} \cdot \begin{pmatrix} r_{11} & r_{12} & r_{13} & -t_x \\ r_{21} & r_{22} & r_{23} & -t_y \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_m \\ y_m \\ z_m \\ 1 \end{pmatrix} \tag{2}$$

Finally, we will include in the camera model terms that take into account the geometric distortion. This distortion appears as a change in position of the image points, as a result of the various types of imperfections in the design, manufacture and assembly of the lenses. So that:

$$\begin{aligned} u' &= u + \delta_u \\ v' &= v + \delta_v \end{aligned} \tag{3}$$

where u and v represent the distortion-free coordinates, u' and v' are the observed coordinates with distortion, δ_u and δ_v represent the geometrical distortion of the coordinates u' and v' respectively. We consider three types of geometric distortion. The first of them caused by imperfections in the shape of the lenses and that is manifested by a positional error of radial type (radial distortion) while the second and third are caused by errors in the lens mounting, reflected by a positional error of radial and tangential type (decentering and prism distortion). Mathematically, the previous types of distortion can be modeled as:

$$\begin{aligned} \delta_u &= k_1 \Delta u \left(\Delta u^2 + \Delta v^2\right) + k_2 \Delta u \left(\Delta u^2 + \Delta v^2\right)^2 + p_1\left(3\Delta u^2 + \Delta v^2\right) + 2 p_2 \Delta u \Delta v + s_1\left(\Delta u^2 + \Delta v^2\right) \\ \delta_v &= k_1 \Delta v \left(\Delta u^2 + \Delta v^2\right) + k_2 \Delta v \left(\Delta u^2 + \Delta v^2\right)^2 + 2 p_1 \Delta u \Delta v + p_2\left(\Delta u^2 + 3\Delta v^2\right) + s_2\left(\Delta u^2 + \Delta v^2\right) \end{aligned} \tag{4}$$

Where $\Delta u = u - x_0$ and $\Delta v = v - y_0$ represent the horizontal and vertical distances to the principal point of the image (point without geometric distortion) and $k_1, k_2, p_1, p_2, s_1, s_2$ are the geometric distortion coefficients. Fig. 2(a) and Fig. 2(b) show the effects on an image caused by the above parameters.

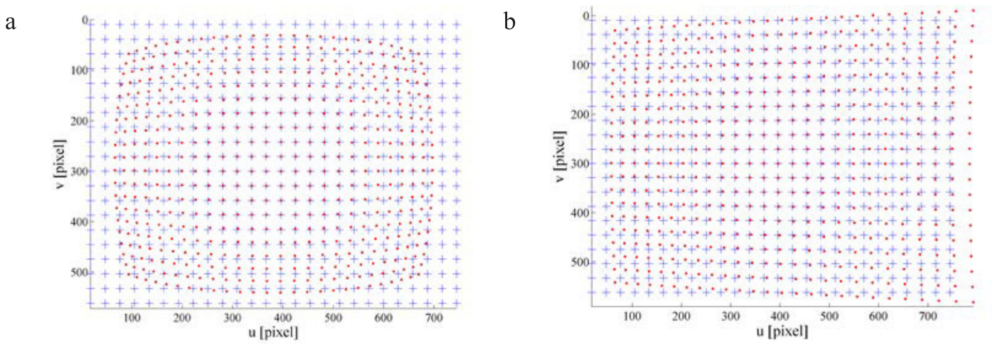

Fig. 2. (a) Radial distortion caused by a coefficient $k_1 = -1 \cdot 10^{-6}$; (b) Decentering distortion caused by a coefficient $p_1 = 1 \cdot 10^{-4}$.

3. Calibration procedure

For the calibration of the optical system we will use a grid distortion target. This standard is an metrological element generally of glass or an opaque material, in one of its surface has deposited a template (typically chromium oxide) with geometrical elements (points, lines, ...) whose horizontal and vertical pitch is known.

Through the image of this standard, in our case a dot grid, we may obtain the camera intrinsic parameters and also determine if the optical system has some type of geometric distortion, employing the camera model defined above.

The calibration procedure is divided into the following steps:

3.1. Image Acquisition

The image obtained by the optical measuring machine, which is represented in matrix form as $I_{ADQ}(i,j)$ is transformed to gray scale.

3.2. Correction and normalization of the image intensity.

There are different causes that produce noise in the currents recorded by the CCD's (Fig 3(a)) (differences in response of the pixels to the luminous flux, CCD imperfections, imperfections in the lenses of the optical system, ambient illumination, ...). Generally, the intensity recorded by each pixel of a CCD proposed by Santo et al. (2004) can be modeled as:

$$I_{ADQ}(i,j) = I_{co}(i,j) + r(i,j) I_0(i,j) \tag{5}$$

Where $I_{ADQ}(i,j)$ represents the matrix of image acquired intensities, $I_{co}(i,j)$ the matrix that contains the portion of the intensity due to the noise non-dependent and temperature-dependent, $r(i,j)$ is the matrix that takes into account the non-uniformity of response of each pixel due to spatial variations and $I_0(i,j)$ is the matrix of image intensities without noise.

The assessment of $I_{co}(i,j)$ is performed experimentally by acquiring n images $I_{D_k}(i,j)$ while the CCD image is not exposed to any incident light. The matrix $r(i,j)$ is obtained experimentally via the acquisition of m images $I_{FF}(i,j)$ of an illuminated surface. Subsequently the average value of intensity for each of the pixels of previous images will be calculated. Consequently, the expression for obtaining the intensity of the corrected image is:

$$I_0(i,j) = \frac{I_c(i,j)}{r(i,j)} = \frac{\bar{I}_{FF_{\max}}}{\bar{I}_{FF}(i,j)} I_c(i,j) = \frac{1}{r(i,j)} \left[I_{ADQ}(i,j) - \bar{I}_D \right] \tag{6}$$

The employ of the above equation produces images with less noise and transition zones between light and dark areas more defined. These transition zones (defined by an intensity gradient) which will allow the detection of the edge.

3.3. Edge detection

Employing the corrected image, we will proceed to determine the pixels that define the dots edges of the grid distortion target. In previous studies conducted by Maresca et al. (2010), it was determined that the edge detection technique based on the Canny filter for metrological images, obtain the most effective results in terms of probability, certainty and precision. Additionally a thresholding technique shown by Gonzalez et al. (2008) has been used to enhance the detection of the edge.

3.4. Determination of the dot center of the grid distortion target

We will adjust the pixels of each of the i dots of the grid distortion target to a circumference (Fig. 3(b)), using an orthogonal regression method. The orthogonal deviation d_{ki} of a set point (u_{ki}, v_{ki}) to the adjust circumference of center (u_{ci}, v_{ci}) and radius r_{ci} is:

$$d_{ki} = \sqrt{(u_{ki} - u_{ci})^2 + (v_{ki} - v_{ci})^2} - r_{ci} \tag{7}$$

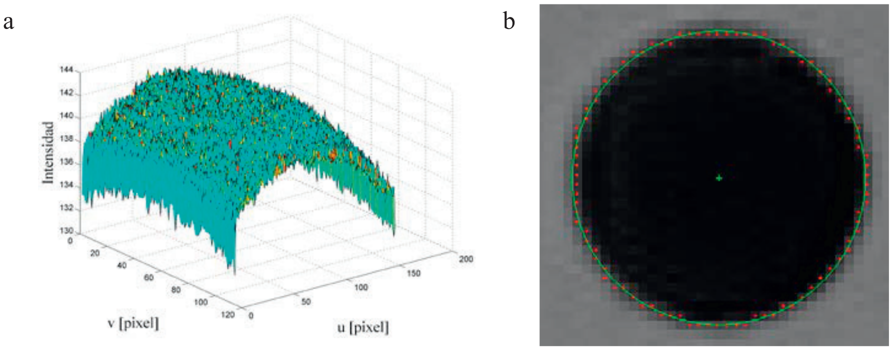

Fig. 3. (a) 3D representation of a non-corrected intensity of an image; (b) Dot of the grid distortion target edge pixels adjusted to a circumference.

3.5. Mathematical resolution of the calibration procedure

Using the coordinates of the i centers obtained in the previous step, we proceed to obtain the matrix of the camera $\mathbf{P}$. The resolution method is divided into two stages:

In the first stage, we assume the correspondence between a set of 3D points and their 2D counterparts, considering that the optical system has no distortion. Such correspondence exists between the coordinates of the center of grid distortion target dots $\mathbf{X}_{cp} = (x_{cp}, y_{cp}, z_{cp})^T$ (expressed in the "world coordinate system") and the center of this dot in the "digital image system" $\mathbf{U}_c = (u_c, v_c)$ (Fig. 4).

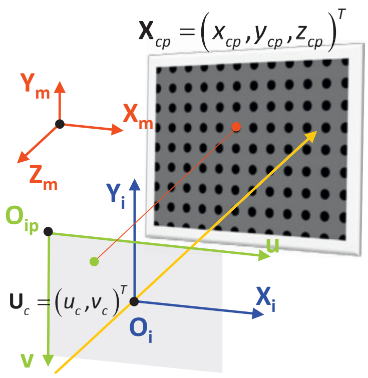

Fig. 4. Correspondence between the coordinates of the center of a grid distortion target dot and the center of this point in the "digital image system".

Considering that it has to fulfill the relation defined in equation 2, and employing a linear solution method we obtain an initial solution $\mathbf{P}_0$ of the camera matrix. We solve it using a least squares method:

$$\begin{bmatrix} x_{cp_1} & y_{cp_1} & z_{cp_1} & 1 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{cp_n} & y_{cp_n} & z_{cp_n} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_{cp_1} & y_{cp_1} & z_{cp_1} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & x_{cp_n} & y_{cp_n} & z_{cp_n} & 1 \end{bmatrix} \cdot \begin{pmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \end{pmatrix} = \begin{pmatrix} u_{c_1} \\ \vdots \\ u_{c_n} \\ v_{c_1} \\ \vdots \\ v_{c_n} \end{pmatrix} \quad \Rightarrow \quad \mathbf{A} \cdot \mathbf{p} = \mathbf{c} \tag{8}$$

Where $\mathbf{A}$ is the coefficient matrix of the least squares system, $\mathbf{p}$ is the column vector of unknowns and $\mathbf{c}$ is the column vector of independent terms.

Before proceeding with the second stage, we will study the eigenvalues of matrix $\mathbf{A}$. This is done using different images of the grid distortion target, acquired with different magnifications. It is found in the determination of the eigenvalues of the system matrix $\mathbf{A}$, that for all cases studied, the least squares system is bad-conditioned (the eigenvalues vary from each other by several orders of magnitude). It is proposed to make a change of variables or normalization of the coordinates $\mathbf{X}_{cp} = \left(x_{cp}, y_{cp}, z_{cp}\right)^T$ and $\mathbf{U}_c = \left(u_c, v_c\right)$. Such change of variables consists of a translation, so the origin is the centroid of the image, followed by a scaling of the coordinates, so that the mean distance of the points to the origin will be equal to $\sqrt{2}$ in the case of 2D coordinates and equal to $\sqrt{3}$ in the case of 3D coordinates.

Because in the calibration procedure we use a two-dimensional grid distortion target which is parallel (or almost parallel) to the plane of the image, all points $\mathbf{X}_m$ of the target (world coordinate system) show identical coordinate z_m. By normalizing the coordinate $\mathbf{X}_m$ into $\tilde{\mathbf{X}}_m$ we verify that the coordinate $\tilde{z}_m$ is always equal to zero, regardless of the taken value. Therefore it was decided to eliminate the *z* coordinate so that the camera model established in point 2 is rewritten as:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_m \\ y_m \\ 1 \end{pmatrix} \tag{9}$$

expressed in reduced matrix form as:

$$\begin{aligned} \mathbf{U}_{hpi} &= \mathbf{K}\left[\mathbf{R} \mid -\mathbf{t}\right] \cdot \mathbf{X}_{hm} \\ \mathbf{U}_{hpi} &= \mathbf{P} \cdot \mathbf{X}_{hm} \end{aligned} \tag{10}$$

where $\mathbf{U}_{hpi} = \left(u, v, 1\right)^T$ are the coordinates in pixels of the “digital image system”, $\mathbf{X}_{hm} = \left(x_m, y_m, 1\right)^T$ are the coordinates of a point in the “world coordinate system”, and the matrix $\mathbf{P}$ is equal to:

$$\mathbf{P} = \begin{pmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & -t_x \\ r_{21} & r_{22} & -t_y \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$

This camera model has 11 parameters and 3 constraints of orthogonality in the rotation matrix $\mathbf{R}$. As an evolution of the affine camera model, the principal point of the image $\left(x_0, y_0\right)$ is not defined, so the coordinates of this must be assumed or removed from the equation. In a first approximation we will considered this point coincident with the center of the image.

The matrix $\mathbf{P}$ is a square matrix and for its definition is invertible, allowing conversion of the points from the "digital image coordinate system" to the "world coordinate system". The inverse matrix of $\mathbf{P}$ will be known as $\mathbf{Q}$, so that:

$$\begin{aligned}\mathbf{X}_{hm} &= \mathbf{Q}\cdot\mathbf{U}_{hpi}\\ \mathbf{X}_{hm} &= \left[\mathbf{R} \;\;|\;\; -\mathbf{t}\right]^{-1}\cdot\mathbf{K}^{-1}\cdot\mathbf{U}_{hpi}\end{aligned} \tag{12}$$

After this modification, the camera model is established and the system of equations defined by equation 8 is solved, having previously performed the change of variable proposed.

$$\tilde{\mathbf{A}}\cdot\tilde{\mathbf{p}} = \tilde{\mathbf{c}} \tag{13}$$

After the first calculation stage and based on the solution $\tilde{\mathbf{p}}$ and using a linear iterative resolution process we will solve the entire camera system, including the determination of distortion coefficients. We will minimize the normalized distance between the center of grid distortion target dots $\tilde{\mathbf{X}}_{cp_i} = \left(\tilde{x}_{cp_i}, \tilde{y}_{cp_i}\right)^T$ and the center of this points observed in the "digital image system" once it has been corrected its possible geometric distortion and have been transformed to the "world coordinate system" $\tilde{\mathbf{X}}_{c_i} = \tilde{\mathbf{Q}}\cdot\tilde{\mathbf{U}}_{c_i}$, with $\tilde{\mathbf{U}}_{c_i} = \left(\tilde{u}_{c_i}{}' - \tilde{\delta}_{u_i}{}', \tilde{v}_{c_i} - \tilde{\delta}_{v_i}{}'\right)$, where $\tilde{u}_c{}'$ and $\tilde{v}_c{}'$ are the raw coordinates with distortion. We tray to minimize:

$$\min\sum_{i=1}^{n} d\left(\tilde{\mathbf{X}}_{cp_i}, \tilde{\mathbf{X}}_{c_i}\right) \tag{14}$$

From this equation it deduces that the parameters to be optimized are the matrix $\tilde{\mathbf{Q}}$, the geometric distortions, $\tilde{\delta}_{u_i}{}'$ and $\tilde{\delta}_{v_i}{}'$, i.e., $\left(\tilde{q}_{11}, \tilde{q}_{12}, \tilde{q}_{13}, \tilde{q}_{21}, \tilde{q}_{22}, \tilde{q}_{23}, \tilde{k}_1, \tilde{k}_2, \tilde{p}_1, \tilde{p}_2, \tilde{s}_1, \tilde{s}_2, \tilde{x}_0, \tilde{y}_0\right)$.

After solving the least squares problem defined by equation 14 and after undoing the normalization performed to the coordinates the calibration matrix $\mathbf{P}$ and the geometric distortion coefficients will be obtained.

4. TESA Visio 300 calibration

Once developed the calibration procedure is necessary to calibrate the equipment TESA Visio 300 using images obtained with it. We will identify intrinsic and extrinsic parameters of the camera for different magnifications, the geometric distortion parameters, as well as a series of parameters used to evaluate the correctness of the resolution method: Quadratic Residual Distance (QRD), Mean Residual Distance (MRD), Maximum Residual Distance (MaxRD) and Standard Deviation of the Residuals Distances (SDRD) used by Luo et al. (2006). During the calibration, the temperature of the measurement area is in the range 20 ± 1 °C. A diffuse bright field transmitted lightning at 50% of its maximum light intensity value was used. Different magnifications (1X, 2X, 3X and 4.5X) were employed.

For each considered magnification 4 images were taken from different zones of the grid distortion target, so that we can minimize systematic errors. As an example we show in table 1 the results obtained for a 4.5X magnification.

Table 1. Calibration results for 4.5X magnification

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
$1/\alpha_x$ [μm/pixel]	1.903	x_0 [pixel]	382.7	p_1	$1.0\cdot10^{-7}$	QRD [μm^2]	2.3
$1/\alpha_y$ [μm/pixel]	-1.901	y_0 [pixel]	250.1	p_2	$-4.5\cdot10^{-7}$	MRD [μm]	0.1
s	-0.101	k_1	$9.3\cdot10^{-9}$	s_1	$-6.8\cdot10^{-7}$	MaxRD [μm]	0.3
----	----	k_2	$-2.2\cdot10^{-14}$	s_2	$2.9\cdot10^{-7}$	SDRD [μm]	0.06

5. Calibration camera and geometric distortion parameters uncertainty calculation

In view of the calculation model for the camera parameters previously developed, a numerical method, in this case the document JCGM (2008) (Monte Carlo method), is the best solution for the calculation of uncertainties. The main stages to be followed to use the Monte Carlo method are:

5.1. Define the input variables

These will be the coordinates of the centers of grid distortion target dots obtained from the calibration certificate, as well as the coordinates of the pixels that define the edges of grid distortion target dots observed in the image.

5.2. Define the output variables

Taking into account equation 12, the output quantities are: $\mathbf{K}^{-1}, x_0, y_0, k_1, k_2, s_1, s_2, p_1, p_2$ that is, the inverse matrix of the camera calibration, the principal point of the image and the geometric distortion coefficients. These magnitudes are correlated, since for its calculation we use the same input variables.

5.3. Assigning the Probability Distribution Function (PDF) of the input quantities

For the two input variables previously defined, we will establish their PDF. For the coordinates of the centers of grid distortion target dots, and employing the information of the calibration certificate, these coordinates correspond to a Gaussian distribution. For the coordinates of the pixels that define the edges of the grid distortion target dots observed in the image, and considering the work of Anchini et al.(2007), it is assumed that the distribution of these coordinates may respond to a triangular distribution, that respect to the most probable value of the pixel coordinate may vary ± 1 pixel. These values of the distribution must take integer values.

5.4. Propagation

To make the propagation of model, the model will be replicated a number of times M equal to 20000. Due to the correlation of the output quantities it is necessary to determine the matrix of correlation coefficients of these magnitudes.

In table 2 is shown by way of example some of the results obtained for a 4.5X magnification.

Table 2. Uncertainties calculation results of the camera calibration parameters

Parameter	Standard uncertainty	Shortest 95% coverage interval	
		Lower limit	Upper limit
$1/\alpha_x$ [μm/pixel]	$2.953\cdot10^{-6}$	0.001898	0.001905
$1/\alpha_y$ [μm/pixel]	$6.011\cdot10^{-6}$	-0.001903	-0.001896
k_1	$1.724\cdot10^{-9}$	$-2.882\cdot10^{-8}$	$3.120\cdot10^{-8}$

6. Conclusions

We have been developed a:

- Camera model for metrological optical equipment whose measurement plane XY is parallel (or almost parallel) to the image plane of the camera. This model contains all intrinsic and extrinsic parameters of the camera.

- Calibration procedure camera by using a grid distortion pattern. This procedure is able to correct the geometric distortions introduced by the "optical system" as well as determining the extrinsic and intrinsic parameters of the camera.

- Procedure for calculating the uncertainty of the camera parameters based on the Monte Carlo method.

Machine under study has been calibrated using the procedures and models developed in this work and found that the camera has minimal geometric distortion for any magnifications in which it has been calibrated.

References

Anchini, R., Liguori, C., Paolillo, A., 2007. Evaluation of the uncertainty of edge-detector algorithms. Instrumentation and Measurement, IEEE Transactions 56, 681–688.

Gonzalez, R. C., Woods, R. E., 2008. Digital Image Processing. Pearson, New Jersey.

Hartley, R., Zisserman, A., 2004. Multiple View Geometry in Computer Vision. Cambridge University Press , Cambridge

Joint Committee for Guides in Metrology (JCGM), 2008. JCGM 101:2008 Evaluation of measurement data — Supplement 1 to the "Guide to the expression of uncertainty in measurement" — Propagation of distributions using a Monte Carlo method.

Luo, H., Zhu, L., Ding, H., 2006. Camera calibration with coplanar calibration board near parallel to the imaging plane. Sensors and Actuators A: Physical 132, 480 – 486.

Maresca, P., Gómez, E., Caja, J., Barajas, C., Berzal, M., 2010. Use of coordinate measuring machines and digital optical machines for the geometric characterization of circumference arcs using the minimum zone method. Measurement 43, 822–836.

Santo, M. D., Liguori, C., Paolillo, A., Pietrosanto, A., 2004. Standard uncertainty evaluation in image-based measurements. Measurement 36, 347 – 358.

Tsai, R., 1987. A versatile camera calibration technique for high-accuracy 3d machine vision metrology using off-the-shelf tv cameras and lenses. Robotics and Automation. IEEE Journal 3, 323–344.

Weng, J., Cohen, P., Herniou, M., 1992. Camera calibration with distortion models and accuracy evaluation. IEEE Transactions on pattern analysis and machine intelligence 14, 965–980.