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Thermodynamics of the Schwarzschild and the Reissner–Nordström black holes with quintessence

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Abstract

In this paper, we study the thermodynamics of the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence. By using the thermodynamical laws of the black holes, we derive the thermodynamic properties of these black holes and we compare the results with each other. We investigate the mass, temperature and heat capacity as functions of entropy for these black holes. We also discuss the equation of state of the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence.

1. Introduction

Black hole thermodynamics is one of the interesting subjects in modern cosmology and is the area of study that seeks to reconcile the laws of thermodynamics with the existence of black hole event horizons which is widely studied in the literature. The seminal connections between black holes and thermodynamics were initially made by Hawking and Bekenstein [1]. Black holes behave as thermodynamic objects which emit radiation from the event horizon by using the quantum field theory in curved space–time, named as Hawking radiation with a characteristic temperature proportional to their surface gravity at the event horizon and they have an entropy equal to one quarter of the area of the event horizon in Planck units [1]. As we know, the main

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laws of thermodynamics have close analogies in the physics of black holes. For example, the second law of thermodynamics is analogous to the second law of black hole dynamics (area law) which implies that the surface of a black hole cannot decrease. The Hawking temperature, entropy and mass of the black holes satisfy the first law of thermodynamics [1].

The expansion of the Universe is a long-established fact and there are significant astronomical evidences that the universe is expanding at an accelerating rate. The current cosmological observation predicts the existence of some form of energy which permeates all of space with a large negative pressure [2–6], called dark energy which constitutes about 70 percent of the energy density of the universe.

Dark energy is a complete mystery and the evidence for it is indirect and understanding the origin of this negative pressure is one of the biggest efforts in cosmology today. There are two proposed forms for dark energy. The first and the simplest explanation for dark energy is the cosmological constant [7] with a constant equation of state $\omega_q = -1$ and the second is the dynamical scalar field models such as quintessence [8], chameleon [9], K-essence [10], tachyon [11], phantom [12] and dilaton [13]. Basically, the difference between these models returns to the magnitude of ω_q which is the ratio of pressure to energy density of dark energy and for quintessence $-1 < \omega_q < -\frac{1}{3}$.

Black holes surrounded by dark energy are believed to play the crucial role in cosmology and one of the important characteristics of a black hole is its thermodynamical properties and also it is interesting to know how does the dark energy affect the thermodynamics of the black holes. Quintessence as one candidate for the dark energy is defined as an ordinary scalar field coupled to gravity [14]. Kiselev [15] by considering the Einstein's field equations for a black hole charged or not and surrounded by quintessence, derived a new solution related to ω_q.

In the present work, we study the thermodynamics of the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence matter by using the solution obtained by Kiselev [15] and we derive the thermodynamic properties of these black holes and we compare the results with each other.

The outline of this paper is as follows: In section 2, we briefly review the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence. In section 3, we discuss the thermodynamic quantities of these black holes, while a conclusion is given in section 4.

2. Schwarzschild and Reissner–Nordström black holes surrounded by quintessence

Kiselev derived a static spherically symmetric exact solution of Einstein equations for a black hole surrounded by the quintessence [15]. The geometry of this black hole can be expressed as,

$$ds^2 = -g(r)dt^2 + g(r)^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{1}$$

where for the Schwarzschild black hole, $g(r)$ is given by,

$$g(r) = 1 - \frac{2M}{r} - \frac{c}{r^{3\omega_q+1}} \tag{2}$$

and for the Reissner–Nordström black hole, $g(r)$ is defined as,

$$g(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{c}{r^{3\omega_q+1}} \tag{3}$$

where M and Q are the mass and charge of the black hole. ω_q is the quintessential state parameter which has the range $-1 < \omega_q < -\frac{1}{3}$ and c is the positive normalization factor dependent on $\rho_q = -\frac{c}{2}\frac{3\omega_q}{r^{3(1+\omega_q)}}$, and ρ_q is the density of quintessence which is always positive.

In this paper, we are going to study the thermodynamics of the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence in detail corresponding to the choice of $\omega_q = -\frac{2}{3}$. Then for the Reissner–Nordström black hole we have,

$$g(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - cr \tag{4}$$

which due to the complexity of the equations and simplify them, we consider the case,

$$Q^2 = \frac{2}{27}\frac{-1 + 9Mc + \sqrt{-(6Mc-1)^3}}{c^2}. \tag{5}$$

Then $g(r)$ for the Reissner–Nordström black hole leads to,

$$g(r) = 1 - \frac{2M}{r} + \frac{\frac{2}{27}\frac{-1+9Mc+\sqrt{-(6Mc-1)^3}}{c^2}}{r^2} - cr \tag{6}$$

and therefore the Reissner–Nordström black hole for $6Mc < 1$ has the following horizons,

$$r_{in} = \frac{1}{3}\frac{-\frac{1}{2}(-(6Mc-1)^3)^{\frac{1}{6}} + \frac{1}{2}\frac{6Mc-1}{(-(6Mc-1)^3)^{\frac{1}{6}}} + 1 + \frac{1}{2}i\sqrt{3}((-(6Mc-1)^3)^{\frac{1}{6}} + \frac{6Mc-1}{(-(6Mc-1)^3)^{\frac{1}{6}}})}{c} \tag{7}$$

$$r_{out} = \frac{1}{3}\frac{(-(6Mc-1)^3)^{\frac{1}{6}} - \frac{6Mc-1}{(-(6Mc-1)^3)^{\frac{1}{6}}} + 1}{c}, \tag{8}$$

where $i = \sqrt{-1}$. For the Schwarzschild black hole surrounded by quintessence, $Q = 0$ and this black hole for $8Mc < 1$ has two horizons as,

$$r_{in} = \frac{1 - \sqrt{1 - 8Mc}}{2c} \tag{9}$$

$$r_{out} = \frac{1 + \sqrt{1 - 8Mc}}{2c} \tag{10}$$

Fig. 1 shows the difference between horizons of the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence matter.

By considering $\omega_q = -\frac{2}{3}$, the density of quintessence as a function of c at the event horizon of the black hole can be expressed as,

$$\rho_q = \frac{c}{r_{out}}. \tag{11}$$

The density of quintessence as a function of c at the event horizon for the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence matter is shown in Fig. 2. This figure illustrates that the density of quintessence for the Schwarzschild black hole surrounded by quintessence is higher than for the Reissner–Nordström black hole.

3. Thermodynamic quantities of the black hole

By using the thermodynamical laws of the black holes, we can derive the thermodynamic properties of the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence. The relation between the Einstein equation and the first law of thermodynamics has been generalized to the cosmological context.

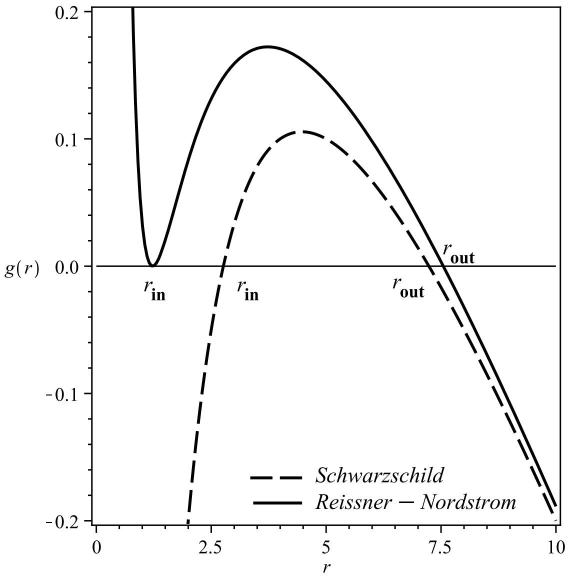

Fig. 1. Horizons of the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence matter with $M = 1$ and $c = 0.1$.

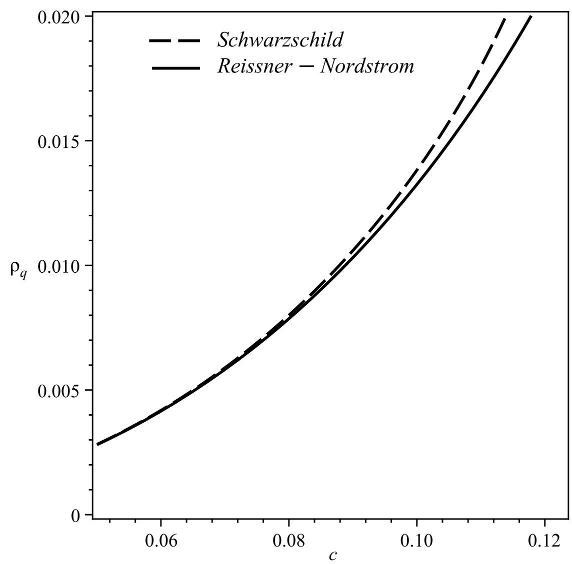

Fig. 2. Density of quintessence ρ_q as a function of c for the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence with $M = 1$.

The black holes have an entropy equal to one quarter of the area of the event horizon and we know that the entropy can be written as [1],

$$S = \frac{A}{4} = \frac{4\pi r_{out}^2}{4} = \pi r_{out}^2 \tag{12}$$

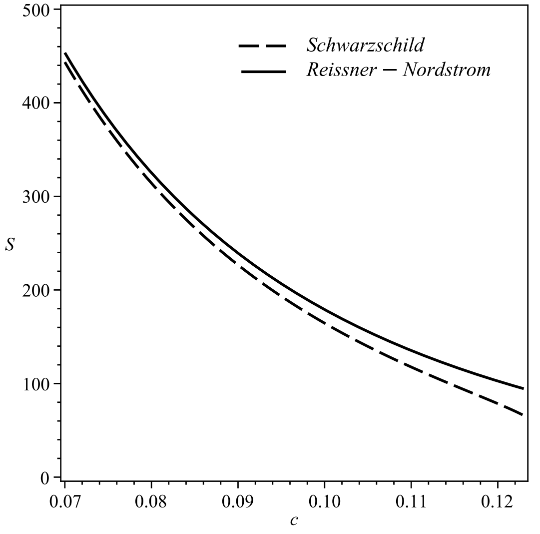

Fig. 3. Entropy as a function of c for the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence with $M = 1$.

The graphs for the entropy as a function of c for the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence matter are given in Fig. 3. The entropy with the same c parameter is higher for the Reissner–Nordström black hole surrounded by quintessence in comparison with the Schwarzschild black hole surrounded by quintessence.

We can establish the relation between the density of quintessence ρ_q and the entropy of the black hole from (11) and (12) as,

$$\rho_q = c\sqrt{\frac{\pi}{S}} \tag{13}$$

which behavior of the density of quintessence ρ_q as a function of entropy with the different values of c is plotted in Fig. 4 which all of them represent decreasing function.

The relation between mass of the Reissner–Nordström black hole surrounded by quintessence and its horizon radius with $\omega_q = -\frac{2}{3}$ can be expressed as,

$$M = \frac{1}{2}\left(r_{out} + \frac{Q^2}{r_{out}} - cr_{out}^2\right) \tag{14}$$

By using Eq. (12), the above equation leads to,

$$M = \frac{1}{2}\left(\sqrt{\frac{S}{\pi}} + Q^2\sqrt{\frac{\pi}{S}} - c\frac{S}{\pi}\right) \tag{15}$$

where for the Schwarzschild black hole $Q = 0$, and for the Reissner–Nordström black hole Q^2 is given by Eq. (5). Then we can obtain the mass of the Reissner–Nordström black hole surrounded by quintessence as a function of S and c from the following equation,

$$M = RootOf\left(\sqrt{\frac{S}{4\pi}} - c\frac{S}{2\pi} + \sqrt{\frac{\pi}{S}}\frac{-1 + 9Xc + \sqrt{-(6Xc-1)^3}}{27c^2} - X = 0\right). \tag{16}$$

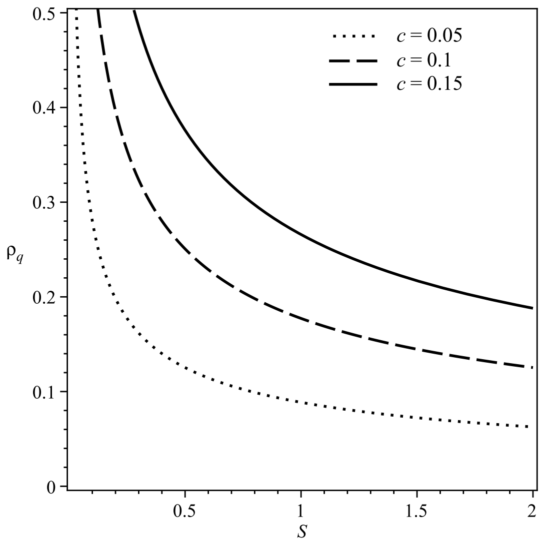

Fig. 4. Behavior of ρ_q as a function of S with the different values of c.

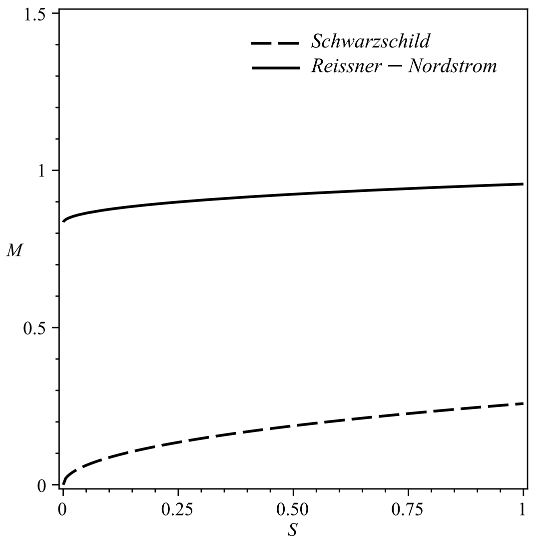

Fig. 5. Variation of M as a function of S for the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence with $c = 0.12$.

Fig. 5 shows the numerically plotting variation of mass as a function of entropy for the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence. From this figure we can see that the mass of the Reissner–Nordström black hole surrounded by quintessence with the same entropy is higher than for the Schwarzschild black hole surrounded by quintessence.

The first law of the black hole thermodynamics can be expressed as [1],

$$dM = TdS + \Phi dQ \tag{17}$$

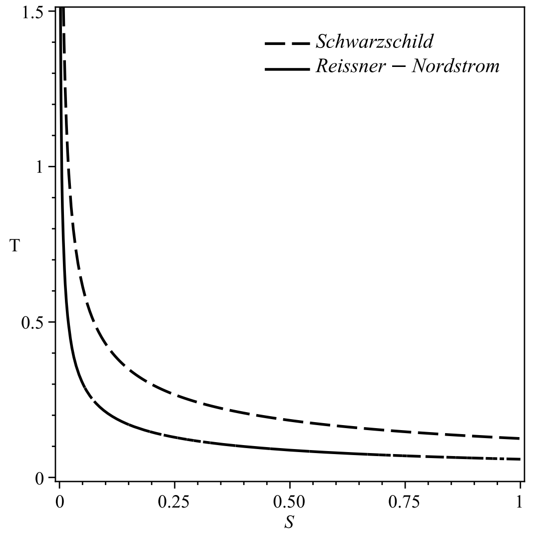

Fig. 6. Temperature T as a function of S for the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence with $c = 0.1$.

where T and Φ are the temperature and electrostatic potential of the black hole. We can derive the temperature and heat capacity of the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence from the following equations,

$$T = \frac{\partial M}{\partial S} \tag{18}$$

$$C = T\frac{\partial S}{\partial T}. \tag{19}$$

The graphs for the temperature T and heat capacity C as a function of S for the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence matter are given in Fig. 6 and Fig. 7. These figures indicate that the temperature with the same S is higher for the Schwarzschild black hole surrounded by quintessence in comparison with the Reissner–Nordström black hole surrounded by quintessence, while the heat capacity of the Reissner–Nordström black hole surrounded by quintessence with the same entropy is higher than for the Schwarzschild black hole surrounded by quintessence.

The relation between pressure P and c parameter can be written as [16],

$$P = -\frac{c}{8\pi} \tag{20}$$

and volume V of the black hole with $\omega_q = -\frac{2}{3}$, becomes [16],

$$V = 4\pi r_{out}^2 \tag{21}$$

then by using the equations (12) and (21), the entropy can be expressed as,

$$S = \frac{V}{4}. \tag{22}$$

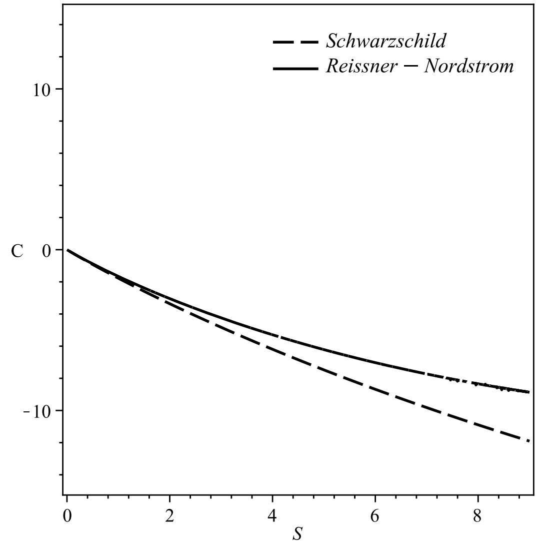

Fig. 7. Heat capacity C as a function of S for the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence with $c = 0.1$.

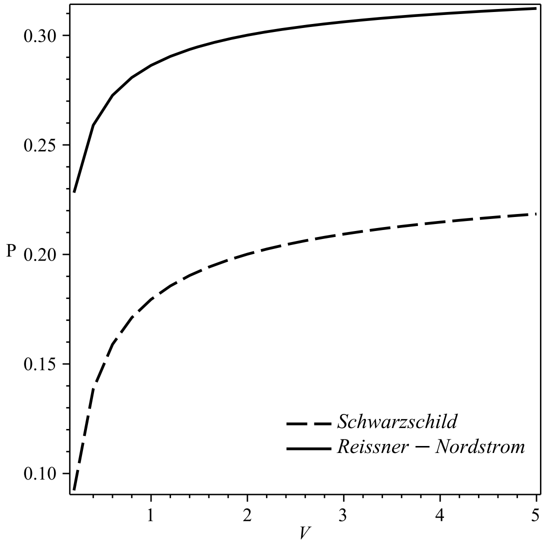

Fig. 8. $P - V$ isotherms for the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence with $T = 1$.

By considering $c = -8\pi P$, $S = \frac{V}{4}$ and rewriting the equation of temperature T as a function of P and V, we can obtain the equation of state $P - V$ for the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence. Fig. 8 shows the $P - V$ isotherms for these black holes. The pressure P with the same volume V is higher for the Reissner–Nordström black hole surrounded by quintessence in comparison with the Schwarzschild black hole surrounded by quintessence.

4. Conclusion

In this paper, by using the thermodynamical laws of the black holes, we have investigated the thermodynamic properties of the Schwarzschild and the Reissner–Nordström black holes surrounded by quintessence and we compare the results with each other. We showed that the density of quintessence for the Schwarzschild black hole surrounded by quintessence is higher than for the Reissner–Nordström black hole surrounded by quintessence. It is shown that the entropy with the same c parameter is higher for the Reissner–Nordström black hole surrounded by quintessence in comparison with the Schwarzschild black hole surrounded by quintessence. The variation of mass, temperature and heat capacity as functions of entropy are plotted. We also have shown that the mass and heat capacity of the Reissner–Nordström black hole surrounded by quintessence with the same entropy are higher than for the Schwarzschild black hole surrounded by quintessence, while the temperature with the same S is higher for the Schwarzschild black hole surrounded by quintessence in comparison with the Reissner–Nordström black hole surrounded by quintessence. Finally, we have plotted the $P - V$ isotherms for these black holes and we showed that the pressure with the same volume is higher for the Reissner–Nordström black hole surrounded by quintessence in comparison with the Schwarzschild black hole surrounded by quintessence.

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