Beltrami went to Venice in 1871 to help his wife recover from some sort of crisis, and wrote to Hoüel (letter 37) that "I never cease to be pained by the fatal indifference of the new generation to anything that is serious and a little difficult." Plus ça change? Beltrami was then 35.

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This is a magnificent book, and a very scholarly book as well. First, it is a joy to admire the reproductions of the scroll; reading the accompanying text, one is captured by the explanations and information which treat much more than only the scroll. What is it all about? The scroll, preserved at the Topkapi Palace Museum Library and reproduced in its entity in this volume, is a pattern book from the workshop of a master builder. It was probably compiled in the late 15th or 16th century somewhere in western or central Iran, probably in Tabriz, a city which once rivaled the Timurid capitals of Samarqand and Herat in architectural splendor. Because of the nearly complete destruction of monuments in Tabriz, the scroll's patterns might be seen as precious mementos of a now-lost archaeological record. How the scroll found its way into the Ottoman imperial treasury collection is not easy to answer. The Ottomans conquered Tabriz several times, on which occasions many skilled artisans and scholars as well as treasures and manuscripts were taken from Tabriz to Istanbul. The scroll could also have been brought to Istanbul by such personages as the Timurid astronomer–mathematician Ali Kuşci, who joined the court at Istanbul in 1472 with a large manuscript collection including mathematical treatises. Another possibility is that it belonged to Timurid–Turkmen decorators and workmen who were invited to the court in the 1470s. If the Topkapi scroll had been used in the office of the chief architect, it would probably have disappeared together with all the architectural drawings cited in the Ottoman sources. Having been deposited in the Inner Treasury of the Topkapi Palace and then forgotten, thanks to its minor relevance to classical Ottoman architecture, it is preserved in superb condition. The implications of the scroll for architectural practice in the late Timurid–Turkmen and early Safavid worlds are of great importance. The Topkapi scroll is a unique document.

Necipoğlu's book has won an award from the U.S. Society of Architectural Historians and was named the best new scholarly book in the field of architecture and urban planning by the American Association of Publishers. It contains encyclopaedic knowledge on the history of practical mathematics in the Arab world and Islamic architecture and should be read by everyone studying Islamic architecture. But what does "Islamic architecture" mean? As we can define "Christian architecture" mainly by churches, chapels, monasteries, etc., so we can define "Islamic architecture" by the principal forms of Islamic buildings, not only, but mainly religious buildings. The oldest of these buildings are mosques which date from the first beginnings of Islam. Style and methods of construction change from generation to generation, especially as regards the choice of material, the gates, the façades and minarets, the profile of the arches in the interior, and the ornamentation. But the ground-plan of the mosque remains largely the same. This principle is valid for most types of religious buildings.

Besides the Madrasa (the second important religious building), monasteries, drinking fountains and elementary schools, the Mausoleum, or Qubba, is an important Islamic monument. From the earliest traceable time, most qubbas have had their own style: a cubic room on a square base with a vaulted roof. The most famous building deviating from the traditional plan is the Dome of the Rock in Jerusalem. The problem of erecting a cupola on a square base finds in Muslim architecture a variety of solutions. The space of transition from the square, or polygon, to the circle is occupied either by planks covered with stucco, or by cornertrompes made at first of bricks and later of stone, or by stalactite pendentives called muqarnas. These pendentives are also found on the lower parts of domes and in the vaulting of arches, especially in the vault of the Ivan used as an entrance passage. Muqarnas is a typical element of Islamic architecture. Mohammad al-Asad (p. 349) describes the muqarnas as follows:

> The muqarnas is a vaulting system based on the replication of units arranged in tiers, each of one supports another one corbeled on top of it. The final result is a stairlike arrangement that is sometimes referred to as honeycomb or stalactite vaulting. The units are made of wood, brick, plaster, or stone and can be painted, or, as in the case of the brick or plaster ones, covered with glazed tiles. Muqarnas compositions can be located in different parts of a building, articulating a column capital, supporting a minaret's balcony, or vaulting over an entry portal, niche, or hall. Muqarnas vaults are usually part of a double-shell arrangement and are therefore visible only from the inside of a building. In some cases, as in the mausoleums of Nur al-Din in Damascus (1172) and Imam Dur in Samarra (circa 1085), the muqarnas is also reflected on the outside. . . . The muqarnas can fulfill a structural role. The muqarnas vaults of the main dome of the fourteenth-century Cairene madrasa of the Sultan Barquq (c. 1382–1398), for example, function as pendentives providing a transitional zone between the square base and the circular dome. Most muqarnas vaults, however, are purely decorative, and their units are suspended from, or connected to, a structural element.

Al-Asad's explanations are accompanied by a computer-generated model. These pictures greatly facilitate the understanding of muqarnas structures. They show how a three-dimensional complex structure originates based on a two-dimensional geometrical grid.

The most essential feature in Islamic architecture is the ornament, the decoration of walls and cupolas, in Western culture referred to as the "Arabesque." Besides the ornament consisting of foliage only, geometrically intertwined bands are similarly combined with foliage. The effect aimed at is the creation of a contrast: a linking up of the positive patterns by means of smaller portions of the surface. A wealth of polygonal shapes or stars appear as the fixed points in a mass of small irregular polygons. The systems most favored are those founded on polygons or stars with an odd number of angles, e.g., pentagons, heptagons, nonagons, or stars with seven and more, even fifteen points. A certain advanced stage of the power of geometrical vision is an indispensable presupposition of this kind of composition.

What we find in the Topkapi scroll are such patterns, patterns for ornaments and patterns to be used as a ground-plan for muqarnas. The scroll is a high-level design book for architects, builders, and artisans. The present book is mainly concerned with the cultural and historical contexts in which these patterns were created, together with their implications for aesthetic theory.

The book is organized in five parts: Part 1, The Scroll Tradition; Part 2, The Discourse on the Geometric "Arabesque"; Part 3, The Geographical, Chronological, and Semantic Horizons of the Geometric Mode; Part 4, Geometry and the Contribution of Mathematical Sciences; and Part 5, Geometry and Aesthetic Theory. Moreover, it contains the color reproduction of the Topkapi Scroll, a catalog of pattern types and drawings in the scroll, a geometric analysis of the muqarnas by Mohammad al-Asad, a map of the Islamic world, and an extensive bibliography. Part 4 is the most interesting from the point of view of the history of mathematics, but the other parts also make for fascinating reading. In fact, it took the reviewer a long time to finish the review, always wanting to pursue details in the different chapters. The book should be an ideal base for a high-level seminar.

The Topkapi scroll, as laid down in Part 1, is the best-preserved example of its kind, with far-reaching implications for the theory and praxis of geometric design in Islamic architecture and ornament. There are no known Islamic architectural working drawings from the pre-Mongol era despite occasional textual references to plans. The earliest known example of a construction plan is a 50 cm stucco plate showing the projection of a quarter muqarnas vault which was found at the Takht-i Suleiman. This plate from the 1270s was the basis on which Ulrich Harb explained the reconstruction of the collapsed vault. Fourteenth-century sources frequently mention architectural drawings produced either on clay tablets or on paper. In the 15th century, Timurid world drawings seem to have been more widely used than ever before. Their extensive use had become essential by the increasing elaboration of geometric design. Up until Necipoğlu's discovery of the Topkapi scroll the earliest known examples of such architectural drawings were a collection of fragmentary post-Timurid design scrolls of 16th-century Samarqand paper, retained at the Uzbek Academy of Sciences in Tashkent. These scrolls almost certainly reflect the sophisticated Timurid drafting methods of the 15th century. In 1876, the English architect, C. Purdon Clarke, brought back from Teheran some scrolls and working drawings from the 18th and 19th centuries that he had collected following the death of the official state architect, Mirza Akbar; these scrolls are now preserved in the Victoria and Albert Museum. In 1981 similar material, still in the hands of the master-artisan, was examined by W. K. Chorbachi in two Arab towns. These scrolls were not only the basic reference manual but also served as a design book. The reviewer visited a workshop at Fez/Morocco a few years ago, where the artisans used a construction plan for a muqarnas on a 1–1 base. The pieces cut out for constructing the muqarnas could actually be put on the draft so that the cross-section of the element, i.e., the cross-section of the beam, matched exactly the figure on the draft. The Timurid scrolls show a decisive switch to the far more complex radial muqarnas with an increasing variety of polygons and star polygons. Also, the Akbar scrolls are more elaborate than the

20th-century Fez drawing. Despite their simplicity, however, the more recent scrolls testify to a relatively unbroken tradition of architectural practice in Central Asia from at least the Timurid period onward. A continuous tradition from the 13th-century Takht-i Suleiman plate to the 20th-century Fez drawing is evident. As in the Ilkhanid period, 700 years earlier, the plane projection of the elements in the Moroccan plan consists of simple geometrical figures: squares, half-squares, rhombuses, half-rhombuses, rectangles, almonds, bipeds.

The contents of the Topkapi and Tashkent scrolls support the commonly expressed view that the key to Timurid–Turkmen architecture lies not only in its fascination with complicated vaulting systems but also in its extensive surface decoration. The numerous two-dimensional geometric patterns and epigraphic compositions in these scrolls condense complex compositions into shorthand formulas meant to act as guidelines for the simpler working methods employed on the construction site. They thus provide a valuable glimpse into the processes of design and execution. The contents of the Topkapi scroll, which resemble those of a pattern book, can be seen as an index of the unprecedented Timurid–Turkmen emphasis on surface decoration, an emphasis that turned the flat façades of buildings into stage props for the display of virtuoso ornamental panels and fragmented vaults into multifaceted compartments with no structural role. The Topkapi scroll reflects a “painterly” aesthetic of architecture informed by the cultural prestige of drawings on paper.

The secondary literature on Islamic geometric ornament is reviewed in Part 2. The first chapter, Chapter 4 of the book, analyzes the 19th- and early 20th-century European literature on ornament within which the discussion of Islamic geometric patterns often was embedded. The European fascination with Islamic ornament started in the Middle Ages with the German Benedictine monk, Theophilus, who was a practicing craftsman himself. In his 12th-century technical manual, *De diversis artibus*, he included a section on metalwork explaining “whatever Arabia adorns with repoussé or cast work, or engravings in relief.” Arabesques appeared in pattern books published from the early 16th century onward. This fascination reached its peak in the 19th century and came to play a central role in the critical discourse on design in 19th-century Europe. The relevant publications are discussed and special attention is given to the first serious study, *The Grammar of Ornament*, published by Owen Jones in 1856. Other important authors include Racinet, Bourgoin, Viollet-le-Duc, and Prisse d’Avennes. From the early 20th century onward, the study of Islamic art and architecture developed into a specialized academic field in which orientalist, archeological, and museum or collections research led to an unprecedented accumulation of information. These academic studies, which grew out of the orientalist discourse, increasingly turned away from the ethnoracial categories encountered in 19th- and early 20th-century writings and replaced them with regional, dynastic, and chronological–stylistic ones. These recent studies on geometric ornament are discussed in Chapter 5. Important authors are Herzfeld, Ernst Kühnel, Massignon, Marçais, Ettinghausen, and Oleg Grabar. There is also a tendency to attribute mystic meanings to the arabesque, as found in the work of Titus Burckhardt, Nasr, or Gombrich.

Necipoğlu focuses on the geometric mode, the main subject of the Topkapi scroll, in Part 3. She outlines the scroll’s spatial and temporal horizons together with its contextual associations, as far as is possible given the serious gaps in the early Islamic archeological record. Even though there is no evidence in the primary sources to support the stereotyped notion that nonfigural visual idioms were considered the "essence" of the arts in Islamic lands, religious constraints did place an unprecedented emphasis on them. In the Islamic world, the preference for providing monuments with a decorative surface is almost as old as the formation of the earliest architectural traditions. The decorative schemes used for the early monuments consisted primarily of vegetal designs showing Mediterranean influences, with geometric patterns playing only a limited role in their compositions. The ubiquity of geometric design was a later development, generally dated to the 10th century. It is believed to have had its origins in Baghdad, the primary cultural center of the Islamic world at the time. Chronologically, it corresponds to the remarkable developments which Muslims achieved in the disciplines of mathematics and geometry. These geometrically organized two-dimensional decorative patterns reflect one of the most regularized applications of geometry in the architecture of the Islamic world. They are organized according to rectilinear or radial grids, in which the circle and its polygonal and star-shaped derivatives play a prominent role. Although modular units are highly regularized, considerable flexibility exists in organizing whole compositons, since the arabesque consists of multidirectional repetitions of basic units. Geometric principles were also used for the generation of various three-dimensional architectural elements, such as the muqarnas. The formal organization of the muqarnas is closely linked to that of two-dimensional geometric patterns, and a muqarnas composition can be viewed as a stereometric projection of these patterns.

The author finds it only natural to assume that the *girih*, a highly codified mode of geometric patterning with a distinctive repertoire of algebraically definable elements, was preferred for some reason over other forms of nonfigural design. The particular kind of geometry used lies at the heart of the problem. The *girih* mode, with its two- and three-dimensional formulations compiled in surviving examples of pattern scrolls, is characterized by its self-consciously limited vocabulary of familiar, almost emblematic, star-and-polygon compositions generated by invisible grid systems that eliminated a broad spectrum of alternative geometric designs. What, Necipoğlu asks, made it so appealing to the sensibilities of the medieval Islamic world and what were the circumstances of its invention? Her answer is that the *girih* mode suddenly flourished in the context of the Sunni revival during the hegemony of the Great Seljuqs in late 10th-century Baghdad, from which it spread to other courts. The notable resistance to the geometric mode in areas dominated by the legacy of the Fatimid and the Spanish Umayyad caliphs provides support for this thesis. It is therefore not a coincidence that geometric patterns and the muqarnas spread from the east to North Africa and Spain during the reign of the Almoravids (1056–1147). In these countries the two- and three-dimensional geometric patterns underwent an internal development, culminating in the refined polychromatic revetment aesthetic of the Alhambra. Necipoğlu outlines how the *girih* mode was developed in a specific time and place and spread from there to other places, with its high point falling between the early 11th and mid-13th centuries and its last creative impulse manifested in the 14th- to early 16th-century monuments of Nasrid Spain, Mamluk Egypt, and the Timurid–Turkmen east. The Topkapi scroll, datable to the late 15th- or 16th-century Iranian world, therefore represents a final regional culmination of the *girih*.

During the 10th and 11th centuries, the professional mathematicians of the Baghdad school disseminated theoretical advances in their field by simplified practical manuals addressing the application of mathematics to everyday needs. This is discussed in Part 4 of the book. It was the popularization of the mathematical sciences that facilitated the application of geometry to design. The mathematician and astronomer, Abu al-Wafa' al-Buzjani (940–998), who had emigrated from his hometown in Nishapur to Baghdad in 959–960, was an important scholar involved in this systematic process of popularization that continued well into the 11th century. During his more than two decades of political experience in Buyid Baghdad, this celebrated mathematician had developed close connections with administrators, secretaries, land surveyors, merchants, architects, artisans, and calligraphers whose special needs he addressed in several practice-oriented manuals and in theoretical and practical mathematics courses that had many auditors. One of these practical manuals, entitled *About That Which the Artisan Needs to Know of Geometric Constructions*, seems to have been based on applied geometry excercises solved in class meetings. The manual addressed the perception that practitioners in various fields of design required better grounding in the methods of constructive geometry. His practical geometry outlines basic mechanical methods for constructing, proportionally subdividing, and symmetrically multiplying geometric figures, further simplified by the use of only a single opening of the compass. It reduces complex problems that involve conic sections, such as the trisection of an angle or the doubling of a cube, to simple mechanical procedures providing approximate solutions. Its direct relevance for the composition of Islamic geometric patterns was first noted by Prisse d'Avennes, followed by several modern scholars. Prisse d'Avennes mentions Franz Woepcke, who studied the early Persian translation included in a mathematical anthology preserved at the Bibliothèque nationale, Paris. Appended to this Persian translation is the thus far only known practical manual that provides "how-to" instructions for drawing two-dimensional *girih* patterns, the anonymous treatise *On Interlocking Similar or Congruent Figures*. This 20-page "how-to" manual, providing instructions for the construction of 61 repeat units for planar geometric patterns, is of particular interest because it shows how the general principles outlined in the practical geometries of al-Buzjani and others were used in developing interlocking star-and-polygon compositions. Despite their striking parallels, the Topkapi scroll and *On Interlocking* . . . embody different design repertories consistent with their respective times of compilation. The latter document, for example, does not include any radial arch-net or muqarnas vault projections, so characteristic of the Topkapi and Tashkent scrolls. Necipoğlu concludes that the Topkapi scroll is a corpus of schemata accumulated over the centuries that reflect the collective memory of master builders who endlessly refined early medieval *girihs* but still possessed a creative geometric imagination, nourished by manuals of practical geometry, workshop drawings, and empirical experience acquired during apprenticeship.

The many Arabic treatises on conic sections, written after Apollonius of Perga's *Conica* was translated, often deal with problems of practical application. Thabit ibn Qurra (d. 901) wrote a treatise on the mensuration of paraboloids, which includes a discussion of the "parabolic dome" created by rotating different sections of a parabola, accompanied by drawings of dome profiles. Treatises on the mensuration of parabolas and paraboloids were written by such mathematicians as Ibrahim b. Sinan (909–946), al-Sijzi (fl. 969–999), Abu Sahl al-Quhi (fl. 980–1000), and Ibn al-Haytham (965–ca. 1040), some of them including sections on arches, vaults, and domes. As Necipoğlu observes, their contribution to architectural practice and to the decorative arts awaits assessment by historians of science so that the ways in which theory and praxis interacted can be understood more clearly.

In his treatise, *Key of Arithmetic*, the Timurid astronomer and mathematician, al-Kashi (d. 1429), also includes an extensive chapter on mensuration, dealing with the areas of plane and solid geometric figures after which he treats architectonic structures. Rather than present instructions on how to design architectonic structures, al-Kashi taught the method for calculating their surface areas by means of approximate values. In addition to facilitating estimates of wages and building materials before construction, al-Kashi's formulas may also have been used in appraising the price of a building after its completion. It has been reported that the chief royal architect in Safavid Iran determined the value of completed buildings by measuring their walls in cubits, a value on the basis of which he was paid a fixed percentage.

In Part 5 is shown how the conception of architecture as a dramatic interplay of structure, ornament, color, sculptural effects, textures, and light presupposed the subjective processes of visual perception, which was consistently stressed in the primary sources. Despite their high status throughout Islamic history, architecture and the decorative arts did not generate a separate body of nonmathematical theoretical writing from which one could deduce their aesthetic principles. In the Islamic world, concepts of beauty were often embedded in metaphysical discussions. The relevance of such philosophical texts for architectural and artisanal production has not yet been explored systematically. Besides proportion, the aesthetics of light also played an important role in medieval descriptions of beauty, among which that provided in Ibn al-Haytham's *Optics* is the most comprehensive. Ibn al-Haytham defined beauty not as an absolute tribute but as a complex interaction between 22 properties. Only two of these, light and color (a quality of light), were by themselves capable of producing beauty, that is, producing "in the soul an effect such that the form appears beautiful." He added to these a third factor capable of causing beauty: "Now beauty may consist in something other than either of the two things we have mentioned, and that is proportionality and harmony." Several views of beauty are expounded, and Necipoğlu concludes that the so-called arabesque (whether geometric, vegetal, calligraphic, or figural) can be seen as the offshoot of a classical notion of beauty recast in abstract terms. The aesthetics of proportion and light, most likely inherited from the last remnants of the school of Alexandria, left its enduring stamp on medieval Islamic visual culture.

Though rooted in the traditional *girih* mode, the Timurid–Turkmen revetment aesthetic codified in the Topkapi scroll took on a highly distinctive appearance, becoming a recognizable stamp of identity associated with the ruling elite's architectural patronage. The *girih* mode was one of the canonical visual idioms that dominated the Islamic lands during the early (950–1250) and late (1250–1500) middle periods. It simultaneously provided an overall sense of visual unity to the World of Islam and highlighted the vigorous diversity within that world.

In the next section, the pattern types occurring in the scroll are arranged in four groups with subgroups for group III and IV. For all drawings of the scroll, a detailed description is given. This enhances our enjoyment and understanding when looking at the reproductions and "produces beauty."

The reviewer wishes the book many readers who will not only admire the beautifully reproduced scroll but also study the accompanying text. This requires some effort but is very rewarding and enriching.

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