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Incompressible SPH based on Rankine source solution for water wave impact simulation

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Abstract

Smoothed Particle Hydrodynamics (SPH) is a Lagrangian meshless particle method. It was originally developed to simulate astrodynamics but has been extended to model dynamics problems with violent motions in many areas. This paper is based on ISPH and its pressure time history is transformed into another equation based on a Rankine source solution. In the new formulation of ISPH, the governing equation for pressure does not include any derivatives of unknown functions and so overcomes the problems associated with direct numerical approximation to second derivatives in existing ISPH formulation, and some important numerical handling techniques will also be included, like free surface particle identification method and solid boundary discrete scheme. The newly improved method will be applied to model various wave and wave impact interaction with different angle slopes, and the results of comparison with experimental data are also given. According to the comparison of pressure time history, this new method can get a good agreement with experimental results.

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1. Introduction

Wave impact problems are very common in marine and ocean engineering, which can help us to know more characters and behaviors of these complex phenomena and it can also helps us to improve the design of practical cases. The high-speed wave impact between a body and water is an important practical problem, whether due to

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wave impact on a dam, deck or due to a moving body such as ship or platform. As the free surface becomes very complex and the pressure exerted are difficult to predict. In this paper, a new SPH model is undertaken to investigate the wave impact with violent free surface.

There are largely two different formulations in literature for Smoothed Particle Hydrodynamics (SPH). The first one is weakly compressible SPH (WCSPH)[1], in which water is considered as slightly compressible and its pressure is related to its density through an equation of state with artificially specified sound speed. The second formulation is incompressible SPH (ISPH)[2], in which water is considered as incompressible and having constant density with pressure found by solving a boundary value problem about it. The ISPH has also been widely applied in the field of water wave dynamics. However, the second order derivatives of pressure needs need to be approximated when discretizing the Poisson equation. In all publications found so far in literature about ISPH, the second derivatives are directly approximated using a scheme similar to that for finite difference method. No matter what scheme to be used, direct numerical approximation to second derivatives always has a difficulty with accurately modeling unknown functions, in particular when particles are randomly distributed. Distribution of particles always becomes random when modeling violent waves even they are regularly distributed at the start of simulation. Therefore, it is obviously advantageous to eliminate use of direct numerical approximation to second derivatives when solving the pressure Poisson equation in ISPH formulation.

The distinction of this paper, compared with other papers on ISPH lies in that the pressure Poisson equation is first transformed into another equation based on a Rankine source solution using the same idea as what is employed in Meshless Local Petrov-Galerkin Method based on Rankine Source Solution (MLPG_R)[3]. In the new formulation of ISPH, the governing equation for pressure does not include any derivatives of unknown functions and so overcomes the problems associated with direct numerical approximation to second derivatives in existing ISPH formulation. This new formulated ISPH is named as ISPH_R in this paper.

In this paper, the details of improved SPH method based the Rankine source decrease order technique will be introduced, and some important numerical handling techniques will also be included, like free surface particle identification method and solid boundary discrete scheme. The newly improve method will be applied to modeling various wave and wave impact interaction with different angle slopes, and the results of comparison with experimental results are also given. More smoothed pressure time history will be obtained based on this new SPH method. At last the features of water wave impact load will be concluded with different factors, such as different slope angles. Although these results are based on 2D case, this method can be extended easily to 3D cases.

2. Improved SPH method

The main difference between the existing ISPH method and the new method named as ISPH_R method lies in the approach to discretization of the pressure Poisson equation defined. The principle idea of the new approach comes from another meshless method called as the Meshless Local Petrov-Galerkin Method based on Rankine Source Solution (MLPG_R) [4][5], that is, reformulating Poisson equation into another form which does not include any derivative of unknown function. For this purpose, it is integrated over a small sub-domain Ω_I surrounding a particles after multiplied by Rankine source solution φ, reads

$$\int_{\Omega_I} \varphi \nabla^2 p_{t+\Delta t} d\Omega_I = \int_{\Omega_I} [\gamma \frac{\rho - \rho^*}{\Delta t^2} + (1-\gamma)\frac{\rho}{\Delta t} \nabla \cdot \mathbf{u}^*] \varphi \, d\Omega_I \quad (1)$$

where φ can be chosen as

$$\varphi = \frac{1}{2\pi} \ln(r / R_I) \text{ for 2D cases} \quad (2)$$

that satisfies $\nabla^2 \varphi = 0$, in Ω_I except for the center and $\varphi = 0$, on $\partial\Omega_I$, which is boundary of Ω_I and R_I is its radius. The radius is usually smaller than the distance between two particles. After some mathematical manipulations, Eq.(1) becomes the following form

$$\int_{\partial\Omega_I} \vec{n}\cdot(p_{t+\Delta t}\nabla\varphi)dS - p_{t+\Delta t} = \gamma\frac{\rho_I-\rho_I^*}{\Delta t^2}\frac{R_I^2}{4} + (1-\gamma)\int_{\Omega_I}\frac{\rho}{\Delta t}\mathbf{u}^*\cdot\nabla\varphi d\Omega \quad (3)$$

More details of mathematical manipulations can be found in Ma and Zhou [5]. It has been noted by the cited paper that the increment of the density ($\rho-\rho^*$) assumed to a constant within the sub-domain and so equal to its value at Particle i when Eq. (3) is derived. This may not cause unacceptable error not only because the density should not change much due to the change in the intermediate position of the particle as pointed above but also because the small error caused due to the assumption is further reduced by multiplying the coefficient γ that is normally chosen in a range of 0~0.3, taken as 0.1 in this paper.

2.4.1. Solitary boundary conditions

On solid boundaries, the following conditions (e.g. Ma and Zhou [5]) should be satisfied

$$\mathbf{u}\cdot\mathbf{n}=\mathbf{U}\cdot\mathbf{n} \qquad \mathbf{n}\cdot\nabla p=\rho\left(\mathbf{n}\cdot\mathbf{g}-\mathbf{n}\cdot\dot{\mathbf{U}}+\nu\mathbf{n}\cdot\nabla^2\mathbf{u}\right) \quad (4)$$

where $\mathbf{n}$ is the unit normal vector of the solid boundaries, $\mathbf{g}$ is the vector of gravitational acceleration, $\mathbf{U}$ and $\dot{\mathbf{U}}$ are the velocity and acceleration of the solid boundaries, respectively. It is noted that the traditional SPH does not need the condition in Eq. (4) as it does not need to solve the boundary value problem for pressure. In ISPH method, the condition in Eq. (4) is necessary. It is obvious that one must compute the term $\nabla^2\mathbf{u}$ when applying this condition in Eq. (4), which needs to estimate the second order derivative at the rigid boundary. To avoid the computation of the second order derivative if possible, Ma and Zhou [5] gave an alternative one as follows:

$$\mathbf{n}\cdot\nabla p=\frac{\rho}{\Delta t}\mathbf{n}\cdot(\mathbf{u}^*-\mathbf{U}) \quad (5)$$

This one is used in this paper.

The condition on free surface is very simple, which is stated that the pressure of water on the its free surface is equal to the atmospheric pressure which can be taken as zero, i.e., $p=0$. In the traditional SPH method, this condition is automatically satisfied as long as the density on the free surface is estimated correctly. However, in the ISPH method, this condition has to be imposed when solving the boundary value problem. In order to impose this condition, one needs to know which particles are on the free surface. At each time step, particles are checked in the following sequence:

(a) No inner particle in the influence domain except for i
(b) $\alpha_i \le 0.90$ and $fsp\_a(i)=1$
(c) $\alpha_i > 0.90$, $fsp\_a(i)=1$ and $fsp\_b(i)=0$
(d) $\alpha_i > 0.90$, $fsp\_a(i)=1$ and $fsp\_c(i)=0$
(e) $\alpha_i \le 0.90$, $NumB \le 2$ and $NumC \le 2$

If any of expressions is true during checking, Particle i is identified as a free surface particle. More details of free surface identification method can refer to Ref.[6].

3. Numerical tests

3.1. Dam breaking

In ocean engineering, wave impacting on marine structures, ship, platform, etc. often appears in rough sea conditions. Extremely large impulsive impact force and violent free surface flow arise during wave impact. A relatively simple model, dam-breaking, is commonly used to investigate the complicated wave impact phenomenon, which is the benchmark model for violent flow and wave impact simulation. L and H are the width and height of the dam in initial stage respectively. Firstly, it adopts $L/H=1$, Fig.1 gives the water wave front and column height of dam breaking. The experiment data can refer to Matin [7]. In order to show more general cases, Fig.2 gives the results of snapshots of wave profiles and pressure distribution at different time for $L/H=2$. Figure 3 give the pressure time history of one fixed point for different particle numbers, different time steps. At the comparison with experimental results are also given at last. According to the results, more particle number or smaller particle size can make the pressure time history more smooth and accuracy. The time stepping length is not very sensitive for time pressure history, so general it can choose the bigger one. More details of the numerical model setting can refer to Ref. [8].

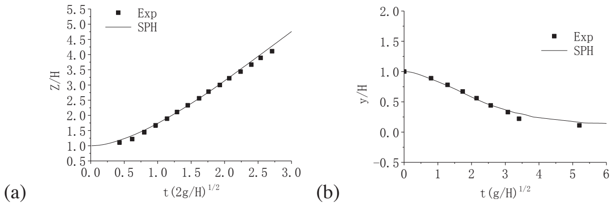

Fig. 1. Water wave front comparison (a); water wave height comparison (b).

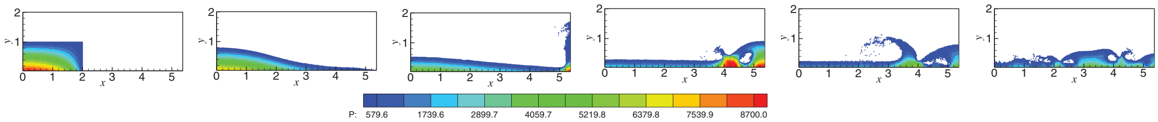

Fig. 2. Snapshots of dam breaking flow at different time steps.

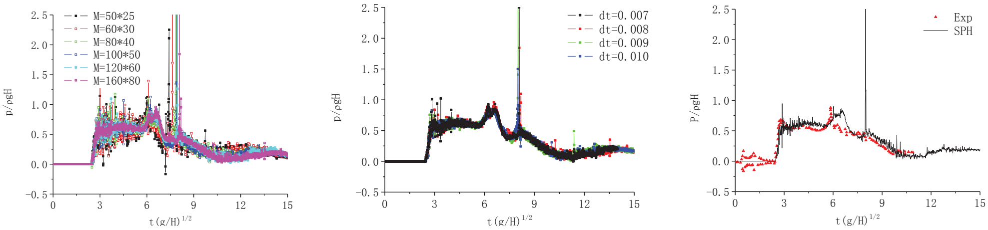

Fig. 3. Pressure time history comparisons of different particle numbers (left), different stepping length (middle), experimental results(right).

3.2. Wave impact towards different angle slopes

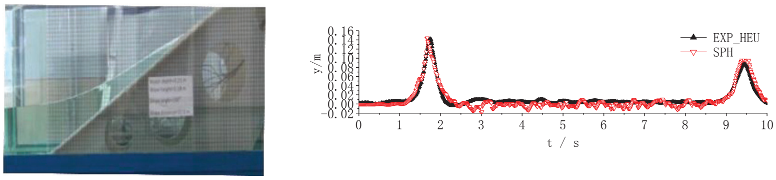

Fig.4 Experiment snapshot of wave impact of angle=300 (left) and wave elevation comparison (right)

In order to give the wave impact results, the model in above section can make some a little corrections. The damping zone on the right side can be substituted by a slope, and the slope angle α can be changed to different values. In this section, the tank length l =10.0m, the water depth h_d =0.25m, there are a wave elevation gauges installed in the tank, which is fixed near the piston wave maker, the distance between the initial position and wave elevation gauge is 2.0 m. On the slope surface, there are some pressure sensors, but in this section only one pressure sensor, which is fixed on the slope surface and the water depth is 0.2m are considered. The solitary wave is generated by a piston-type wave maker according to the theory given by $x_p = h/k[\tanh\chi(\tau) + \tanh k\lambda]$, where h is the wave height, $k = \sqrt{3h/4}$, $\chi(\tau) = k[c\tau - x_p(\tau) - \lambda]$, and the dimensionless celerity $c = \sqrt{1+h}$. There also includes an experiment for the same wave impact model, which is carried out in Harbin Engineering University (HEU). Figure 4 gives the snapshot of experimental model fixed and wave impact phenomena. Figure 4 also gives the comparison of the results of wave elevation comparison with experimental data. Fig.5 give the numerical results of wave impact with different slope angles, which can refer to 30°, 90° and 150°. It gives the wave profile and pressure distribution on different angle slopes. Pressure time history of different angles is also given. When the wave impact on small angle slopes(30^0 or 90^0), the pressure time history can get a good agreement with experimental results. But for the large angle slopes (150°), the amplitude of maximum pressure time history is still very good, but some details of pressure time history is still needed to improved.

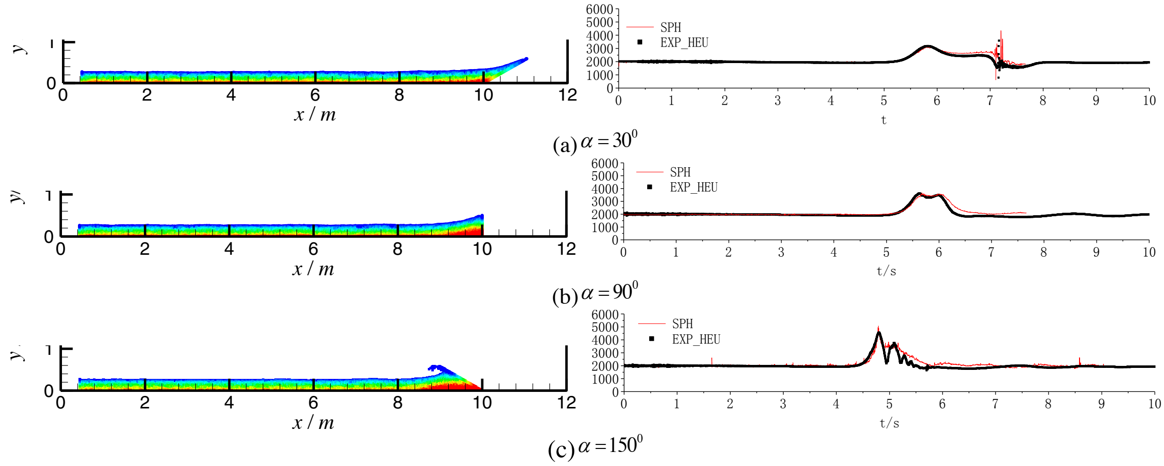

Fig.5 Pressure distribution of different slope angles and wave impact pressure comparison

4. Conclusions

This paper introduced a new incompressible SPH based on Rankin source solution (shorted as ISPH_R) to simulate 2D violent waves. The method adopts the Rankin solution to decrease the order of the derivatives in the Poisson equation defining the boundary value problem about pressure. In the cases about dam-breaking and wave impact problems, the results of the ISPH_R method are in good agreement with available experimental data, it is expected to be hold for general cases because the feature of no derivatives in governing equation for pressure adopted in this paper is applied for all the cases. This method can be extended to three dimensional cases but the relevant work will be presented in our future work.

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