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Renormalization group equations as 'decoupling' theorems

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Abstract

We propose a simple derivation of renormalization group equations and Callan–Symanzik equations as decoupling theorems of the structures underlying effective field theories.

1. Introduction

It is well known that any known quantum field theory could at best be an effective field theory (EFT) that include ingredients as many as possible for the description of certain phenomena. Various divergences (UV, IR and other types) arise in an EFT simply due to its simplification made at the scales widely separated from the dominating ones. In other words, an EFT is usually not a complete framework for accounting for the quantum fluctuations over any distances, it fails for certain modes underlying the 'effective ones'.

The conventional ways to deal with the divergences within EFT framework are renormalization (for UV divergences) and factorization (for IR and/or collinear divergences): the true noncalculable contributions from the underlying structures could be separated and put into some EFT ingredients (operators, couplings or matrix elements, etc.) that could be determined later. Such tricky procedures lead to the celebrated renormalization group equations (RGE). However, in the conventional treatment, an artificial regularization (as simple and temporary substitutes for the true underlying structures) must be employed in the course of calculations and the associated tricky procedures seem to obscure the physical rationality behind the treatments, though the final results should be independent of such regularization procedures.

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In this Letter, I wish to give a rather simple (to some extent, less rigorous) derivation of RGE and the Callan–Symanzik equation, solely based on the simple picture that there are some elaborate details underlying the effective theories in use for certain purpose. In other words, the scale hierarchy substantiating the EFT construction is the only starting point. It is hoped that such simple lines of arguments could help the further exploration of the more fundamental implications of the EFT reconstructions as well as various factorization approaches that prevail in physics literature.

2. Effective versus underlying structures

To proceed, we take the existence of a well-defined formulation of the theory underlying (UT) the EFTs as a natural fact or postulate. In fact, we could take the well-defined formulation of an EFT as a result of reconstruction or projection procedure from the related sectors in the complete UT. The hierarchy between the scales in the formulation discriminates the parameters into effective and underlying ones, respectively. Such hierarchy should automatically facilitate a well-defined expansion in terms of the ratios like $\Lambda_{\mathrm{EFT}}/\Lambda_{\mathrm{UT}}$ (for UV underlying structures) or $\Lambda_{\mathrm{UT}}/\Lambda_{\mathrm{EFT}}$ (for IR or other non-UV underlying structures), so that the resulting formulation is expressed in terms of the effective parameters and some possible 'agent' constants (only arise in the loop contributions) in the limit that the underlying details are apparently 'decoupled'. No unphysical divergences should appear in the course of such reconstruction or projection procedure.

For convenience, we introduce the symbol $\check{P}_{\mathrm{EFT}}$ to assume all the elaborate procedures of the reconstruction or projection of an EFT out of a complete underlying theory, which should contain at least the following three operations: (1) projecting into the subspace of EFT; (2) averaging over the associated underlying dynamical processes (integrating out); (3) taking the decoupling limits with respect to the typical underlying constants, $\{\sigma\}$. With the help of this projection symbol, we could easily identify the technical origin of various divergences in EFT. To see this point, we employ the path integral formalism.

According to the above arguments, a well-defined generating functional for an EFT should be obtained from the EFT projection on the generating functional for UT, $\check{P}_{\mathrm{EFT}}$.

$$Z_{\mathrm{EFT}}\big([J_{\mathrm{EFT}}]\big) \equiv \check{P}_{\mathrm{EFT}} \int d[\Phi] \exp\big\{i\mathcal{S}\big([\Phi],[g];\{\sigma\}\big\|[J_{\mathrm{EFT}}]\big)\big\}, \tag{1}$$

with $[g]$ being the 'effective' parameters. That means, the path integral should be performed in the presence of the underlying structures. If one perform the projection first before the path integral, an ill-defined EFT is resulted,

$$\int d[\phi] \exp\big\{i\mathcal{S}\big([\phi],[g]\big)\big\}_{\text{ill-defined}} = \int \check{P}_{\mathrm{EFT}} d[\Phi] \exp\big\{i\mathcal{S}\big([\Phi],[g];\{\sigma\}\big)\big\}. \tag{2}$$

Thus, the appearance of various divergences in an EFT implies that path integral and the EFT projection $\check{P}_{\mathrm{EFT}}$ do not commute.

In conventional approaches, the role of the sophisticated projection procedure $\check{P}_{\mathrm{EFT}}$ is played by the $\mathcal{R}$ operation procedure in the well-known BPZH program when there is only UV divergences. That is, $\mathcal{R}$ operation could be seen as one tricky realization of the projection operation $\check{P}_{\mathrm{EFT}}$ (in the case of UV divergences alone), which could in turn be put into the forms with multiplicative renormalization,

$$\check{P}_{\mathrm{EFT}}(\hat{O}) \Rightarrow \mathcal{R}(\hat{O}) \Rightarrow Z_O^{-1}\hat{O}. \tag{3}$$

3. Canonical scaling with underlying structures

Now let us consider a general vertex function (1PI) $\Gamma^{(n)}([p],[g];\{\sigma\})$ that is well defined in UT with $[p]$, $[g]$ denoting the external momenta and the Lagrangian couplings (including masses) in an EFT and $\{\sigma\}$ denoting the underlying parameters or constants. Now it is easy to see that such a vertex function must be a homogeneous function of all its dimensional arguments, that is

$$\Gamma^{(n)}\big([\lambda p],\big[\lambda^{d_g} g\big];\big\{\lambda^{d_\sigma}\sigma\big\}\big)=\lambda^{d_{\Gamma^{(n)}}}\Gamma^{(n)}\big([p],[g];\{\sigma\}\big), \tag{4}$$

where $d_{\dots}$ refers to the canonical mass dimension of any parameters involved.

The corresponding equation in EFT could be obtained through the application of the projection $\check{P}_{\mathrm{EFT}}$ to both sides of Eq. (4),

$$\Gamma^{(n)}\big([\lambda p],\big[\lambda^{d_g} g\big];\big\{\lambda^{d_{\bar c}}\bar c\big\}\big)=\lambda^{d_{\Gamma^{(n)}}}\Gamma^{(n)}\big([p],[g];\{\bar c\}\big). \tag{5}$$

Here some constants $\{\bar c\}$ must appear as the agents of $\{\sigma\}$ to maintain the dimension balance between $\{\sigma\}$ and the EFT couplings and masses. Note that $\{\bar c\}$ only appear in the loop diagrams of EFT.

The differential form for Eq. (4) reads

$$\Big\{\lambda\partial_\lambda+\sum d_g g\partial_g+\sum d_\sigma\sigma\partial_\sigma-d_{\Gamma^{(n)}}\Big\}\Gamma^{(n)}\big([\lambda p],[g];\{\sigma\}\big)=0. \tag{6}$$

Since $\sum d_g g\partial_g=\Theta$ (the trace of the stress tensor), the alternative form of Eq. (6) reads

$$\Big\{\lambda\partial_\lambda+\sum d_\sigma\sigma\partial_\sigma-d_{\Gamma^{(n)}}\Big\}\Gamma^{(n)}\big([p],[g];\{\sigma\}\big)=-i\Gamma^{(n)}_{\Theta}\big([0;\lambda p],[g];\{\sigma\}\big). \tag{7}$$

Obviously only dimensional constants contribute to the scaling behavior. Eq. (6) or Eq. (7) is just the most general UT version of the EFT scaling laws. They differ from naive EFT scaling laws only by the *canonical* contribution from the underlying structures ($\sum d_\sigma\sigma\partial_\sigma$). This is just the origin of EFT scaling anomalies.

To see this point, we first note the consequence of applying $\check{P}_{\mathrm{EFT}}$ to $\sum d_\sigma\sigma\partial_\sigma$,

$$\check{P}_{\mathrm{EFT}}\Big\{\sum d_\sigma\sigma\partial_\sigma\Big\}\Gamma^{(n)}\big([\cdots];\{\sigma\}\big)=\Big\{\sum d_{\bar c}\bar c\partial_{\bar c}\Big\}\Gamma^{(n)}\big([\cdots];\{\bar c\}\big). \tag{8}$$

Then, it is straightforward to see that, within EFT, $\sum d_{\bar c}\bar c\partial_{\bar c}$ has to be expanded into the insertion of appropriate EFT operators ($[I_{O_i}]$, 'elementary' or composite) with appropriate coefficients (δ_{O_i}). Thus, we arrive at the following decoupling theorem:

$$\check{P}_{\mathrm{EFT}}\Big\{\sum d_\sigma\sigma\partial_\sigma\Big\}=\sum d_{\bar c}\bar c\partial_{\bar c}=\sum_{O_i}\delta_{O_i}I_{O_i}. \tag{9}$$

Note that each δ_{O_i} must at least be a function of EFT couplings $[g]$ and $\{\bar c\}$. At present stage, we do not exclude nonlocal operators from the set of $[O_i]$, which might be more relevant in the presence of IR or other non-UV divergences. Thus Eq. (9) is a rather general form of 'decoupling theorem' for any sort of underlying structures, UV or IR.

As the final step, it is easy to classify these operators into the kinetic operators (for the EFT fields $[\phi]$), the coupling operators (with couplings $[g]$), and 'composite' ones, $[O_N]$, that do not appear in the EFT Lagrangian

$$\sum_{O_i}\delta_{O_i}I_{O_i}=\sum_g\delta_g g\partial_g+\sum_\phi\delta_\phi\hat I_\phi+\sum_{O_N}\delta_{O_N}\hat I_{O_N}. \tag{10}$$

Now with Eqs. (8), (10) we can turn the primary decoupling theorem in Eq. (9) and the full scaling law in Eq. (6) into the following forms,

$$\Big\{\sum_{\bar c} d_{\bar c}\bar c\partial_{\bar c}-\sum_{O_N}\delta_{O_N}\hat I_{O_N}-\sum_g\delta_g g\partial_g-\sum_\phi\delta_\phi\hat I_\phi\Big\}\Gamma^{(n)}\big([\lambda p],[g];\{\bar c\}\big)=0, \tag{11}$$

$$\Big\{\lambda\partial_\lambda+\sum_{O_N}\delta_{O_N}\hat I_{O_N}+\sum_g(d_g+\delta_g)g\partial_g+\sum_\phi\delta_\phi\hat I_\phi-d_{\Gamma^{(n)}}\Big\}\Gamma^{(n)}\big([\lambda p],[g];\{\bar c\}\big)=0. \tag{12}$$

Here Eqs. (11), (12) are only true for the complete sum of all graphs (or up to a certain order). It is obvious that the *canonical* contributions from the underlying structures become the *anomalies* in terms of EFT parameters. δ_{O_i} is just the anomalous dimension for the operator O_i in EFT. Again the operators contributing to the scaling anomalies should contain the ones corresponding to IR or other non-UV singularities.

4. Novel perspective of RGE and Callan–Symanzik equation

In this section we limit our attention to a special type of theories beset only with UV divergences: the ones without the scaling anomalies $\sum_{\{O_N\}} \delta_{O_N}([g]; \{\bar{c}\}) \hat{I}_{O_N}$, i.e., the renormalizable theories in conventional terminology, and all the operators are now local. Then, Eqs. (11) and (12) become simpler

$$\left\{ \sum_{\bar{c}} d_{\bar{c}} \bar{c} \partial_{\bar{c}} - \sum_g \delta_g g \partial_g - \sum_\phi \delta_\phi \hat{I}_\phi \right\} \Gamma^{(n)}\big([\lambda p], [g]; \{\bar{c}\}\big) = 0, \tag{13}$$

$$\left\{ \lambda \partial_\lambda + \sum_g (d_g + \delta_g) g \partial_g + \sum_\phi \delta_\phi \hat{I}_\phi - d_{\Gamma^{(n)}} \right\} \Gamma^{(n)}\big([\lambda p], [g]; \{\bar{c}\}\big) = 0. \tag{14}$$

These equations just correspond to the usual RGE and Callan–Symanzik equation (CSE) for renormalizable theories. We could turn these equations into more familiar forms. For this purpose, we note that all the agent constants could be parametrized in terms of a single scale $\bar{\mu}$ and a series dimensionless ones $(\bar{c}_0)$. In the conventional programs, they are first predetermined through renormalization conditions, finally transformed into the physical parameters [1] or fixed somehow [2].

4.1. RGE and CSE as decoupling theorems

In Eqs. (13) and (14) only the insertion of kinetic operators appears unfamiliar. To remove this unfamiliarity, let us note that $\sum_\phi \delta_\phi \hat{I}_\phi$ lead to the following consequences [3]:

$$\delta_g \to \bar{\delta}_g \equiv \left(\delta_g - n_{g;\phi} \frac{\delta_\phi}{2} - n_{g;\psi} \frac{\delta_\psi}{2} \right), \qquad \Gamma^{(n_\phi, n_\psi)} \to (1 + \delta_\psi)^{n_\psi/2} (1 + \delta_\phi)^{n_\phi/2} \Gamma^{(n_\phi, n_\psi)}, \tag{15}$$

with $n_{g;\phi}$ and $n_{g;\psi}$ being respectively the number of bosonic and fermionic field operators contained in the vertex with coupling g.

Then Eqs. (13) and (14) take the following forms:

$$\left\{ \bar{\mu} \partial_{\bar{\mu}} - \sum_g \bar{\delta}_g g \partial_g - \sum_\phi n_\phi \frac{\delta_\phi}{2} - \sum_\psi n_\psi \frac{\delta_\psi}{2} \right\} \Gamma^{(n_\phi, n_\psi)}\big([p], [g]; \big\{\bar{\mu}; \big(\bar{c}_0^i\big)\big\}\big) = 0, \tag{16}$$

$$\left\{ \lambda \partial_\lambda + \sum_g D_g g \partial_g + \sum_\phi n_\phi \frac{\delta_\phi}{2} + \sum_\psi n_\psi \frac{\delta_\psi}{2} - d_{\Gamma^{(n_\phi, n_\psi)}} \right\} \Gamma^{(n_\phi, n_\psi)}\big([\lambda p], [g]; \big\{\bar{\mu}; \big(\bar{c}_0^i\big)\big\}\big) = 0, \tag{17}$$

with $D_g \equiv \bar{\delta}_g + d_g$. Eqs. (16), (17) take the familiar forms of RGE and CSE. Here, all the constants (finite!) survive or arise from the 'decoupling' limit implied by the projection procedure. Thus, we could naturally interpret these equations as 'decoupling' theorems for the canonical scaling laws with underlying structures.

The RGE and CSE for the generating functional read

$$\left\{ \bar{\mu} \partial_{\bar{\mu}} - \sum_g \delta_g g \partial_g - \sum_\phi \delta_\phi \hat{I}_\phi \right\} \Gamma^{1\text{PI}}\big([\phi], [g]; \big\{\bar{\mu}; \big(\bar{c}_0^i\big)\big\}\big) = 0, \tag{18}$$

$$\left\{\sum_\phi \int d^D x\,[d_\phi - x\cdot\partial_x)\phi(x)]\frac{\delta}{\delta\phi(x)} + \sum_g D_g g\partial_g + \sum_\phi \delta_\phi \hat{I}_\phi - D\right\}\Gamma^{1\mathrm{PI}}\big([\phi],[g];\{\bar{\mu};(\bar{c}_0^i)\}\big)=0 \quad (19)$$

with D denoting the spacetime dimension. We note that the operator trace anomalies [4] could be readily read from Eq. (19),

$$g_{\mu\nu}\Theta^{\mu\nu} = \sum_g \delta_g g\partial_g \mathcal{L}_{\mathrm{EFT}} + \sum_\phi \delta_\phi \hat{O}_{\mathrm{kinetic}}(\phi). \quad (20)$$

The right-hand side could be further simplified after using motion equations.

4.2. RGE and CSE for composite operators

For the vertex functions in the presence of composite operators, the only complication lies in the contribution from the related composite operators (more could show up than those in the vertex functions), that is the general cases in Eqs. (11), (12),

$$\left\{\bar{\mu}\partial_{\bar{\mu}} - \sum_{O_N}\bar{\delta}_{O_N}\hat{I}_{O_N} - \sum_g \bar{\delta}_g g\partial_g - \sum_\phi n_\phi \frac{\delta_\phi}{2} - \sum_\psi n_\psi\frac{\delta_\psi}{2} - \sum_{i=A,\ldots}\bar{\delta}_{O_i}\right\}\Gamma^{(n)}_{O_A,\ldots}\big([\lambda p],[g];\{\bar{\mu};(\bar{c}_0^i)\}\big)=0, \quad (21)$$

$$\left\{\lambda\partial_\lambda + \sum_{O_N}\bar{\delta}_{O_N}\hat{I}_{O_N} + \sum_g D_g g\partial_g + \sum_\phi n_\phi\frac{\delta_\phi}{2} + \sum_\psi n_\psi\frac{\delta_\psi}{2} - D_{\Gamma^{(n)}_{O_A,\ldots}}\right\}\Gamma^{(n)}_{O_A,\ldots}\big([\lambda p],[g];\{\bar{\mu};(\bar{c}_0^i)\}\big)=0. \quad (22)$$

Here $D_{\Gamma^{(n)}_{O_A,\ldots}} = d_{\Gamma^{(n)}_{O_A,\ldots}} - \sum_{i=A,\ldots}\bar{\delta}_{O_i}$, while the contributions from other composite operators ($\sum_{O_N}\bar{\delta}_{O_N}\hat{I}_{O_N}$) could be further put into the familiar forms as the nondiagonal anomalous dimensions: $\sum_{O_N}\bar{\delta}_{O_N}\hat{I}_{O_N}\Gamma^{(n)}_{O_A,\ldots} = \sum_{[O_N],[i=A,\ldots]}\bar{\delta}_{O_N O_i}\Gamma^{(n)}_{O_A,\ldots}$. Thus, the final forms for Eqs. (21), (22) read

$$\left\{\bar{\mu}\partial_{\bar{\mu}} - \sum_g\bar{\delta}_g g\partial_g - \sum_\phi n_\phi\frac{\delta_\phi}{2} - \sum_\psi n_\psi\frac{\delta_\psi}{2} - \sum_{[O_N],[i=A,\ldots]}\bar{\delta}_{O_N O_i} - \sum_{O_A,\ldots}\bar{\delta}_{O_i}\right\}$$
$$\times\,\Gamma^{(n)}_{O_A,\ldots}\big([\lambda p],[g];\{\bar{\mu};(\bar{c}_0^i)\}\big)=0, \quad (23)$$

$$\left\{\lambda\partial_\lambda + \sum_g D_g g\partial_g + \sum_\phi n_\phi\frac{\delta_\phi}{2} + \sum_\psi n_\psi\frac{\delta_\psi}{2} + \sum_{[O_N],[i=A,\ldots]}\bar{\delta}_{O_N O_i} - D_{\Gamma^{(n)}_{O_A,\ldots}}\right\}$$
$$\times\,\Gamma^{(n)}_{O_A,\ldots}\big([\lambda p],[g];\{\bar{\mu};(\bar{c}_0^i)\}\big)=0. \quad (24)$$

4.3. Underlying structures and the notion of renormalization

As remarked above, the EFT parameters $[g]$ and the UT agents $\{\bar{c}\}$ should be 'derived' from UT (they could not be derived from EFT!). Thus, in EFT, they have to be determined or fixed somehow through physical boundaries or data [1] or through sensible procedures [2].

Now to see the origin of the notion of renormalization, we solve the equation for scaling law, Eq. (17). This could be conveniently achieved through the introduction of 'running' parameters $[\bar{g}(\lambda)]$ for $[g]$ based on Coleman's bacteria analogue [5]. Then the solution of Eq. (17) can be found as the solution of the following equation,

$$\left\{\lambda\partial_\lambda + \sum_{\bar{g}}\big[d_{\bar{g}} + \delta_{\bar{g}}\big([\bar{g}];\{\bar{c}\}\big)\big]\bar{g}\partial_{\bar{g}} + \sum_\phi \delta_\phi\big([\bar{g}];\{\bar{c}\}\big)\hat{I}_\phi - d_{\Gamma^{(n)}}\right\}\Gamma^{(n)}\big([\lambda p],[\bar{g}];\{\bar{c}\}\big)=0 \quad (25)$$

with $\bar{g}(=\bar{g}([g];\lambda))$ satisfying the following kind of equation,

$$\lambda\partial_\lambda\big\{\bar{g}\big([g];\lambda\big)/\lambda^{d_g}\big\}=-\big\{d_{\bar{g}}+\delta_{\bar{g}}\big([\bar{g}];\{\bar{c}\}\big)\big\}\bar{g}\big([g];\lambda\big)/\lambda^{d_g}, \tag{26}$$

with the natural boundary condition: $\bar{g}([g];\lambda)|_{\lambda=1}=g$ for each EFT parameter. The EFT couplings $[g]$ should be finite 'bare' parameters as they are in principle defined in the underlying theory. Now the notion renormalization arises in EFT with the rescaling $[p]\to[\lambda p]$: the EFT couplings $[g]$ (defined in UT!) get 'renormalized', $[g]\to[\bar{g}([g];\lambda)]$. Accordingly, one could define the 'renormalization' constants (finite again!) as $z_g([g];\lambda)\equiv\bar{g}([g];\lambda)/g$. Thus renormalization is a notion in EFT associated with the rescaling, whose genuine origin is the contributions from underlying structures.

The renormalization constants for operators (kinetic or composite) could be introduced in similar manner [3], including the cases with 'mixing' [6]. As a result, in the underlying theory point of view, the 'renormalization' constants are finite and could be introduced afterwards as byproducts, not as compulsory components. We suspect that this simple scenario might be helpful in more complicated EFTs, e.g., the Standard model, especially in its sectors with unstable fields and with flavor mixing.

5. Appelquist–Carazzone decoupling theorem and underlying structures

Now let us discuss the decoupling theorem a la Appelquist–Carazzone [7] from the underlying structures' perspective.

5.1. Decoupling and repartition

First let us note that, in the underlying theory perspective, the EFT parameters and the underlying parameters are grouped or partitioned into two separate sets by the reference scale that naturally appear in any physical processes (e.g., center energy in a scattering) according to the relative magnitudes: the effective set $[g]$ and the underlying set $\{\sigma\}$. When an EFT field 'becomes' too heavy to directly participate the EFT dynamics, it only induces a new partition between the effective and underlying parameters, with the union of the two sets kept 'conserved' in the course of decoupling:

$$[g]\cup\{\sigma\}=[g]'\cup\{\sigma\}',\qquad [g]'\equiv[g]/[M_H],\qquad \{\sigma\}'\equiv\{\sigma\}\cup[M_H]. \tag{27}$$

This repartition yields a new EFT that differs from the original one by a very massive field, hence a new set of agent constants $\{\bar{c}\}'$ is generated from this repartition. In terms of the scaling behavior, that means

$$\sum_\sigma d_\sigma\sigma\partial_\sigma+\sum_g d_g g\partial_g={\sum_\sigma}' d_\sigma\sigma\partial_\sigma+{\sum_g}' d_g g\partial_g, \tag{28}$$

or, equivalently,

$$\sum_{\bar{c}} d_{\bar{c}}\bar{c}\partial_{\bar{c}}+\sum_g d_g g\partial_g\Longrightarrow\sum_{\bar{c}'} d_{\bar{c}'}\bar{c}'\partial_{\bar{c}'}+{\sum_g}' d_g g\partial_g. \tag{29}$$

Then from the configuration of the parameters described in Eq. (27), the decoupling of a heavy EFT field could be formally be taken as the following two operations: (i) repartitioning the EFT and underlying parameters; (ii) taking the low energy limit with respect to the new underlying parameters, including the heavy EFT field's mass. This is the mathematical formulation of EFT field decoupling.

5.2. Practical decoupling of EFT fields

From Section 4.3, we have seen that, the underlying parameters or their agents scale contribute to the 'running' or 'renormalization' of the EFT parameters. Thus the repartition will alter the contents of 'running': what to scale as underlying parameters versus what to scale as EFT parameters:

(a) Before decoupling, namely, M_H is an EFT parameter and does not vary together with the underlying parameters or their agents. Then the factor $\ln \frac{\bar{c}^2}{M_H^2}$ contributes to the running of any EFT objects

$$\delta_\lambda \left[\ln \frac{\bar{c}^2}{M_H^2} \right] = \ln \frac{[(1+\delta\lambda)\bar{c}]^2}{M_H^2} - \ln \frac{\bar{c}^2}{M_H^2} \neq 0; \tag{30}$$

(b) After decoupling, M_H becomes underlying and varies homogeneously with $\{\bar{c}\}$, then

$$\delta_\lambda \left[\ln \frac{\bar{c}^2}{M_H^2} \right] = \ln \frac{[(1+\delta\lambda)\bar{c}]^2}{[(1+\delta\lambda)M_H]^2} - \ln \frac{\bar{c}^2}{M_H^2} = 0. \tag{31}$$

That is, taking as a member of the underlying parameters or their agents, M_H will cancel out some agents' contributions to the 'running'. Accordingly, the anomalous dimensions of the EFT parameters will alter by a finite amount ($\Delta\bar{\delta}_g$) due to decoupling

$$\bar{\delta}_g \Rightarrow \bar{\delta}'_g = \bar{\delta}_g + \Delta\bar{\delta}_g. \tag{32}$$

From Eqs. (28) or (29), we see that, one could also work with the EFT containing M_H, only adding some unnecessary technical complexities. This is also a well-known fact.

It is not difficult to see that our arguments above amounts to provide a relatively 'physical' rationale to the 'subtraction' solution of decoupling [8]. Of course different boundary conditions are needed across the threshold, which must lead to certain matching conditions for the 'running' parameters [9]. We hope our understanding could also be useful in the heavy quark effects in deeply inelastic scattering [10] and other important phenomenologies.

6. Discussion and summary

Of course, all the results presented here are not new. However, we still feel the way we derived these results seems more general and more natural. This underlying theory perspective might be of helps in deepening our understanding of the EFT methods. This is relevant to all the quantum theories, as any known theory is in fact an effective theory to some extents.

Of course, we did not touch the cases with concrete IR or other non-UV singularities. In such cases the underlying structures at large distances (hence often nonperturbative) must be incorporated (expanded) in a controllable way. Considering the complications in various factorization formulations, we refrain here from a naïve extrapolation of the scaling law. But the abstract operator form of the decoupling theorems (or scaling laws) in Eq. (9) (or Eq. (12)) is valid independent of such concrete details. The next step for deriving decoupling theorems or scaling laws in the case of non-UV singularities is to elaborate the contents of the operators and their anomalous dimensions that are responsible for non-UV singularities. We hope this line of investigations would lead to a different approach to the problem of non-UV singularities[2] and help to clarify the universal contents of factorization and/or its violation.

[2] In QCD, this means an approach for tackling the nonperturbative dynamics.

In summary, we derived the scaling laws in any EFT assuming the existence of nontrivial underlying structures with renormalization group equation and Callan–Symanzik equation being interpreted as 'decoupling' theorems of the underlying structures. The Appelquist–Carazzone theorem was briefly discussed in this underlying structures perspective.

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References

[1] G. Sterman, An Introduction to Quantum Field Theory, Cambridge Univ. Press, Cambridge, 1993.
[2] P.M. Stevenson, Phys. Rev. D 23 (1981) 2916;
G. Grunberg, Phys. Rev. D 29 (1984) 2315;
S.J. Brodsky, G.P. Lepage, P.B. Mackenzie, Phys. Rev. D 28 (1983) 228.
[3] J.-F. Yang, hep-th/0311219;
J.-F. Yang, hep-th/9908111.
[4] S.L. Adler, J.C. Collins, A. Duncan, Phys. Rev. D 15 (1977) 1712;
J.C. Collins, A. Duncan, S.D. Joglekar, Phys. Rev. D 16 (1977) 438.
[5] S. Coleman, Aspects of Symmetry, Cambridge Univ. Press, Cambridge, 1985, Chapter 3.
[6] H. Kluberg-Stern, J.B. Zuber, Phys. Rev. D 12 (1975) 54;
N.K. Nielsen, Nucl. Phys. B 97 (1975) 527;
N.K. Nielsen, Nucl. Phys. B 120 (1977) 212.
[7] T. Appelquist, J. Carazzone, Phys. Rev. D 11 (1975) 2262.
[8] S. Weinberg, Phys. Lett. B 91 (1980) 51;
L.J. Hall, Nucl. Phys. B 178 (1981) 75;
B. Ovrut, H. Schnitzer, Nucl. Phys. B 179 (1981) 381;
B. Ovrut, H. Schnitzer, Nucl. Phys. B 189 (1981) 509;
W. Bernreuther, W. Wetzel, Nucl. Phys. B 197 (1982) 228.
[9] G. Rodrigo, A. Santamaria, Phys. Lett. B 313 (1993) 441;
K.G. Chetyrkin, B.A. Kniehl, M. Steinhauser, Phys. Rev. Lett. 79 (1997) 2184.
[10] See, e.g., M.A. Aivazis, J.C. Collins, F.I. Olness, W.K. Tung, Phys. Rev. D 50 (1994) 3102;
M. Buza, Y. Matiounine, J. Smith, R. Migneron, W.L. van Neerven, Nucl. Phys. B 472 (1996) 611;
R.S. Thorne, R.G. Roberts, Phys. Rev. D 57 (1998) 6871;
M. Krämer, F.I. Olness, D.E. Soper, Phys. Rev. D 62 (2000) 096007.